Problem 1. Consider a plug flow reactor in which the following reactions occur: \(2A \rightarrow B\), \(B \rightarrow 2A\), \(B \rightarrow C\). The reaction rates per unit volume are \(r_1 = k_1C_A\), \(r_2 = k_2C_B\) and \(r_3 = k_3C_B\), respectively. The reactor is operated under steady-state conditions with constant volumetric flow rate \(q\). The cross-sectional area of the reactor is \(A_c\). The variable \(z\) is the axial distance along the reactor, with \(z = 0\) denoting the reactor entrance.

1. Derive the following model equations by writing differential mass balances with \(\frac{A_c}{q} = 1\), \(k_1 = 2\), \(k_2 = 1\), \(k_3 = 3\), \(C_{A_i} = 10\) and and \(C_{B_i} = 0\):

\[
\frac{dy}{dz} = \begin{bmatrix}
-4 & 2 \\
2 & -4
\end{bmatrix} y = Ay, \quad y(0) = \begin{bmatrix}
10 \\
0
\end{bmatrix}
\]

where: \(y = [C_A \ C_B]^T\).

2. Find the eigenvalues and eigenvectors of the matrix \(A\) in part 1.

3. Determine the reactor length \(L\) necessary to maximize \(C_B\).

Problem 2. Consider a continuous stirred tank reactor in which the following reversible reaction occurs: \(A \rightarrow B\), \(B \rightarrow A\). The reaction rates per unit volume of the forward and reverse reactions are \(r_f = k_fC_A\) and \(r_r = k_rC_B\), respectively. The reactor has two inlet streams, with the first stream having volumetric flow rate \(q_1\) and concentrations \(C_{A1}\) and \(C_{B1} = 0\) and the second stream having volumetric flow rate \(q_2\) and concentrations \(C_{A2}\) and \(C_{B2}\). The reactor has an outlet stream with volumetric flow rate \(q_3\) and concentrations \(C_{A3}\) and \(C_{B3}\). The reactor operates isothermally with constant volume \(V\) and density \(\rho\).

1. Derive the following model equations:

\[
\begin{align*}
\frac{dC_{A3}}{dt} &= \frac{q_1}{V}C_{A1} + \frac{q_2}{V}C_{A2} - \left(k_f + \frac{q_1}{V} + \frac{q_2}{V}\right)C_{A3} + k_rC_{B3} \\
\frac{dC_{B3}}{dt} &= \frac{q_2}{V}C_{B2} + k_fC_{A3} - \left(k_r + \frac{q_1}{V} + \frac{q_2}{V}\right)C_{B3}
\end{align*}
\]

2. Given the parameter values \(k_f = 2\), \(k_r = 1\), \(\frac{q_1}{V} = 2\), \(\frac{q_2}{V} = 1\), \(C_{A1} = 1\), \(C_{A2} = 1\) and \(C_{B2} = 6\), formulate and solve a matrix problem \(Ax = b\) to find the steady-state values \(\bar{C}_{A3}\) and \(\bar{C}_{B3}\).

3. Formulate and solve the eigenvalue problem for the matrix \(A\) in part 2.
4. Formulate and solve the model in part 1 as \( \frac{dx}{dt} = Ax + b \) to obtain \( x(t) \).

5. Apply the initial conditions \( C_{A3}(0) = 4 \) and \( C_{B3}(0) = 0 \) to the solution in part 4 to obtain the solution \( C_{A3}(t) \) and \( C_{B3}(t) \).