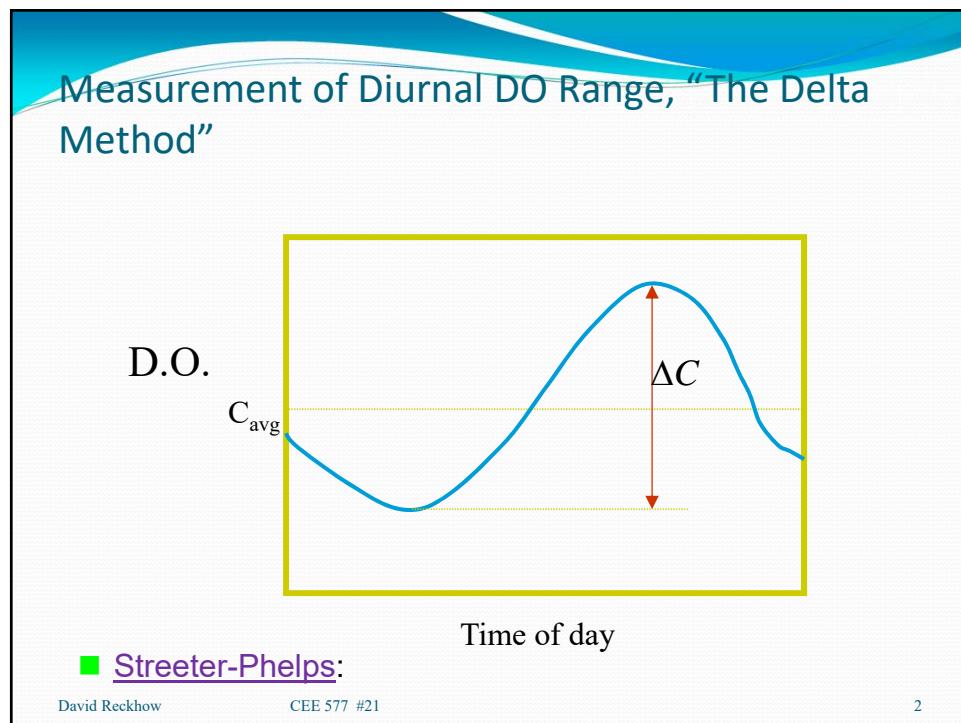


Updated: 6 November 2017

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# CEE 577: Surface Water Quality Modeling

Lecture #21  
Qual2E/K: Algae &Basic Formulation cont.;  
Heat Balance  
(Chapra, L24 & L26)



## DiToro method (1975)

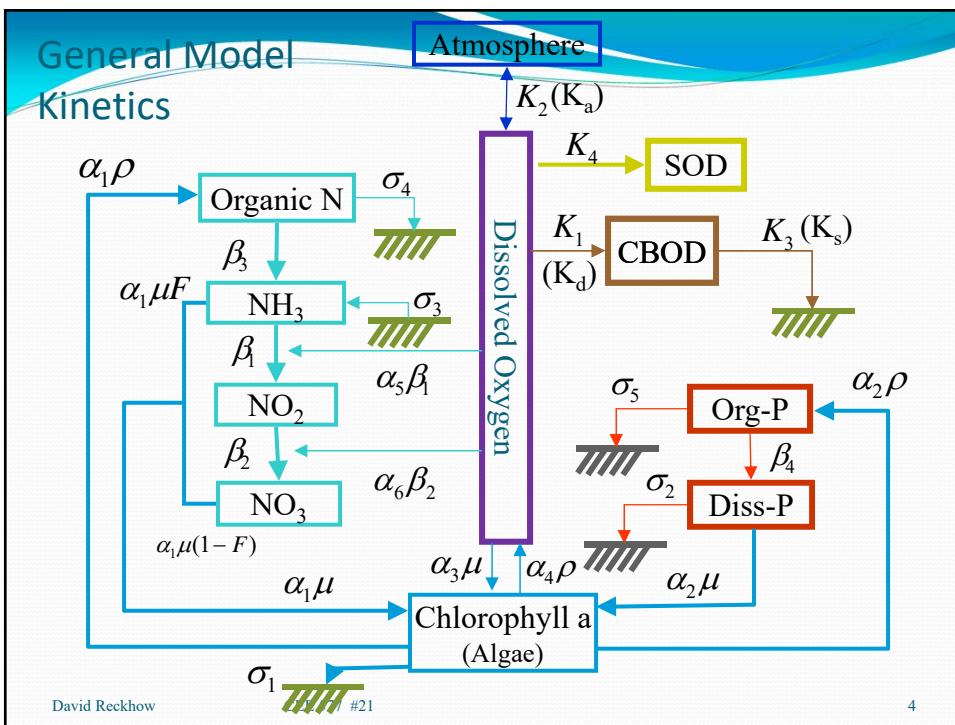
$$P = \left\{ \frac{0.5k_a [1 - e^{-k_a}]}{(1 - e^{-0.5k_a})^2} \right\} \Delta C$$

$$R = P + k_a \overline{D}$$

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# Algal growth model

- General mass balance

$$\frac{dA}{dt} = \mu A - \rho A - \frac{\sigma_1}{z} A$$

- Algal specific growth rate

- multiplicative model
  - limiting nutrient model
  - inverse additive model

$$\mu = \mu_{\max}(FL)(FN)(FP)$$

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## Algal growth model

- General mass balance

$$\frac{dA}{dt} = \mu A - \rho A - \frac{\sigma_1}{z} A$$

Respiration rate ( $d^{-1}$ )

Algal settling rate (ft/d)

- How do we calculate  $\mu$ ?  
Specific growth rate ( $d^{-1}$ )

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## Algal growth model

- Algal specific growth rate

- multiplicative model

$$\mu = \mu_{\max} (FL)(FN)(FP)$$

All based on algal growth limitation factors: numbers between 0 and 1

- limiting nutrient model

- if N is limiting  $\mu = \mu_{\max} (FL)(FN)$

- if P is limiting  $\mu = \mu_{\max} (FL)(FP)$

- inverse additive model

$$\mu = \mu_{\max} (FL) \left[ \frac{2}{\frac{1}{FN} + \frac{1}{FP}} \right]$$

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## Algal growth model

- Algal growth limitation factors

- Nitrogen

$$FN = \frac{N_e}{N_e + K_N}$$

- where the half velocity constant for N is  $K_N$

- and the available nitrogen is:  $N_e = NH_3-N + NO_3-N$

- Phosphorus

$$FP = \frac{P_2}{P_2 + K_P}$$

- and the half velocity constant for P is  $K_P$

- and the available phosphorus ( $P_2$ ) is dissolved-P

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## Algal light functions

- Types
  - Half saturation
  - Smith's function
  - Steel's equation
- Light-depth function

$$FL_z = \frac{I_z}{I_z + K_L}$$

$$FL_z = \frac{I_z}{\sqrt{I_z^2 + K_L^2}}$$

$$FL_z = \frac{I_z}{K_L} \exp\left(1 - \frac{I_z}{K_L}\right)$$

Extinction coefficient ( $\text{ft}^{-1}$ )

$$I_z = I_o e^{-\lambda z}$$

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## The light attenuation factor

$$\phi_l = \frac{2.718f}{k_e H} (e^{-\alpha_1} - e^{-\alpha_0})$$

Where:

$$\alpha_1 = \frac{I_a}{I_s} e^{-k_e H}$$

$$\alpha_0 = \frac{I_a}{I_s}$$

Optimal intensity  
for plant growth  
(250-500 ly/d)

$$\text{And: } k_e = \frac{1.8}{SD}$$

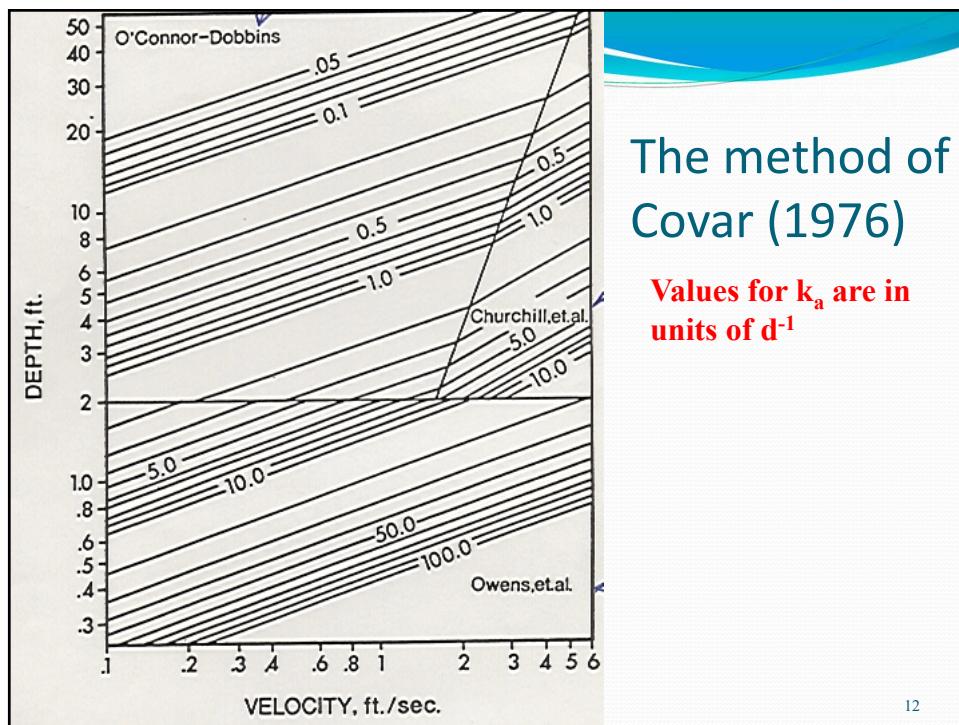
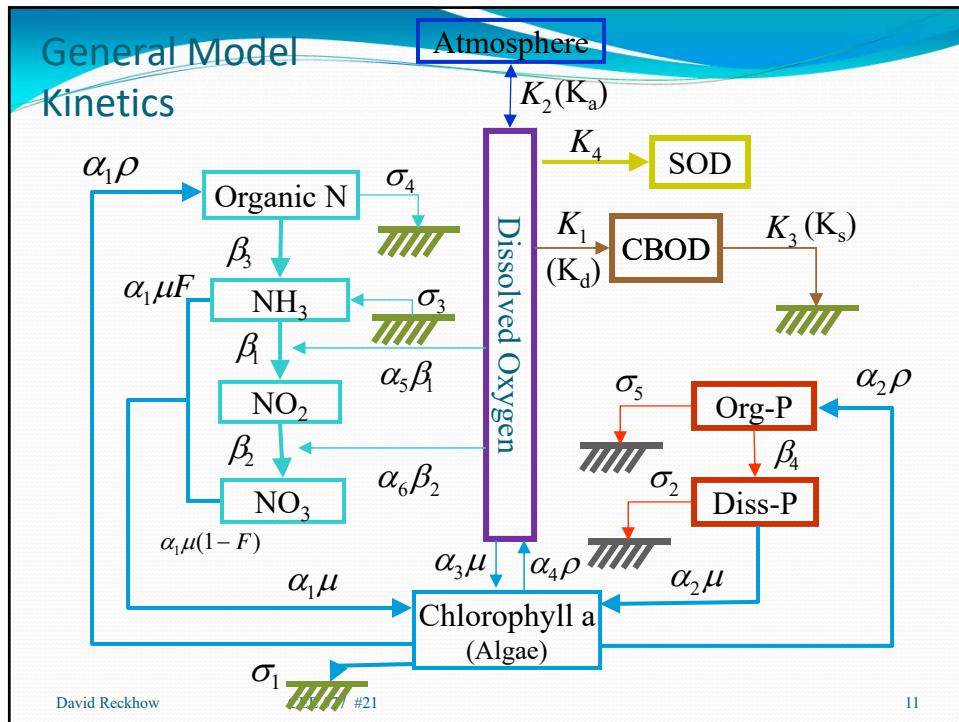
Extinction coefficient ( $\text{m}^{-1}$ )

Avg. daylight intensity

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## Reaeration

- Eight Options in QUALE

- 1 = specify value

- 2 = Churchill Formula

$$k_a = 11.6 \frac{U}{H^{1.67}}$$

- 3 = O'Connor Dobbins Formula

- 4 = Owens Formula

$$k_a = 21.6 \frac{U^{0.67}}{H^{1.85}}$$

$$k_a = 12.9 \frac{U^{0.5}}{H^{1.5}}$$

- 5 = Thackston & Krenkel

- developed for TVA's narrow streams with low DO, requires

"n", generally good equation

- froude number

- shear velocity

$$F = \frac{U^*}{\sqrt{gz}}$$

$$k = 25(1 + F^{0.5}) \left( \frac{U^*}{z} \right)$$

$$U^* = \frac{Un\sqrt{g}}{1.49z^{1.167}}$$

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## Reaeration

- Eight options cont.

- 6 = Langbien & Durum

- for  $z > 2$  ft (synthesis of others)

$$k = 7.62 \frac{U}{z^{1.33}}$$

- 7 = Power Function of Flow

$$k = aQ^b$$

- 8 = Tsivoglou & Wallace

- requires escape coefficient

- $c = 0.054$  ft $^{-1}$  for  $Q = 15\text{-}3000$  cfs

- $c = 0.110$  ft $^{-1}$  for  $Q = 1\text{-}15$  cfs

$$k = 38,900n^2 \frac{U^3}{z^{1.333}} c$$

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## Waste Loading

- Point Sources
    - Municipal WW
    - Industrial WW
    - Tributaries
  - Non-point Sources
    - Agricultural
    - Silvicultural
    - Atmospheric
    - Urban & Suburban Runoff

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# Loading Calculations

## Point Sources - General Concepts

$$W(t) = Q(t) \bullet c(t)$$

### Important Conversion Factors:

$$8.34 \frac{lb \bullet liters}{mg \bullet MG}$$

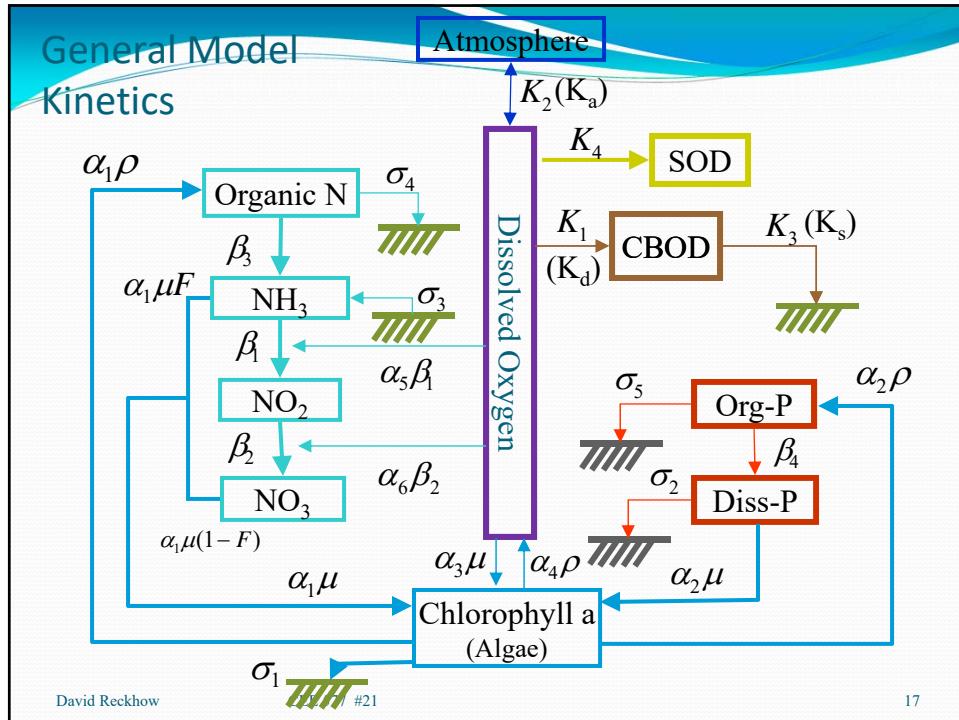
$$5.39 \frac{lb \bullet liters \bullet sec}{mg \bullet ft^3 \bullet day}$$

$$2.45 \frac{Kg \bullet liters \bullet sec}{mg \bullet ft^3 \bullet day}$$

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- [To next lecture](#)