Updated: 23 October 2017

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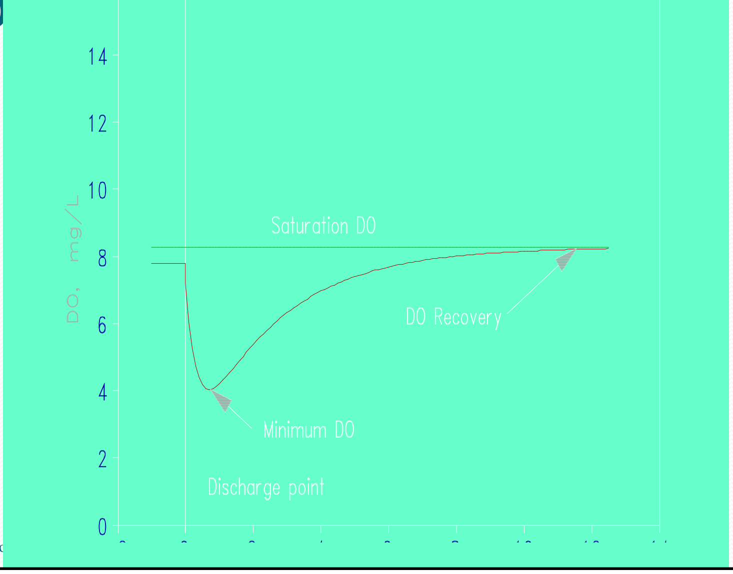
CEE 577: Surface Water Quality Modeling

Lecture #16

Streeter-Phelps: Reaeration & Dams

(Chapra, L22 & L23)

Typical DO Sag Curve



Streeter Phelps Equation

$$V\frac{dD}{dt} = k_dV\boxed{L} - k_aVD \longrightarrow L = L_oe^{-k_dt}$$

This equation can be solved by separation of variables and integration, or by use of an integrating factor. The boundary condition is t = 0 @ D = D_o. This yields the DO sag

$$D = D_oe^{-k_at} + \frac{k_dL_o}{k_a - k_r}\left(e^{-k_rt} - e^{-k_at}\right)$$

where

D = stream deficit at time t, [mg/L]

D_o = initial oxygen deficit (@ t = 0), [mg/L]

And recognizing that: t=x/U

$$D = D_oe^{-k_a x/U} + \frac{k_dL_o}{k_a - k_r}\left(e^{-k_r x/U} - e^{-k_a x/U}\right)$$

Anaerobic Conditions?

Critical Time

- The most stress is placed on the aquatic life in a stream when the DO is at a minimum, or the deficit, D, is a maximum. This occurs when dD/dt = 0. We can obtain the time at which the deficit is a maximum by taking the derivative of the DO sag equation with respect to t and setting it equal to zero, then solving for t. This yields,

$$t_{crit} = \frac{1}{k_a - k_r} \ln\left[\frac{k_a}{k_r}\left(1 - \frac{D_o(k_a - k_r}{k_d L_o}\right)\right]$$

t_{crit} = time at which maximum deficit (minimum DO) occurs, [days]

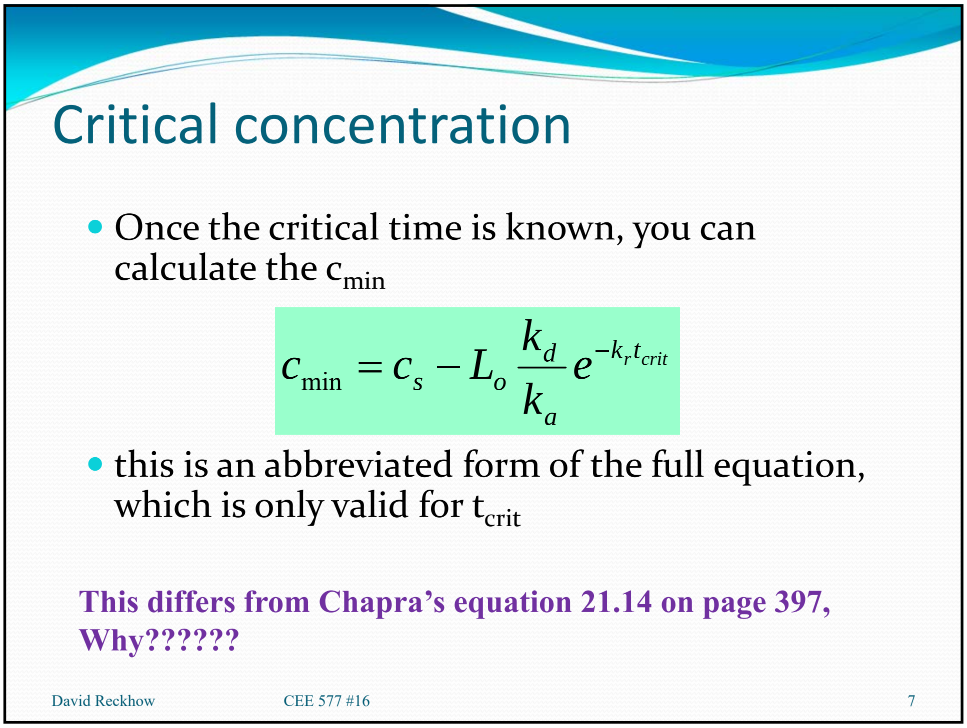
Special Case for D and t_c

- When k_a and k_r are equal:

$$t_c = \frac{1}{k_r}\left(1 - \frac{D_0}{L_o}\right)$$

$$D = \left(L_o k_r t + D_o\right)e^{-k_r t}$$

From Davis & Masten, page 290-291



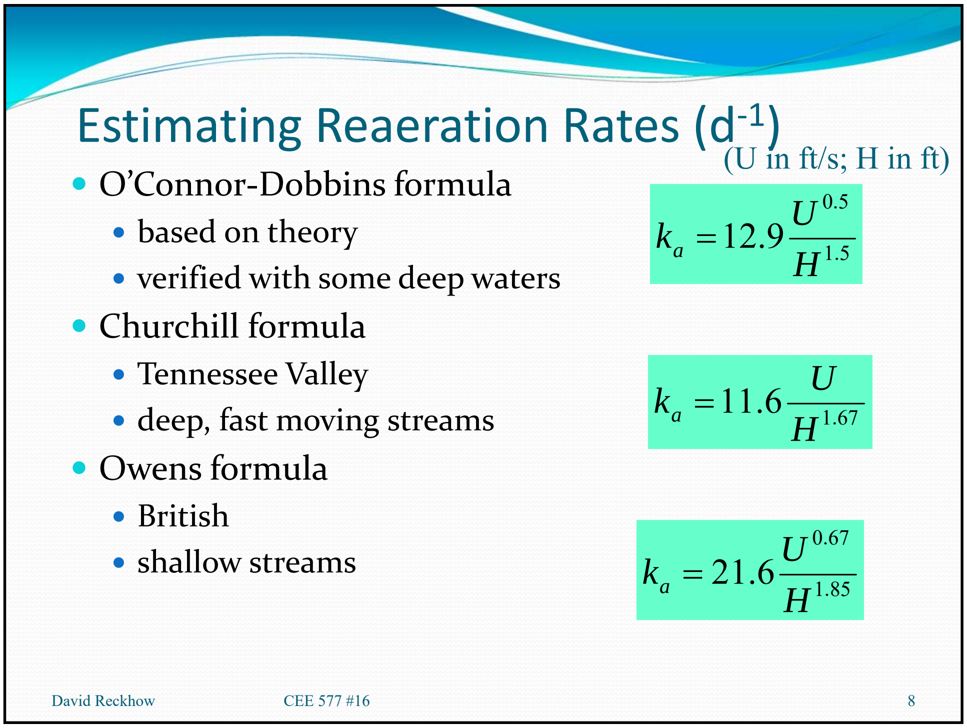
Critical concentration

- Once the critical time is known, you can calculate the $c_{\min}$

$$c_{\min} = c_s - L_o \frac{k_d}{k_a} e^{-k_r t_{crit}}$$

- this is an abbreviated form of the full equation, which is only valid for t_{crit}

This differs from Chapra's equation 21.14 on page 397, Why??????



Estimating Reaeration Rates (d^{-1})

(U in ft/s; H in ft)

- O'Connor-Dobbins formula
  - based on theory
  - verified with some deep waters

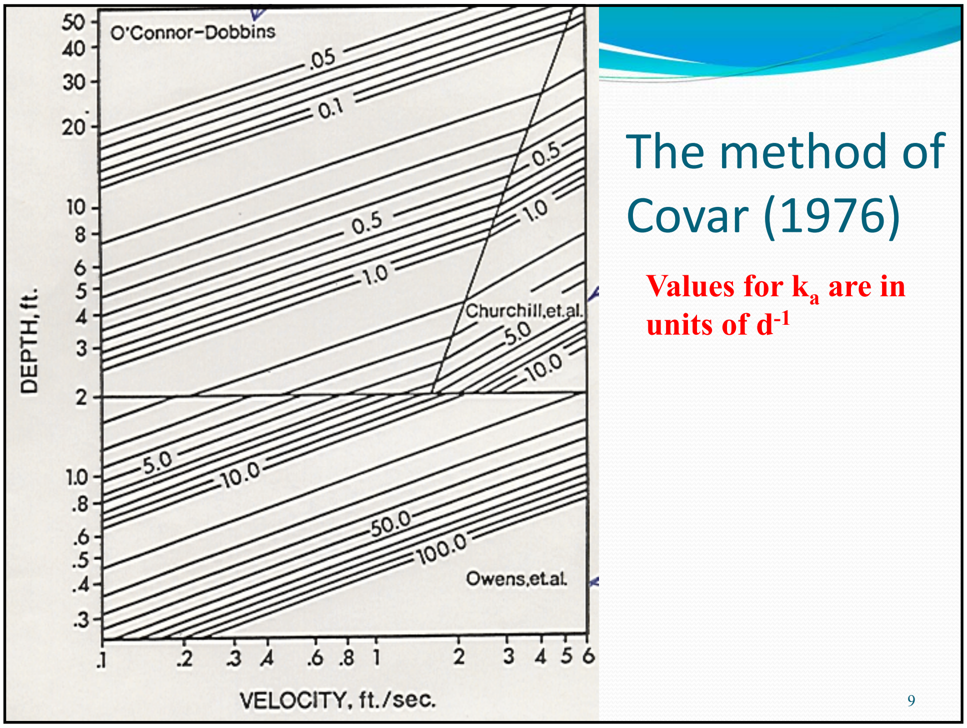
$$k_a = 12.9 \frac{U^{0.5}}{H^{1.5}}$$

- Churchill formula
  - Tennessee Valley
  - deep, fast moving streams

$$k_a = 11.6 \frac{U}{H^{1.67}}$$

- Owens formula
  - British
  - shallow streams

$$k_a = 21.6 \frac{U^{0.67}}{H^{1.85}}$$



The method of Covar (1976)

Values for k_a are in units of d^{-1}

Using the Covar approach

- Determine proper Domain

H (ft)	U (ft/s)	Formula
<2	Any	Owens
>2	$<1.2H^{0.34}$	O'Connor-Dobbins
>2	$>1.2H^{0.34}$	Churchill

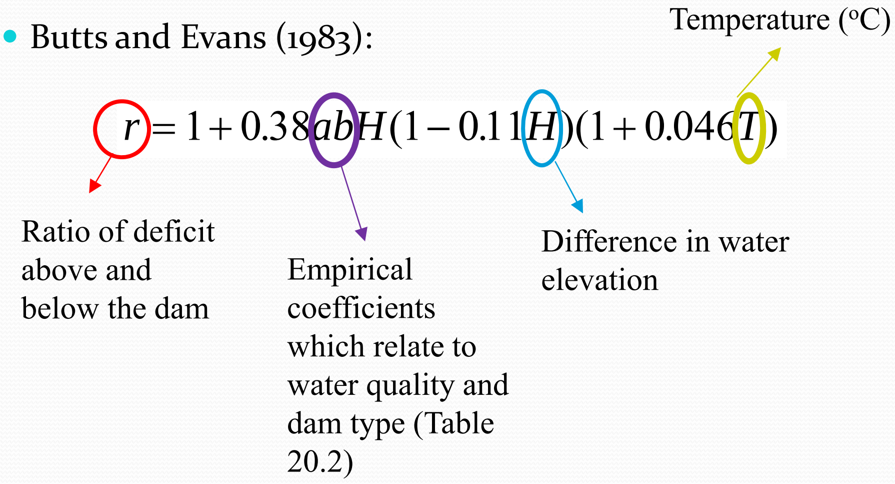
More on k_a estimation

- Tsivoglou & Wallace (1972) method
  - $k_a = 0.88US$, for Q = 10-300 cfs
  - $k_a = 1.8US$, for Q = 1-10 cfs
- Temperature correction
  - θ=1.024

$$k_T = k_{20^o C}\theta^{T-20^o C}$$

Dam Reaeration

- Butts and Evans (1983):



Temperature (°C)

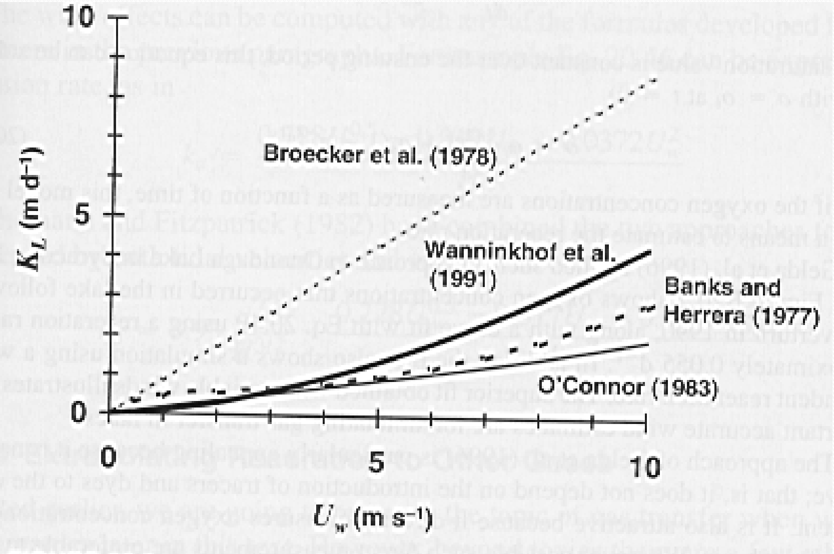
$$\boxed{r} = 1 + 0.38\boxed{ab}H(1-0.11\boxed{H})(1+0.046\boxed{T})$$

Ratio of deficit above and below the dam

Empirical coefficients which relate to water quality and dam type (Table 20.2)

Difference in water elevation

Wind dependent reaeration formulas



- To next lecture