Thermal Pollution

- Concerns
  - direct lethal effects on sensitive plants or animals
  - indirect effects on aquatic ecosystems through effects on the kinetics of biological, physical or chemical processes

- Principal Sources
  - Point sources
    - electric power generating stations (Major)
    - municipal & industrial WWTP’s (minor)
  - Non-point Sources
    - long wave atmospheric radiation ($H_a$)
    - Short wave solar radiation ($H_s$)
Temperature or Heat Modeling

- Mass Balance is written based on thermal energy
  \[ \frac{\partial c}{\partial t} = -\frac{1}{A} \frac{\partial (Qc)}{\partial x} + E \frac{\partial^2 c}{\partial x^2} + k c + \frac{H_N}{z} \]
  Each term is in: BTU/m³/d
  - and we relate this to temperature by: \( c = \rho c_H (T - T_o) \)
    - \( \rho = \) density of water (g/m³)
    - \( c_H = \) heat capacity of water (BTU/g/°C)
    - \( T_o = \) arbitrary “frame of reference” temperature (°C)
  - Substituting in we get:
    \[ \frac{\partial T}{\partial t} = -\frac{1}{A} \frac{\partial (QT)}{\partial x} + E \frac{\partial^2 T}{\partial x^2} + \frac{H_N}{\rho c_H z} \]

Heat Modeling (cont.)

- Determination of \( H_N \)
  - Refer to:
  - Also some treatment in Chapra
    - (Lecture 30)
Total Surface Heat Flux

- Chapra
- Fig 30.3
Numerical Solutions (QUAL2E)

- The problem
  - need to develop numerical estimates for the first and second derivatives of concentration versus distance

\[ \frac{\partial c}{\partial t} = -U \frac{\partial c}{\partial x} + E \frac{\partial^2 c}{\partial x^2} \]

- The solution
  - Finite Difference

Finite Difference

- Euler-Cauchy Approach
  - Forward Difference
    \[ \frac{\partial c}{\partial x}_i \approx \frac{c_{i+1} - c_i}{\Delta x} \]
  - Backward Difference
    \[ \frac{\partial c}{\partial x}_i \approx \frac{c_i - c_{i-1}}{\Delta x} \]
  - Central or Centered Difference
    \[ \frac{\partial c}{\partial x}_i \approx \frac{c_{i+1} - c_{i-1}}{2\Delta x} \]

But less stable
Finite Difference (cont.)

- Formulation: Explicit
  - backward difference
  - forward difference
  - Gives centered difference

\[
\frac{\partial^2 c}{\partial x^2} \approx \frac{\partial c}{\partial x} \bigg|_i - \frac{\partial c}{\partial x} \bigg|_{i-1} \Delta x
\]

\[
\frac{(c_{i+1} - c_i) \Delta x - (c_i - c_{i-1}) \Delta x}{\Delta x^2} \approx \frac{c_{i+1} - 2c_i + c_{i-1}}{\Delta x^2}
\]

The Explicit Approach

- The differential equation:
  \[
  \frac{\partial c}{\partial t} = -U \frac{\partial c}{\partial x} + E \frac{\partial^2 c}{\partial x^2}
  \]

- Becomes:
  \[
  \frac{c_{i+1}^n - c_i^n}{\Delta t} = -U \frac{c_{i+1}^n - c_{i-1}^n}{2 \Delta x} + E \frac{c_{i+1}^n - 2c_i^n + c_{i-1}^n}{\Delta x^2}
  \]

- And rearranging:
  \[
  c_{i+1}^{n+1} = c_i^n + \left( -U \frac{c_{i+1}^n - c_{i-1}^n}{2 \Delta x} + E \frac{c_{i+1}^n - 2c_i^n + c_{i-1}^n}{\Delta x^2} \right) \Delta t
  \]
The QUAL2E approach

- Classical Implicit Backwards Difference

\[ c_i^{n+1} = c_i^n + \left( -U \frac{c_i^{n+1} - c_{i-1}^{n+1}}{\Delta x} + E \frac{c_{i+1}^{n+1} - 2c_i^{n+1} + c_{i-1}^{n+1}}{\Delta x^2} + kc_i^{n+1} + z_i \right) \Delta t \]

- \( k \) is the rate constant for all internal processes and reactions (e.g., biodegradation)
- \( z_i \) is the sum of all internal sources and sinks
  - e.g., SOD
- Solved using matrix subroutines

To next lecture