Lecture #16

Streeter-Phelps: Reaeration & Dams
(Chapra, L22 & L23)
Typical DO Sag Curve

- Saturation DO
- DO Recovery
- Minimum DO
- Discharge point

DO, mg/L
This equation can be solved by separation of variables and integration, or by use of an integrating factor. The boundary condition is \( t = 0 @ D = D_o \). This yields the DO sag

\[
D = D_o e^{-k_a t} + \frac{k_d L_o}{k_a - k_r} \left( e^{-k_r t} - e^{-k_a t} \right)
\]

where

- \( D \) = stream deficit at time \( t \), [mg/L]
- \( D_o \) = initial oxygen deficit (at \( t = 0 \)), [mg/L]
And recognizing that: $t = \frac{x}{U}$

$$D = D_0 e^{-\frac{k_a x}{U}} + \frac{k_d L_o}{k_a - k_r} \left( e^{-\frac{k_r x}{U}} - e^{-\frac{k_a x}{U}} \right)$$
Critical Time

The most stress is placed on the aquatic life in a stream when the DO is at a minimum, or the deficit, D, is a maximum. This occurs when $dD/dt = 0$. We can obtain the time at which the deficit is a maximum by taking the derivative of the DO sag equation with respect to $t$ and setting it equal to zero, then solving for $t$. This yields,

$$t_{crit} = \frac{1}{k_a - k_r} \ln \left( \frac{k_a}{k_r} \left( 1 - \frac{D_o (k_a - k_r)}{k_d L_o} \right) \right)$$

$t_{crit} =$ time at which maximum deficit (minimum DO) occurs, [days]
Special Case for $D$ and $t_c$

- When $k_a$ and $k_r$ are equal:

$$t_c = \frac{1}{k_r} \left( 1 - \frac{D_0}{L_o} \right)$$

$$D = \left( L_o k_r t + D_0 \right) e^{-k_r t}$$

From Davis & Masten, page 290-291
Critical concentration

- Once the critical time is known, you can calculate the $c_{\text{min}}$

\[ c_{\text{min}} = c_s - L_o \frac{k_d}{k_a} e^{-k_r t_{\text{crit}}} \]

- This is an abbreviated form of the full equation, which is only valid for $t_{\text{crit}}$

This differs from Chapra’s equation 21.14 on page 397, Why???????
Estimating Reaeration Rates (d^{-1})

- O’Connor-Dobbins formula
  - based on theory
  - verified with some deep waters

- Churchill formula
  - Tennessee Valley
  - deep, fast moving streams

- Owens formula
  - British
  - shallow streams
The method of Covar (1976)

Values for $k_a$ are in units of $d^{-1}$
Using the Covar approach

- Determine proper Domain

<table>
<thead>
<tr>
<th>$H$ (ft)</th>
<th>$U$ (ft/s)</th>
<th>Formula</th>
</tr>
</thead>
<tbody>
<tr>
<td>&lt;2</td>
<td>Any</td>
<td>Owens</td>
</tr>
<tr>
<td>&gt;2</td>
<td>&lt;1.2$H^{0.34}$</td>
<td>O’Connor-Dobbins</td>
</tr>
<tr>
<td>&gt;2</td>
<td>&gt;1.2$H^{0.34}$</td>
<td>Churchill</td>
</tr>
</tbody>
</table>
More on $k_a$ estimation

- Tsivoglou & Wallace (1972) method
  - $k_a = 0.88 \text{US}$, for $Q = 10-300 \text{ cfs}$
  - $k_a = 1.8 \text{US}$, for $Q = 1-10 \text{ cfs}$

- Temperature correction
  - $\theta = 1.024$

$$k_T = k_{20^\circ C} \theta^{T-20^\circ C}$$
Dam Reaeration

- Butts and Evans (1983):

\[ r = 1 + 0.38abH(1 - 0.11H)(1 + 0.046T) \]

- Ratio of deficit above and below the dam
- Empirical coefficients which relate to water quality and dam type (Table 20.2)
- Difference in water elevation
- Temperature (°C)
Wind dependent reaeration formulas

![Graph showing wind dependent reaeration formulas](image-url)
To next lecture