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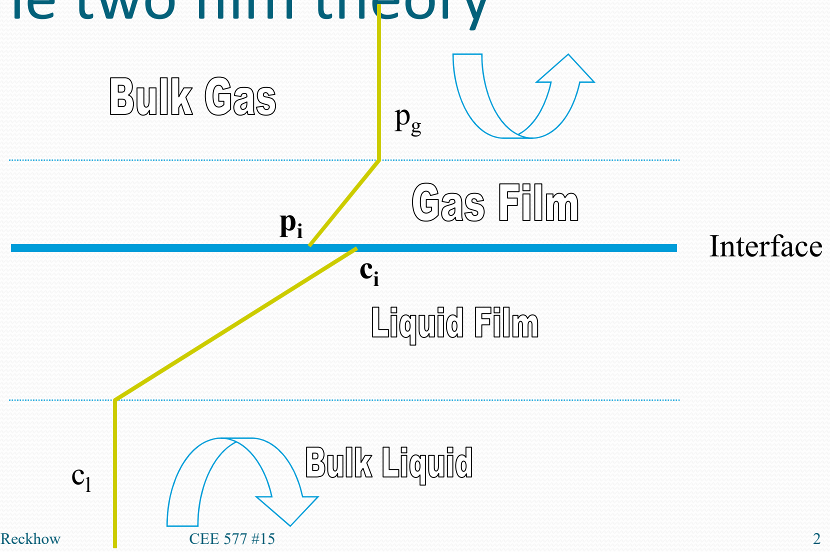
CEE 577: Surface Water Quality Modeling

Lecture #15
Gas Transfer

(Chapra, L20)

The two film theory



Bulk Gas

p_g

Gas Film

p_i

Interface

c_i

Liquid Film

c_l

Bulk Liquid

Two film model

Universal Gas Law

$$\boxed{\frac{n}{V}} = \frac{P}{RT_a}$$

Molar concentration

- Flux from the bulk liquid to the interface

$$J_l = K_l(c_i - c_l)$$

- Flux from the interface to the bulk gas

Mass transfer velocities (m/d)

$$J_g = \frac{K_g}{RT_a}(p_g - p_i)$$

  - And the K's are related to the molecular diffusion coefficients by:

$$K_l = \frac{D_l}{z_l} \qquad K_g = \frac{D_g}{z_g}$$

Two film theory (cont.)

- We want to be able to relate flux to bulk air and water concentrations
  - interface concentrations cannot be directly measured

$$J = v_v\left(\frac{p_g}{H_e} - c_l\right)$$

- to do this we must substitute expressions for the interface concentrations

Whitman's 2 film model (cont.)

- According to Henry's law: $p_i = H_e c_i$
- And relating this back to the bulk concentration

$$p_i = H_e\left(\frac{J_l}{K_l} + c_l\right)$$

Recall: $J_l = K_l(c_i - c_l)$

So: $c_i = \frac{J_l}{K_l} + c_l$

- now solving and equating the fluxes, we get (pg. 371 in text):

$$\frac{1}{v_v} = \frac{1}{K_l} + \frac{RT_a}{H_e K_g}$$

The net transfer velocity across the air-water interface (m/d)

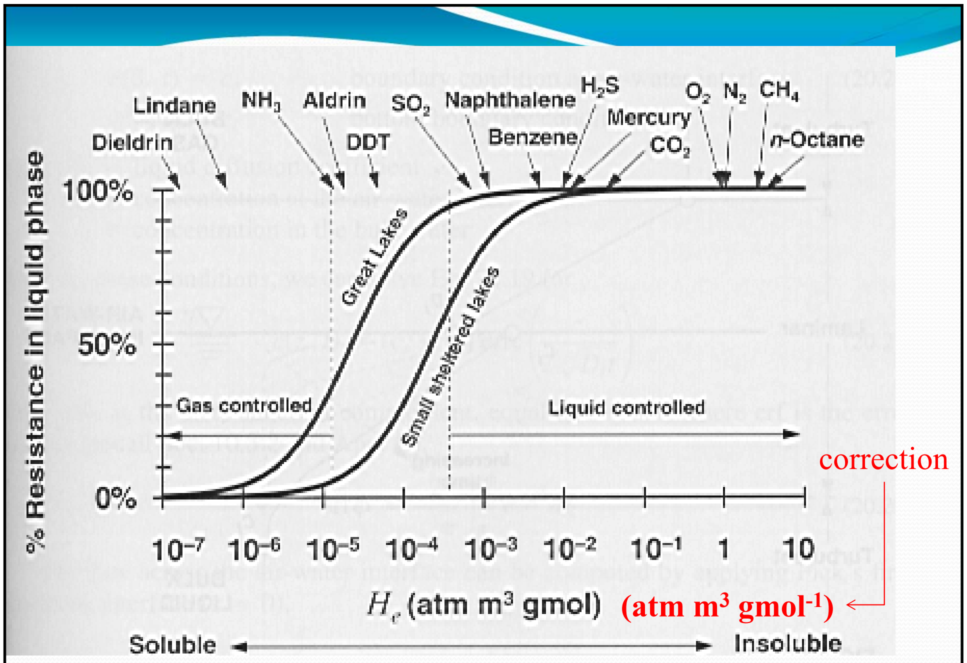
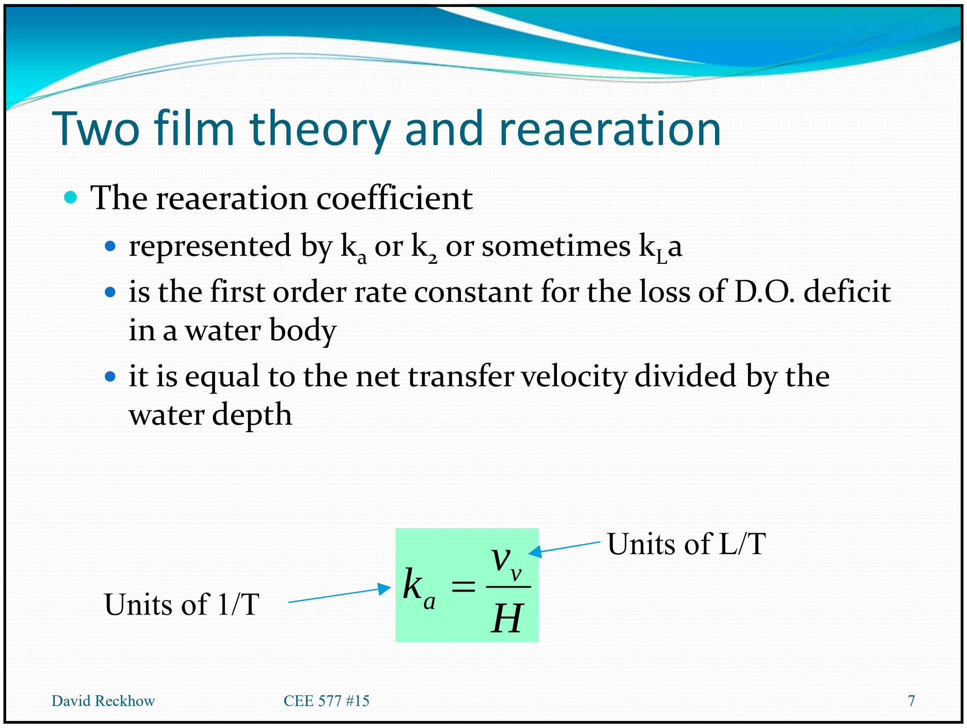


Figure 20.4, page 373 in text.



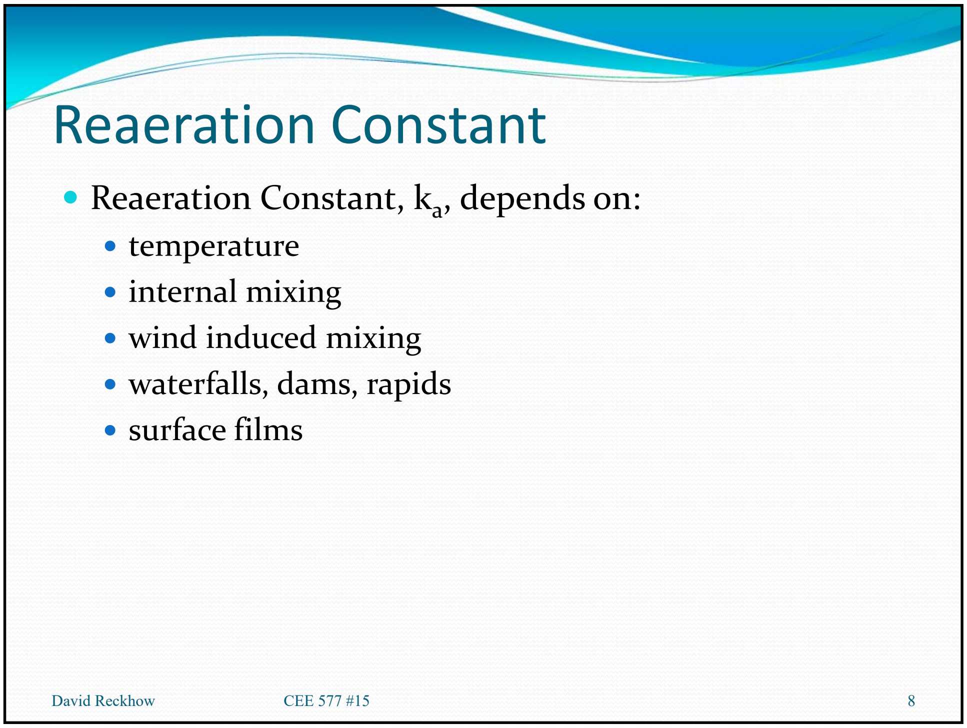
Two film theory and reaeration

- The reaeration coefficient
  - represented by k_a or k_2 or sometimes k_La
  - is the first order rate constant for the loss of D.O. deficit in a water body
  - it is equal to the net transfer velocity divided by the water depth

Units of 1/T

$$k_a = \frac{v_v}{H}$$

Units of L/T



Reaeration Constant

- Reaeration Constant, k_a, depends on:
  - temperature
  - internal mixing
  - wind induced mixing
  - waterfalls, dams, rapids
  - surface films

DO Deficit

$$D = DO_{sat} - DO_{act}$$

where

D = oxygen deficit, [mg/L]

DO_{sat} = saturation value of dissolved oxygen, [mg/L]

DO_{act} = actual dissolved oxygen value for the stream, [mg/L]

DO Deficit Mass Balance

- Let us assume that the rate of oxygen entering the stream through the atmosphere is proportional to the dissolved oxygen deficit in the stream. Similarly, let us assume that the rate of oxygen consumed or leaving the stream is proportional to the amount of organic matter in the stream, expressed as BOD_u (ultimate BOD).

$$\frac{dD}{dt} = k_d L - k_a D$$

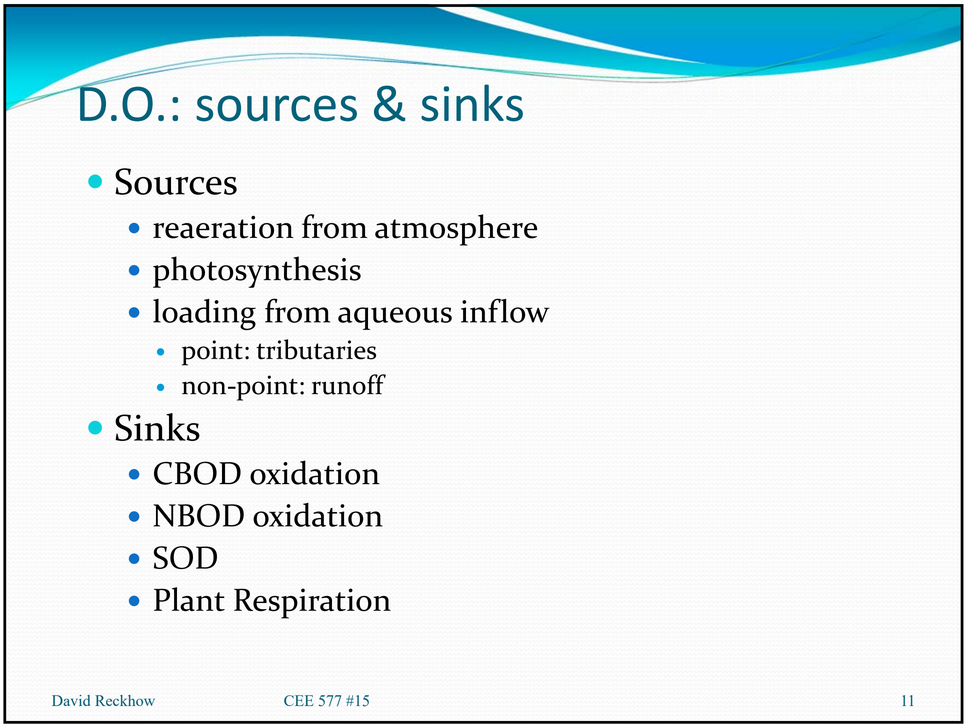
Where:

t = time, [days]

L = ultimate stream BOD, [mg/L]

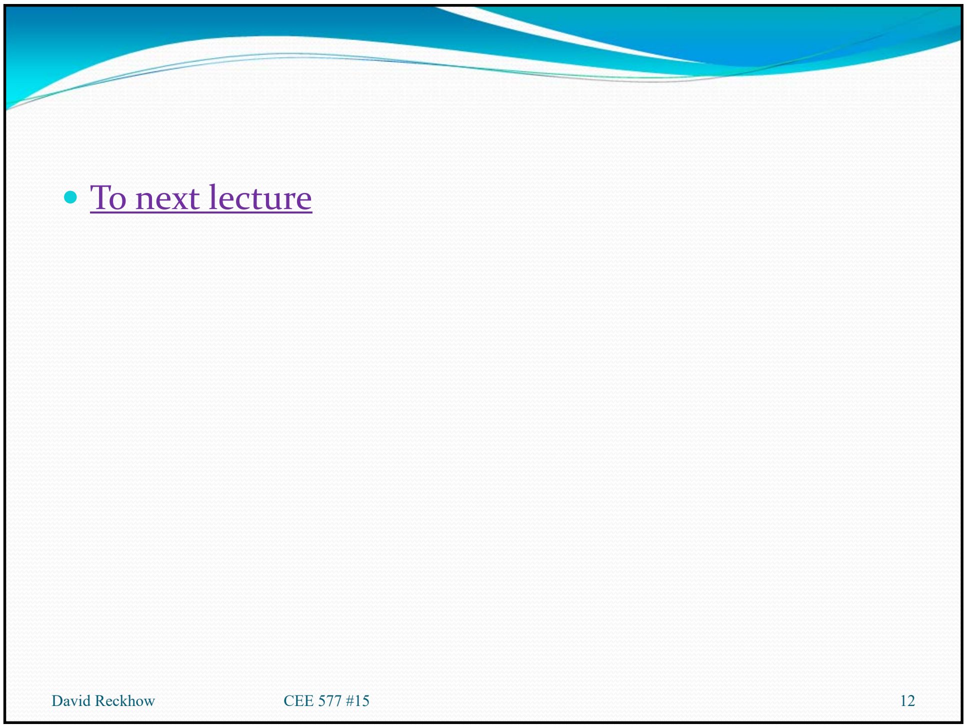
k_d = deoxygenation constant, [day^{-1}]

k_a = reaeration constant, [day^{-1}]



D.O.: sources & sinks

- Sources
  - reaeration from atmosphere
  - photosynthesis
  - loading from aqueous inflow
    - point: tributaries
    - non-point: runoff
- Sinks
  - CBOD oxidation
  - NBOD oxidation
  - SOD
  - Plant Respiration



- To next lecture