

# Introduction to MATLAB

Karl D. Hammond

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The MATLAB suite is an interactive shell-based program and scripting language that enables one to input and perform algebraic operations with significantly less (human) effort than is typically required with compiled languages such as C, C++, and Fortran. MATLAB-based simulations typically run much more slowly than an equivalently programmed simulation in a compiled language, however, due to the fact that the input must be converted to machine language on the fly.

MATLAB was created with vectors and matrices in mind, and much of its utility stems from the very transparent handling of such constructs. Those familiar with C or C++ will find MATLAB's command list to be quite similar, and will quickly adapt.

## 1 MATLAB Eccentricities

The language/interface has several eccentricities that all users should be aware of:

- All variables are case-sensitive (*a* and *A* are different variables)
- Variables are stored in the same space (with the same names) between the MATLAB command line and the SIMULINK GUI.
- All identifiers must be unique. For example, you cannot have a function *and* a variable with the same name.
- Spaces are generally ignored, except when they separate elements such as variable names or matrix elements. Multiple spaces are collapsed.
- There is *no* implied multiplication.
- Entering a command by itself or with a trailing comma (,) causes MATLAB to echo the result of the command to the screen. For example, typing “2 + 2” prints

```
ans =
```

```
4
```

Note that echoing commands makes each command run more slowly as well.

- Entering a command with a trailing semicolon (;) will suppress command echoing. Semicolons should always be used in scripts or any loops to avoid cluttering your screen with garbage during the simulation.
- The current (working) directory in the top of the window must contain all of your script files (\*.m)

## 2 Entering Matrices

MATLAB's facilities for working with matrices are simple. Vectors and matrices are handled identically. To enter a matrix, surround the elements with brackets; separate columns by either spaces or commas, and separate rows by semicolons. For example,

```
>> A = [ 1 2 ; 3 -1 ];
>> A = [1 2;3 -1];
```

One could also use commas:

```
>> A = [ 1, 2; 3, -1 ];
>> A = [1,2;3,-1];
```

(all declarations are equivalent).

To refer to a matrix *element*, use  $A(x,y)$ , where  $x$  and  $y$  are the row and column. If  $A$  is defined as above, then  $A(2,2)$  returns  $-1$ . Using a colon in place of a row or column returns the vector containing the associated row or column:  $A(2,:)$  returns  $[3 -1]$  (a row vector), while  $A(:,2)$  returns  $\begin{bmatrix} 2 \\ -1 \end{bmatrix}$  (a column vector).

## 3 Mathematical Operators

For this section, assume  $A = \begin{bmatrix} 1 & 2 \\ 3 & -1 \end{bmatrix}$ ,  $u = [1 \ 2]$ ,  $v = \begin{bmatrix} 1 \\ 0.5 \end{bmatrix}$

- Addition/subtraction:  $A + A$  yields  $[2 \ 4 ; 6 \ -2]$  (matrix dimensions must agree!)
- Scalar multiplication:  $3 * A$  yields  $[3 \ 6 ; 9 \ -1]$ . Also,  $2 * 3$  yields 6 and  $2 * u$  yields  $[2 \ 4]$ .
- Matrix multiplication:  $A * A$  yields  $[7 \ 0 ; 0 \ 7]$ . Note that  $u * v$  yields 2, but  $v * u$  yields  $[1 \ 2 ; 0.5 \ 1]$  (inner product in the first case, outer product in the second). Matrix dimensions must agree.
- Element-by-element multiplication:  $A .* A$  yields  $[1 \ 4 ; 9 \ 1]$ .  $u .* u$  yields  $[1 \ 4]$ .  $v .* v$  yields  $[1 ; 0.25]$ . Matrix dimensions must agree!
- Transposition:  $A'$  yields  $[1 \ 3 ; 2 \ -1]$ .  $u'$  yields  $[1 ; 2]$  and  $v'$  yields  $[1 \ 0.5]$ .
- Exponents:  $A^3$  yields  $[7 \ 14 ; 21 \ -7]$ . Matrix must be square.
- Concatenation:  $[A \ v]$  yields  $[1 \ 2 \ 1 ; 3 \ -1 \ 0.5]$ .  $[A ; u]$  yields  $[1 \ 2 ; 3 \ -1 ; 1 \ 2]$ .

## 4 Common Functions

MATLAB has literally hundreds of predefined functions. Some common ones are

Purpose	Example	Result
Get help	<code>help eig</code>	(help on <code>eig</code> command)
Determinant	<code>det(A)</code>	-7
Rank	<code>rank(A)</code>	2
Eigenvalues/vectors	<code>eig(A)</code>	[ 2.6458 ; -2.6458 ]
Matrix Inverse	<code>inv(A)</code>	[ 0.1429 0.2857 ; 0.4286 -0.1429 ]
Reduced Row-echelon Form	<code>rref(A)</code>	[ 1 0 ; 0 1 ]
Create transfer function	<code>G=tf(1,[1 0.5])</code> <code>s=tf('s'); G=1/(s+0.5)</code>	$G(s) = 1/(s + 0.5)$ $G(s) = 1/(s + 0.5)$
Create state space system	<code>sys=ss(A,B,C,D)</code>	
Convert SS to transfer function	<code>[num,den] = ss2tf(A,B,C,D)</code>	
Plot step response	<code>step(H)</code>	(plot)
Plot impulse response	<code>impz(H)</code>	(plot)
Plot $x$ vs $t$	<code>plot(t,x)</code>	(plot)
Stack two plots	<code>subplot(2,1,1); plot(t1,x1)</code> <code>subplot(2,1,2); plot(t2,x2)</code>	

## 5 In-class Assignment

Obtain an approximate solution to the linear system

$$\begin{aligned}
 x + 3y - 10z + 20w &= 1 \\
 2x + 3y - z + w &= -1 \\
 3x + 4y + 2z + w &= 3 \\
 -2x + 3y + 4z + 5w &= 5.
 \end{aligned} \tag{1}$$

You should use both matrix inversion ( $\vec{x} = A^{-1}\vec{b}$ ) and Gauss-Jordan elimination ( $\text{rref}[A|\vec{b}]$ ). Compute the characteristic values (eigenvalues) and vectors (eigenvectors) of the associated square matrix. Find the norm of the vector  $\vec{b}$  in the equation  $A\vec{x} = \vec{b}$ .

### Answers

$$\vec{x} = A^{-1}\vec{b} = \begin{bmatrix} 0.8728 \\ -0.7395 \\ 1.2889 \\ 0.7617 \end{bmatrix} \tag{2}$$

$$\text{rref}[A|\vec{b}] = \left[ \begin{array}{cccc|c} 1 & 0 & 0 & 0 & 0.8728 \\ 0 & 1 & 0 & 0 & -0.7395 \\ 0 & 0 & 1 & 0 & 1.2889 \\ 0 & 0 & 0 & 1 & 0.7617 \end{array} \right] \tag{3}$$

$$\lambda = \{0.7297 \pm 8.781i, 1.2598, 8.2808\} \tag{4}$$

$$\vec{E}_{1.2598} = \begin{pmatrix} -0.5842 \\ 0.6 \\ -0.4435 \\ -0.3194 \end{pmatrix} \quad (5)$$

$$\vec{E}_{8.2808} = \begin{pmatrix} 0.6560 \\ 0.2359 \\ 0.5389 \\ 0.4729 \end{pmatrix} \quad (6)$$

$$\vec{E}_{0.7297+8.781i} = \begin{pmatrix} 0.8850 \\ 0.0101 - 0.2086i \\ -0.1045 - 0.2843i \\ -0.0657 + 0.2777i \end{pmatrix} \quad (7)$$

$$\vec{E}_{0.7297-8.781i} = \begin{pmatrix} 0.8850 \\ 0.0101 + 0.2086i \\ -0.1045 + 0.2843i \\ -0.0657 - 0.2777i \end{pmatrix} \quad (8)$$

$$\|\vec{b}\| = 6 \quad (9)$$