An Introduction to Experimental Design for Chemical Engineering Laboratory

Michael A. Henson
Modified by Surita R. Bhatia, 9/8/03
Department of Chemical Engineering
University of Massachusetts

Outline – Part I

- A motivating example
- Introduction
  » The experimental design problem
  » Design objectives
  » Input levels
  » Empirical process models
  » General design procedure
- Basic design techniques
  » Randomized designs
  » Full & fractional factorial designs
  » Plackett-Burman designs
- Minitab exercise #1
A Motivating Example

Olefin Polymerization System

Experimental Considerations

- Operating objectives
  » Maximize productivity
  » Achieve target polymer properties

- Input variables
  » Catalyst & co-catalyst concentrations
  » Monomer and co-monomer concentrations
  » Reactor temperature

- Output variables
  » Polymer production
  » Copolymer composition
  » Molecular weight
Experimental Design

- Problem
  » Determine optimal input values

- Brute force approach
  » Select values for the five inputs
  » Conduct semi-batch experiment
  » Calculate polymerization rate from on-line data
  » Obtain polymer properties from lab analysis
  » Repeat until best inputs are found

- Statistical techniques
  » Allow efficient search of input space
  » Handle nonlinear variable interactions
  » Account for experimental error

Outline – Part I

- A motivating example

- Introduction
  » The experimental design problem
  » Design objectives
  » Input levels
  » Empirical process models
  » General design procedure

- Basic design techniques
  » Randomized designs
  » Full & fractional factorial designs
  » Plackett-Burman designs

- Minitab exercise #1
The Experimental Design Problem

- Design objectives
  » Information to be gained from experiments

- Input variables (factors)
  » Independent variables
  » Varied to explore process operating space
  » Typically subject to known limits

- Output variables (responses)
  » Dependent variables
  » Chosen to reflect design objectives
  » Must be measured

- Statistical design of experiments
  » Maximize information with minimal experimental effort
  » Complete experimental plan determined in advance
Design Objectives

- Comparative experiments
  » Determine the best alternative

- Screening experiments
  » Determine the most important factors
  » Preliminary step for more detailed analysis

- Response surface modeling
  » Achieve a specified output target
  » Minimize or maximize a particular output
  » Reduce output variability
  » Achieve robustness to operating conditions
  » Satisfy multiple & competing objectives

- Regression modeling
  » Determine accurate model over large operating regime

Input Levels

- Input level selection
  » Low & high limits define operating regime
  » Must be chosen carefully to ensure feasibility

- Two-level designs
  » Two possible values for each input (low, high)
  » Most efficient & economical
  » Ideal for screening designs

- Three-level designs
  » Three possible values for each input (low, normal, high)
  » Less efficient but yield more information
  » Well suited for response surface designs
Empirical Models

- **Scope**
  - Three factors ($x_1, x_2, x_3$) & one response ($y$)

- **Linear model**
  \[ y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 \]
  - Accounts only for main effects
  - Requires at least four experiments

- **Linear model with interactions**
  \[ y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \beta_{12} x_1 x_2 + \beta_{13} x_1 x_3 + \beta_{23} x_2 x_3 \]
  - Includes binary interactions
  - Requires at least seven experiments

Empirical Models cont.

- **Quadratic model**
  \[ y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \beta_{12} x_1 x_2 + \beta_{13} x_1 x_3 + \beta_{23} x_2 x_3 + \beta_{11} x_1^2 + \beta_{22} x_2^2 + \beta_{33} x_3^2 \]
  - Accounts for response curvature
  - Requires at least ten experiments

- **Number of parameters/response**

<table>
<thead>
<tr>
<th>Factors</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Linear</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>6</td>
<td>7</td>
</tr>
<tr>
<td>Interaction</td>
<td>4</td>
<td>7</td>
<td>11</td>
<td>16</td>
<td>22</td>
</tr>
<tr>
<td>Quadratic</td>
<td>6</td>
<td>10</td>
<td>15</td>
<td>21</td>
<td>28</td>
</tr>
</tbody>
</table>
General Design Procedure

1. Determine objectives
2. Select input variables & their levels
3. Select output variables
4. Perform experimental design
5. Execute designed experiments
6. Perform data consistency checks
7. Statistically analyze the results
8. Modify the design as necessary

Outline – Part I

- A motivating example
- Introduction
  - The experimental design problem
  - Design objectives
  - Input levels
  - Empirical process models
  - General design procedure
- Basic design techniques
  - Randomized designs
  - Full & fractional factorial designs
  - Plackett-Burman designs
- Minitab exercise #1
Completely Randomized Designs

- Basic features
  - Limited to one factor ($k = 1$)
  - All other factors are ignored completely
  - Arbitrary number of input levels ($L$) & repeated experiments ($n$)
  - Total number of experiments: $N = Ln$
  - Randomized ordering of experiments

- Example
  - $L = 4$, $n = 3 \rightarrow N = 12$
  - Input levels: 1, 2, 3, 4
  - Random ordering: 3, 1, 4, 2, 2, 1, 3, 4, 1, 2, 4, 3

Randomized Block Designs

- Basic features
  - Limited to one primary factor ($x_1$)
  - Other nuisance factors are considered ($x_2$)
  - Total number of experiments: $N = (L_1 L_2 \ldots L_k)n$
  - Key nuisance factors held constant in each block of experiments

- Example
  - $k = 2$, $L_1 = 4$, $L_2 = 3$, $n = 1 \rightarrow N = 12$
  - Design prior to randomization

<table>
<thead>
<tr>
<th>$x_1$</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x_2$</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>3</td>
</tr>
</tbody>
</table>
Latin Square Designs

- Basic features
  - Extensions of randomized block designs
  - Can handle 2-4 nuisance factors
  - Each combination of nuisance factor levels is combined with each primary factor level once

- Advantages
  - Can handle several distinct nuisance factors
  - Yield relatively small number of experiments

- Disadvantages
  - All factors must have the same number of levels
  - Interactions between factors ignored

Full Factorial Designs

- Basic features
  - All permutations of factor levels considered
  - Total number of experiments: \( N = (L_1 L_2 \cdots L_k)^n \)
  - No duplication, same number of levels: \( N = L^k \)

<table>
<thead>
<tr>
<th>Factors</th>
<th>Two-Level Design</th>
<th>Three-Level Design</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>4</td>
<td>9</td>
</tr>
<tr>
<td>3</td>
<td>8</td>
<td>27</td>
</tr>
<tr>
<td>4</td>
<td>16</td>
<td>81</td>
</tr>
<tr>
<td>5</td>
<td>32</td>
<td>243</td>
</tr>
<tr>
<td>6</td>
<td>64</td>
<td>729</td>
</tr>
<tr>
<td>7</td>
<td>128</td>
<td>2187</td>
</tr>
</tbody>
</table>
Full Factorial Designs cont.

- Example: \( k = 3, L = 2, n = 1 \)
  - Requires eight experiments \( (N = L^k = 2^3 = 8) \)
  - Input levels: \(-1 = \) minimum, \(+1 = \) maximum
  - Basic design

<table>
<thead>
<tr>
<th>Run</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>( x_1 )</td>
<td>-1</td>
<td>+1</td>
<td>-1</td>
<td>+1</td>
<td>-1</td>
<td>+1</td>
<td>-1</td>
<td>+1</td>
</tr>
<tr>
<td>( x_2 )</td>
<td>-1</td>
<td>-1</td>
<td>+1</td>
<td>+1</td>
<td>-1</td>
<td>+1</td>
<td>+1</td>
<td>+1</td>
</tr>
<tr>
<td>( x_3 )</td>
<td>-1</td>
<td>-1</td>
<td>-1</td>
<td>-1</td>
<td>+1</td>
<td>+1</td>
<td>+1</td>
<td>+1</td>
</tr>
</tbody>
</table>

- Common modifications
  - Repeat runs for improved statistical data analysis
  - Add center points runs to capture nominal behavior
  - Randomize runs to reduce effects of unpredictable factors

Fractional Factorial Designs

- Terminology
  - Balanced design – all input level combinations have the same number of observations
  - Orthogonal design – the effect of any factor sums to zero across the effect of the other factors

<table>
<thead>
<tr>
<th>Run</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>Sum</th>
</tr>
</thead>
<tbody>
<tr>
<td>( x_1 )</td>
<td>-1</td>
<td>+1</td>
<td>+1</td>
<td>-1</td>
<td>0</td>
</tr>
<tr>
<td>( x_2 )</td>
<td>-1</td>
<td>+1</td>
<td>-1</td>
<td>+1</td>
<td>0</td>
</tr>
<tr>
<td>( x_3 )</td>
<td>-1</td>
<td>-1</td>
<td>+1</td>
<td>+1</td>
<td>0</td>
</tr>
</tbody>
</table>

Minitab software can be used to assist in creating balanced and orthogonal fractional factorial designs

- Basic features
  - Utilize a specified fraction of the full factorial design
  - Both balanced & orthogonal
  - Most useful for determining main effects
  - Can determine interaction and/or quadratic effects
### ½-Fractional Factorial Designs

- Full factorial design: \(k = 3, L = 2, n = 1\)

<table>
<thead>
<tr>
<th>Run</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>(x_1)</td>
<td>-1</td>
<td>+1</td>
<td>-1</td>
<td>+1</td>
<td>-1</td>
<td>+1</td>
<td>-1</td>
<td>+1</td>
</tr>
<tr>
<td>(x_2)</td>
<td>-1</td>
<td>-1</td>
<td>+1</td>
<td>+1</td>
<td>-1</td>
<td>-1</td>
<td>+1</td>
<td>+1</td>
</tr>
<tr>
<td>(x_3)</td>
<td>-1</td>
<td>-1</td>
<td>-1</td>
<td>-1</td>
<td>+1</td>
<td>+1</td>
<td>+1</td>
<td>+1</td>
</tr>
</tbody>
</table>

- Alternative ½-fractional factorial designs

### Other Fractional Factorial Designs

- Two-level designs

<table>
<thead>
<tr>
<th>Factors</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>Full</td>
<td>8</td>
<td>16</td>
<td>32</td>
<td>64</td>
<td>128</td>
</tr>
<tr>
<td>1/2</td>
<td>4</td>
<td>8</td>
<td>16</td>
<td>32</td>
<td>64</td>
</tr>
<tr>
<td>1/4</td>
<td>NA</td>
<td>4</td>
<td>8</td>
<td>16</td>
<td>32</td>
</tr>
<tr>
<td>1/8</td>
<td>NA</td>
<td>NA</td>
<td>8</td>
<td>16</td>
<td>32</td>
</tr>
</tbody>
</table>

- Three-level designs

<table>
<thead>
<tr>
<th>Factors</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>Full</td>
<td>27</td>
<td>81</td>
<td>243</td>
<td>729</td>
<td>2187</td>
</tr>
<tr>
<td>1/3</td>
<td>9</td>
<td>27</td>
<td>81</td>
<td>243</td>
<td>729</td>
</tr>
<tr>
<td>1/9</td>
<td>3</td>
<td>9</td>
<td>27</td>
<td>81</td>
<td>243</td>
</tr>
<tr>
<td>1/27</td>
<td>NA</td>
<td>NA</td>
<td>9</td>
<td>27</td>
<td>81</td>
</tr>
</tbody>
</table>
Center Points

- Represent mid-range values of all factors (indicated with “0”)

- Reasons for adding center points
  » Two-level designs do not have center points
  » Determine process variability
  » Check for response curvature

- Placement of center point runs
  » Generally add 3-5 center point runs to two-level full & fractional factorial designs
  » Beginning & end of experimental design
  » Evenly dispersed throughout the design matrix
  » Not randomized

Center Point Example

- Three factor full factorial design
  » Two replicates of each factorial point
  » Randomization of factorial runs
  » Three additional center points

- Randomized design

<table>
<thead>
<tr>
<th>Run</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
<th>13</th>
<th>14</th>
<th>15</th>
<th>16</th>
<th>17</th>
<th>18</th>
<th>19</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x_1$</td>
<td>0</td>
<td>-1</td>
<td>-1</td>
<td>-1</td>
<td>-1</td>
<td>+1</td>
<td>+1</td>
<td>+1</td>
<td>0</td>
<td>+1</td>
<td>-1</td>
<td>+1</td>
<td>+1</td>
<td>-1</td>
<td>+1</td>
<td>-1</td>
<td>+1</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>$x_2$</td>
<td>0</td>
<td>-1</td>
<td>+1</td>
<td>-1</td>
<td>+1</td>
<td>+1</td>
<td>-1</td>
<td>+1</td>
<td>0</td>
<td>-1</td>
<td>-1</td>
<td>+1</td>
<td>+1</td>
<td>-1</td>
<td>-1</td>
<td>+1</td>
<td>-1</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>$x_3$</td>
<td>0</td>
<td>+1</td>
<td>+1</td>
<td>-1</td>
<td>+1</td>
<td>-1</td>
<td>-1</td>
<td>+1</td>
<td>-1</td>
<td>0</td>
<td>-1</td>
<td>+1</td>
<td>+1</td>
<td>-1</td>
<td>+1</td>
<td>-1</td>
<td>-1</td>
<td>0</td>
<td></td>
</tr>
</tbody>
</table>
Resolution

- Confounding
  - There is not enough data to define higher order effects (e.g., quadratic, cross terms); they are “confounded” by lower order effects (e.g., linear)

- Resolution III designs
  - Main effects confounded with binary interaction effects
  - Useful only for screening

- Resolution IV designs
  - Binary interaction effects confounded with each other
  - Mainly useful for screening

- Resolution V designs
  - Binary effects confounded with ternary effects
  - Useful for determining interaction & quadratic effects

Plackett-Burman Designs

- Basic features
  - Very efficient for large number of factors
  - Yield minimum number of experiments
  - Resolution III designs where main effects are heavily confounded with binary interaction effects

- Two-level designs

<table>
<thead>
<tr>
<th>Factors</th>
<th>11</th>
<th>15</th>
<th>19</th>
<th>23</th>
<th>27</th>
</tr>
</thead>
<tbody>
<tr>
<td>Full factorial</td>
<td>2048</td>
<td>32768</td>
<td>524288</td>
<td>8.39x10^6</td>
<td>1.34x10^8</td>
</tr>
<tr>
<td>Plackett-Burman</td>
<td>12</td>
<td>16</td>
<td>20</td>
<td>24</td>
<td>28</td>
</tr>
<tr>
<td>Linear model parameters</td>
<td>12</td>
<td>16</td>
<td>20</td>
<td>24</td>
<td>28</td>
</tr>
</tbody>
</table>
Outline – Part I

- A motivating example
- Introduction
  » The experimental design problem
  » Design objectives
  » Input levels
  » Empirical process models
  » General design procedure
- Basic design techniques
  » Randomized designs
  » Full & fractional factorial designs
  » Plackett-Burman designs
- *Minitab exercise #1*

Introduction to Minitab

- Background
  » Statistical data analysis software
  » Departmental license
  » Utilized to demonstrate DOE concepts
- Features
  » Data management
  » Data visualization
  » Basic statistics
  » Regression analysis
  » Analysis of variance
  » Design of experiments
  » Multivariate statistics
Minitab Data Analysis Tools

- Data & file management
- Graphical
  - Histograms, scatter plots
  - Box plots, main effects plots, interaction plots
- Basic statistics
  - Chi-squared test and t-tests
  - Confidence intervals
  - Correlation & covariance
- Modeling
  - Linear & nonlinear regression
  - Residual analysis
  - Analysis of variance (ANOVA)

Polymerization Reactor Example

- Input variables
  - Catalyst concentration ($x_1$)
  - Co-catalyst concentration ($x_2$)
  - Ethylene concentration ($x_3$)
  - Propylene concentrations ($x_4$)
  - Reactor temperature ($x_5$)

- Output variables
  - Polymer production ($y_1$)
  - Ethylene composition of polymer ($y_2$)
  - Number average molecular weight ($y_3$)
  - Polydispersity ($y_4$)
Example Data Files

- Numbers represent scaled values of output variables
- Different numbers of replicates and noise in sample data sets
  - Sample set #1 (Noise Free): n = 1
  - Sample sets #2 and #3 (Low and Medium Noise): n = 2
  - Sample set #4 (High Noise): n = 3
- Examples of two-level full & fractional factorial designs, with and without center points
  - Noise Free, full factorial design, no center points:
    n = 1, k = 5, L = 2, N = n(L^k) = 2^5 = 32
  - Medium noise, ½-fractional design, two center points:
    n = 2, k = 5, L = 2, N = ½ [n(L^k)] + 2 = ½ [2(2^5)] + 2 = 34

Noise Free Data

<table>
<thead>
<tr>
<th>Run</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
<th>13</th>
<th>14</th>
<th>15</th>
<th>16</th>
</tr>
</thead>
<tbody>
<tr>
<td>x_1</td>
<td>-1</td>
<td>+1</td>
<td>-1</td>
<td>+1</td>
<td>-1</td>
<td>+1</td>
<td>-1</td>
<td>+1</td>
<td>-1</td>
<td>+1</td>
<td>-1</td>
<td>+1</td>
<td>-1</td>
<td>+1</td>
<td>-1</td>
<td>+1</td>
</tr>
<tr>
<td>x_2</td>
<td>-1</td>
<td>-1</td>
<td>+1</td>
<td>-1</td>
<td>+1</td>
<td>-1</td>
<td>+1</td>
<td>-1</td>
<td>+1</td>
<td>-1</td>
<td>+1</td>
<td>-1</td>
<td>+1</td>
<td>-1</td>
<td>+1</td>
<td>-1</td>
</tr>
<tr>
<td>x_3</td>
<td>-1</td>
<td>-1</td>
<td>-1</td>
<td>-1</td>
<td>+1</td>
<td>+1</td>
<td>-1</td>
<td>-1</td>
<td>+1</td>
<td>-1</td>
<td>-1</td>
<td>+1</td>
<td>+1</td>
<td>-1</td>
<td>-1</td>
<td>-1</td>
</tr>
<tr>
<td>x_4</td>
<td>-1</td>
<td>-1</td>
<td>-1</td>
<td>-1</td>
<td>-1</td>
<td>-1</td>
<td>-1</td>
<td>+1</td>
<td>-1</td>
<td>+1</td>
<td>+1</td>
<td>+1</td>
<td>+1</td>
<td>-1</td>
<td>-1</td>
<td>-1</td>
</tr>
<tr>
<td>x_5</td>
<td>-1</td>
<td>-1</td>
<td>-1</td>
<td>-1</td>
<td>-1</td>
<td>-1</td>
<td>-1</td>
<td>-1</td>
<td>-1</td>
<td>-1</td>
<td>-1</td>
<td>-1</td>
<td>-1</td>
<td>-1</td>
<td>-1</td>
<td>-1</td>
</tr>
<tr>
<td>y_1</td>
<td>-4</td>
<td>-8</td>
<td>-20</td>
<td>-8</td>
<td>4</td>
<td>0</td>
<td>-12</td>
<td>0</td>
<td>4</td>
<td>0</td>
<td>-12</td>
<td>0</td>
<td>12</td>
<td>8</td>
<td>-4</td>
<td>8</td>
</tr>
<tr>
<td>y_2</td>
<td>4</td>
<td>4</td>
<td>4</td>
<td>4</td>
<td>8</td>
<td>8</td>
<td>8</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>4</td>
<td>4</td>
<td>4</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>y_3</td>
<td>20</td>
<td>8</td>
<td>16</td>
<td>4</td>
<td>16</td>
<td>4</td>
<td>12</td>
<td>0</td>
<td>16</td>
<td>4</td>
<td>12</td>
<td>0</td>
<td>12</td>
<td>0</td>
<td>8</td>
<td>-4</td>
</tr>
<tr>
<td>y_4</td>
<td>0</td>
<td>12</td>
<td>8</td>
<td>20</td>
<td>0</td>
<td>12</td>
<td>8</td>
<td>20</td>
<td>0</td>
<td>12</td>
<td>8</td>
<td>20</td>
<td>0</td>
<td>12</td>
<td>8</td>
<td>20</td>
</tr>
</tbody>
</table>
### Design Alternatives

<table>
<thead>
<tr>
<th>Design</th>
<th>Noise Free</th>
<th>Low Noise</th>
<th>Medium Noise</th>
<th>High Noise</th>
</tr>
</thead>
<tbody>
<tr>
<td>Full</td>
<td>32 (A)</td>
<td>64</td>
<td>64</td>
<td>96</td>
</tr>
<tr>
<td>Full w/ center pts</td>
<td>33 (B)</td>
<td>66</td>
<td>66</td>
<td>99</td>
</tr>
<tr>
<td>½-fractional w/ center pts</td>
<td>16 (A)</td>
<td>32 (C)</td>
<td>32 (E)</td>
<td>48 (F)</td>
</tr>
<tr>
<td>½-fractional w/ center pts</td>
<td>17 (B)</td>
<td>34 (D)</td>
<td>34</td>
<td>51</td>
</tr>
<tr>
<td>¼-fractional w/ center pts</td>
<td>8</td>
<td>16 (C)</td>
<td>16 (E)</td>
<td>24 (F)</td>
</tr>
<tr>
<td>¼-fractional w/ center pts</td>
<td>9 (ex.)</td>
<td>18 (D)</td>
<td>18</td>
<td>27</td>
</tr>
</tbody>
</table>

### Design Exercises

- **Preliminaries**
  - Each lab group assigned two design alternatives (previous slide)

- **Minitab Assignment**
  - Perform experimental design in Minitab
  - Import selected data runs from Excel file
  - Perform simple graphical data analysis in Minitab
    - Turn in Main Effect Plots and Binary Interaction Plots
    - Analyze DOE data, propose models for each output variable based on both design alternatives (8 total correlations, and write a short (~1 paragraph) summary of conclusions
    - **Due by Friday, 9/23 in class**
      - Grade will be counted in Participation/Safety portion of first lab
  - Save data file for later use in next class period
Regression Modeling

- Empirical model for two factors
  \[ y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \beta_{12} x_1 x_2 + \beta_{13} x_1 x_3 + \beta_{23} x_2 x_3 + \beta_{11} x_1^2 + \beta_{22} x_2^2 + \beta_{33} x_3^2 \]
  - Bias
  - Main effects
  - Binary interaction effects
  - Quadratic effects

- Regression analysis
  » Fit model to experimental observations
  » Eliminate negligible terms
  » Determine parameter values for significant terms

References

General Analysis of DOE Data