

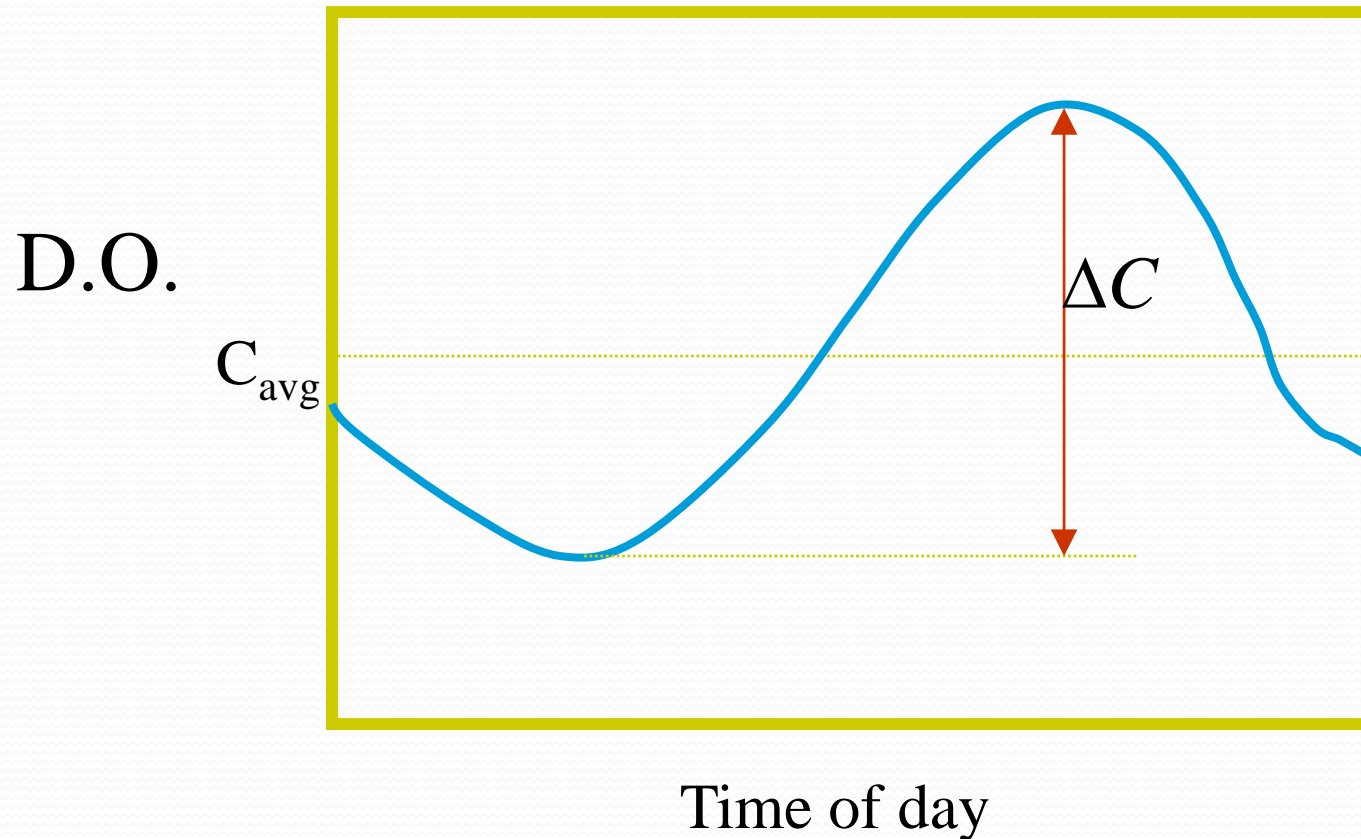
# CEE 577: Surface Water Quality Modeling

Lecture #21

Qual2E/K: Algae & Basic Formulation cont.;  
Heat Balance

(Chapra, L24 & L26)

# Measurement of Diurnal DO Range, “The Delta Method”



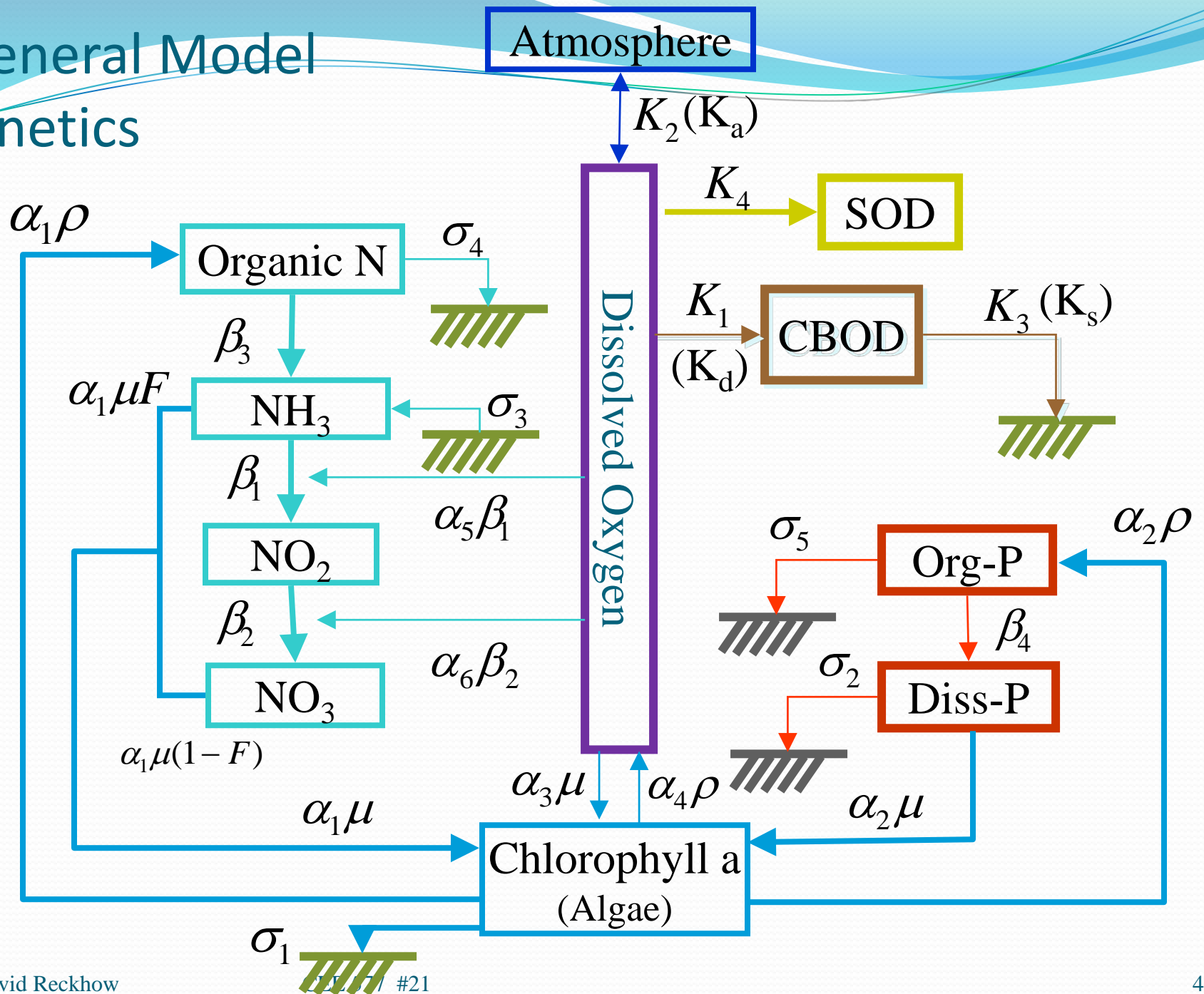
## ■ Streeter-Phelps:

## DiToro method (1975)

$$P = \left\{ \frac{0.5k_a [1 - e^{-k_a}]}{(1 - e^{-0.5k_a})^2} \right\} \Delta C$$

$$R = P + k_a \bar{D}$$

# General Model Kinetics



# Algal growth model

- General mass balance

$$\frac{dA}{dt} = \mu A - \rho A - \frac{\sigma_1}{z} A$$

- Algal specific growth rate
  - multiplicative model
  - limiting nutrient model
  - inverse additive model


$$\mu = \mu_{\max} (FL)(FN)(FP)$$

# Algal growth model

- General mass balance

Respiration rate (d<sup>-1</sup>)

Algal settling  
rate (ft/d)

$$\frac{dA}{dt} = \mu A - \rho A - \frac{\sigma_1}{z} A$$

- How do we calculate  $\mu$ ?  
Specific growth rate (d<sup>-1</sup>)

# Algal growth model

- Algal specific growth rate
  - multiplicative model

$$\mu = \mu_{\max} (FL)(FN)(FP)$$

All based on algal growth limitation factors: numbers between 0 and 1

- limiting nutrient model

- if N is limiting  $\mu = \mu_{\max} (FL)(FN)$

- if P is limiting  $\mu = \mu_{\max} (FL)(FP)$

- inverse additive model

$$\mu = \mu_{\max} (FL) \left[ \frac{2}{1/FN + 1/FP} \right]$$

# Algal growth model

- Algal growth limitation factors

- Nitrogen

$$FN = \frac{N_e}{N_e + K_N}$$

- where the half velocity constant for N is  $K_N$

- and the available nitrogen is:  $N_e = \text{NH}_3\text{-N} + \text{NO}_3\text{-N}$

- Phosphorus

$$FP = \frac{P_2}{P_2 + K_P}$$

- and the half velocity constant for P is  $K_P$

- and the available phosphorus ( $P_2$ ) is dissolved-P



# Algal light functions

- Types
  - Half saturation
  - Smith's function
  - Steel's equation
- Light-depth function

$$FL_z = \frac{I_z}{I_z + K_L}$$

$$FL_z = \frac{I_z}{\sqrt{I_z^2 + K_L^2}}$$

$$FL_z = \frac{I_z}{K_L} \exp\left(1 - \frac{I_z}{K_L}\right)$$

Extinction coefficient ( $\text{ft}^{-1}$ )

$$I_z = I_o e^{-\lambda z}$$

# The light attenuation factor

$$\phi_l = \frac{2.718f}{k_e H} (e^{-\alpha_1} - e^{-\alpha_0})$$

Where:

$$\alpha_1 = \frac{I_a}{I_s} e^{-k_e H}$$
$$\alpha_0 = \frac{I_a}{I_s}$$

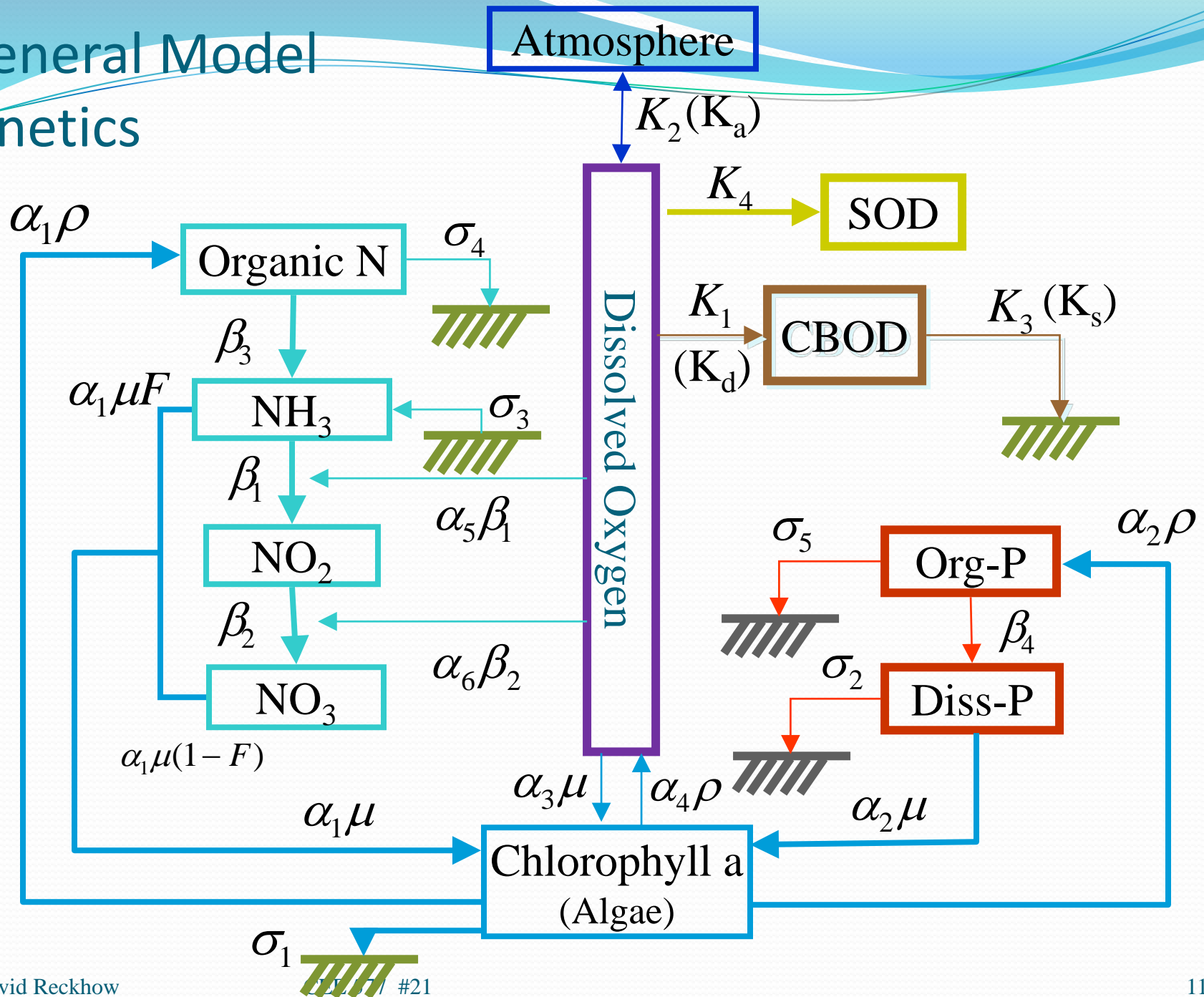
Extinction coefficient ( $\text{m}^{-1}$ )

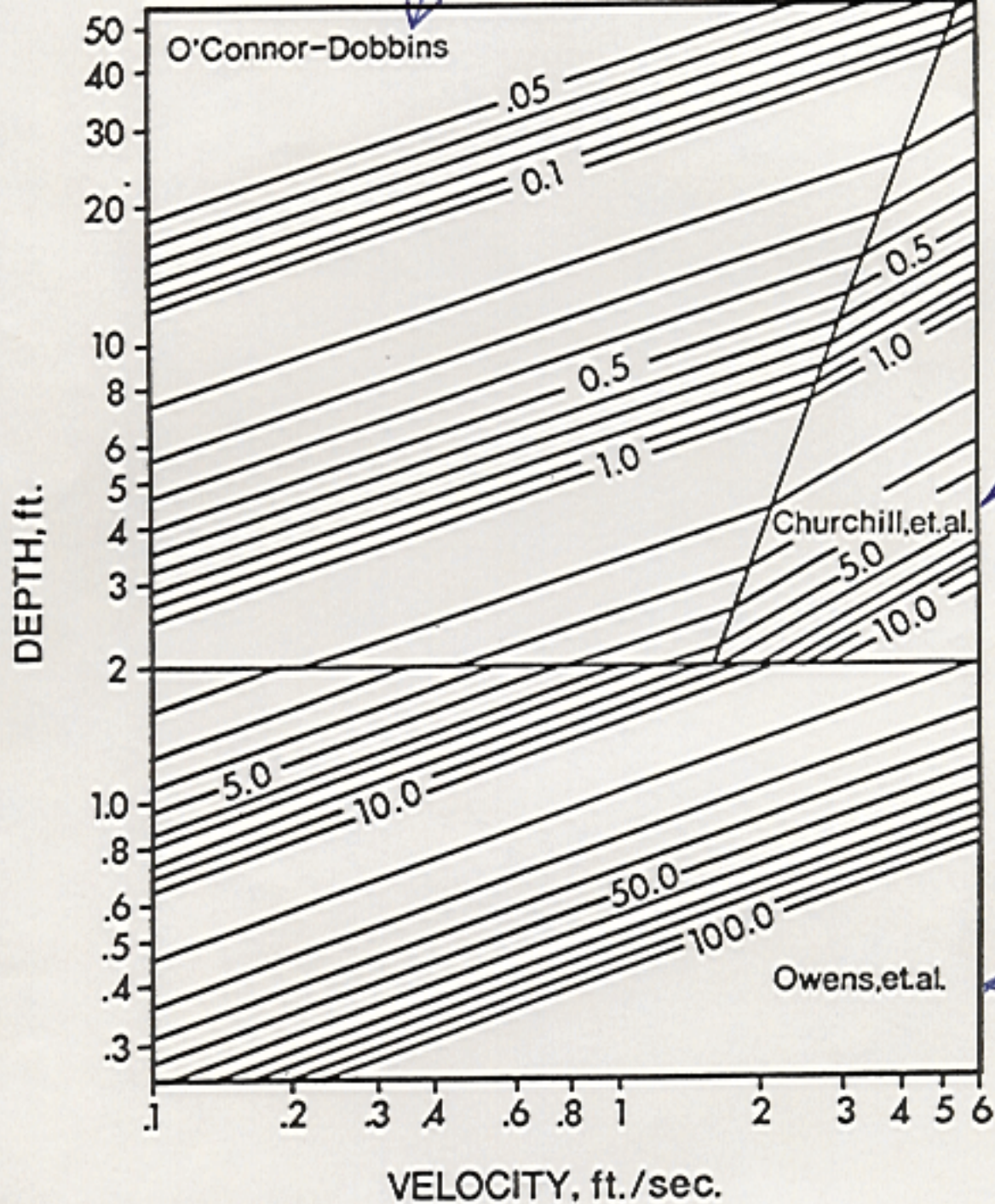
Avg. daylight intensity

And:  $k_e = \frac{1.8}{SD}$

Optimal intensity  
for plant growth  
(250-500  $\mu\text{y/d}$ )

# General Model Kinetics





## The method of Covar (1976)

**Values for  $k_a$  are in units of  $d^{-1}$**

# Reaeration

- Eight Options in QUAL2E

- 1 = specify value

- 2 = Churchill Formula

$$k_a = 11.6 \frac{U}{H^{1.67}}$$

- 3 = O'Connor Dobbins Formula

$$k_a = 12.9 \frac{U^{0.5}}{H^{1.5}}$$

- 4 = Owens Formula

$$k_a = 21.6 \frac{U^{0.67}}{H^{1.85}}$$

- 5 = Thackston & Krenkel

- developed for TVA's narrow streams with low DO, requires "n", generally good equation

- froude number
- shear velocity

$$F = \frac{U^*}{\sqrt{gz}}$$

$$k = 25(1 + F^{0.5}) \left( \frac{U^*}{z} \right)$$

$$U^* = \frac{Un\sqrt{g}}{1.49z^{1.167}}$$

# Reaeration

- Eight options cont.
  - 6 = Langbien & Durum
    - for  $z > 2\text{ft}$  (synthesis of others)
  - 7 = Power Function of Flow
  - 8 = Tsivoglou & Wallace
    - requires escape coefficient
      - $c = 0.054 \text{ ft}^{-1}$  for  $Q = 15\text{-}3000 \text{ cfs}$
      - $c = 0.110 \text{ ft}^{-1}$  for  $Q = 1\text{-}15 \text{ cfs}$

$$k = 7.62 \frac{U}{z^{1.33}}$$

$$k = aQ^b$$

$$k = 38,900n^2 \frac{U^3}{z^{1.333}} c$$

# Waste Loading

- Point Sources
  - Municipal WW
  - Industrial WW
  - Tributaries
- Non-point Sources
  - Agricultural
  - Silvicultural
  - Atmospheric
  - Urban & Suburban Runoff

# Loading Calculations

## Point Sources - General Concepts

$$W(t) = Q(t) \cdot c(t)$$

Diagram illustrating the loading calculation equation  $W(t) = Q(t) \cdot c(t)$  with units:

- $W(t)$  is labeled with  $\text{lb/d or kg/d}$ .
- $Q(t)$  is labeled with  $\text{ft}^3/\text{s or L/d}$ .
- $c(t)$  is labeled with  $\text{mg/L}$ .

Important Conversion Factors:

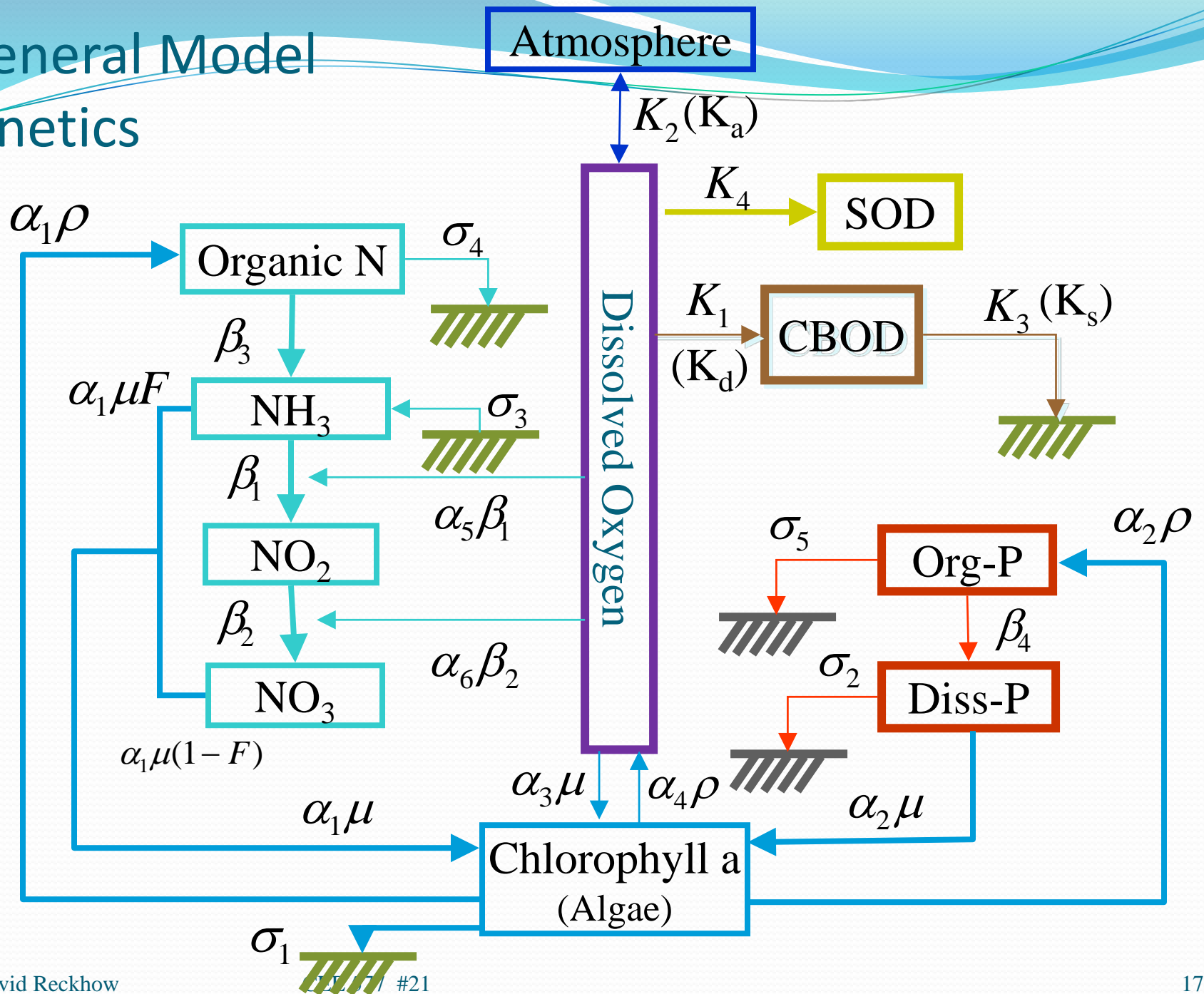
$$8.34 \frac{\text{lb} \cdot \text{liters}}{\text{mg} \cdot \text{MG}}$$

$$5.39 \frac{\text{lb} \cdot \text{liters} \cdot \text{sec}}{\text{mg} \cdot \text{ft}^3 \cdot \text{day}}$$

$$2.45 \frac{\text{Kg} \cdot \text{liters} \cdot \text{sec}}{\text{mg} \cdot \text{ft}^3 \cdot \text{day}}$$



# General Model Kinetics



- To next lecture