Updated: 23 October 2017

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CEE 577: Surface Water Quality Modeling

Lecture #13

BOD and Deoxygenation

(Chapra, L20)

BOD Modeling

"L" is modelled as a simple 1st order decay: $\frac{dL}{dt} = -k_1 L$

Which leads to: $L = L_o e^{-k_1 t}$

And combining with: $BOD_t \equiv y_t = L_o - L_t$

We get: $BOD_t \equiv y_t = L_o (1 - e^{-k_1 t})$

Temperature Effects

Temperature Dependence

- Chemist's Approach: Arrhenius Equation

$$\frac{d(\ln k)}{dT_a} = \frac{E_a}{RT_a^2}$$

$$k_{T_a} = k_{293^o K} e^{E_a (T_a - 293)/RT_a 293}$$

- Engineer's Approach:

$$k_T = k_{20^o C} \theta^{T-20^o C}$$

Often we use:
θ=1.047
for CBOD

NBOD

Nitrogeneous BOD (NBOD)

$$NH_3 + 1.5O_2 \xrightarrow{Nitrosomonas} NO_2^- + H_2O + H^+$$

$$NO_2^- + \frac{1}{2}O_2 \xrightarrow{Nitrobacter} NO_3^-$$

2 moles oxygen/1 mole of ammonia
4.57 grams oxygen/gram ammonia-nitrogen

Like CBOD, the NBOD can be modelled as a simple 1st order decay:

$$\frac{dL^N}{dt} = -k_N L^N$$

NBOD cont.

- The model is then:

$$NBOD_t = L_o^N\left(1 - e^{-k_N t}\right)$$

  - where:

$$L_o^N \equiv NBOD_u = 4.57\left(org - N + NH_3 - N\right)$$

- Nitrifiers
  - very slow generation time (~1 day)
  - sensitive to low D.O.
- NBOD may be very important for non-nitrified, but otherwise highly treated waters

Typical Municipal WW Charact.

Parameter	Typical Wastewater Characteristics, mg/L except pH	U.S. EPA Discharge Standards, mg/L except pH	Typical Concentrations in Lakes or Streams, mg/L except pH
BOD_5	150-300	30	2-10
Total Suspended Solids	150-300	30	2-20
COD	400-600	N/A	5-50
D.O.	0	4-5	4-Sat.
NH_3-N	15-40	*	<1
NO3	0	*	<1
pH	6-8	6-9	6-8

BOD Model

$$\frac{\partial L}{\partial t} = -U\frac{\partial L}{\partial x} - k_r L$$

Where the total removal rate is:

$$k_r = \underbrace{k_d}_{\text{Decomposition rate in the stream}} + \underbrace{k_s}_{\text{Settling rate}}$$

Settling rate

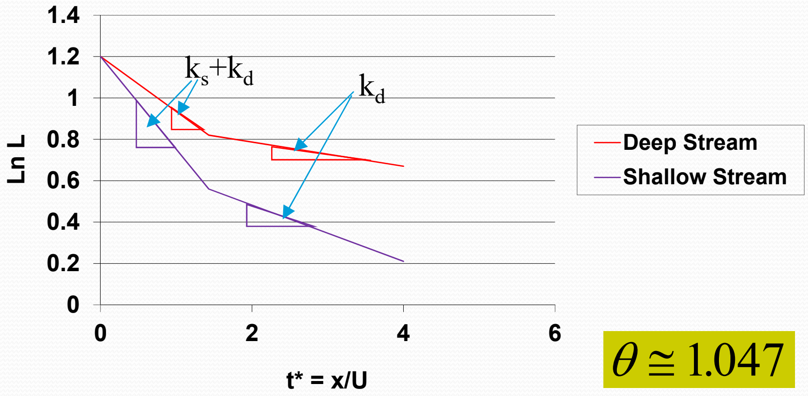
$$= \frac{v_s}{H}$$

Decomposition rate in the stream

At steady state: $\frac{\partial L}{\partial t} = 0$

$$L = L_o e^{-k_r x/U}$$

Estimating the k's



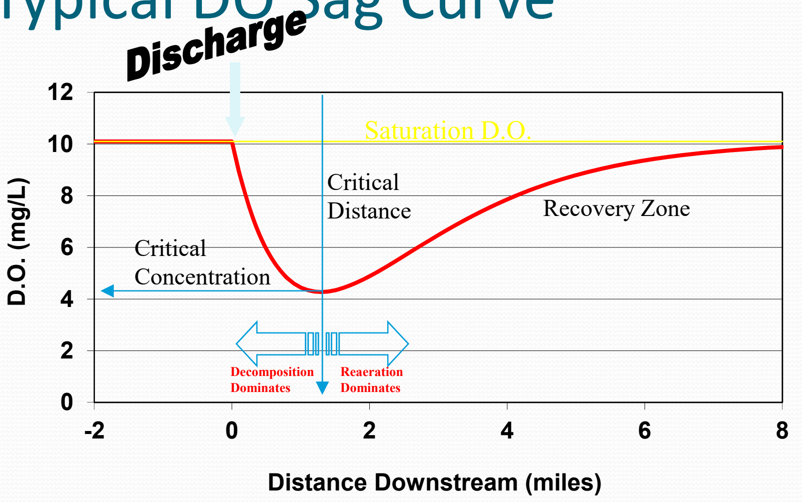
$$\theta \cong 1.047$$

$$k_d = C\left(\frac{H}{8}\right)^{-0.434}, \quad 0 \le H \le 8\,ft$$

$$k_d = C, \quad H > 8\,ft$$

Where: C=0.2 for unstable bottoms
C=0.3 for rocky bottoms

Typical DO Sag Curve



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Lecture #22

(Distributed Systems, time variable. Dye Studies)

Chapra, L10

Plug Flow (time variable)

- Simulating accidental spill, tracer studies

$$\frac{\partial c}{\partial t} = -U\frac{\partial c}{\partial x} - kc$$

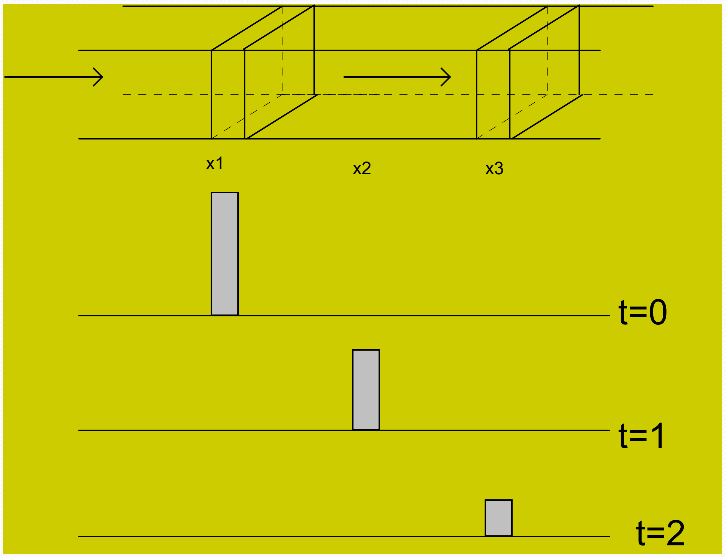
- when a spill causes a concentration of c_o at x=0 and t=0

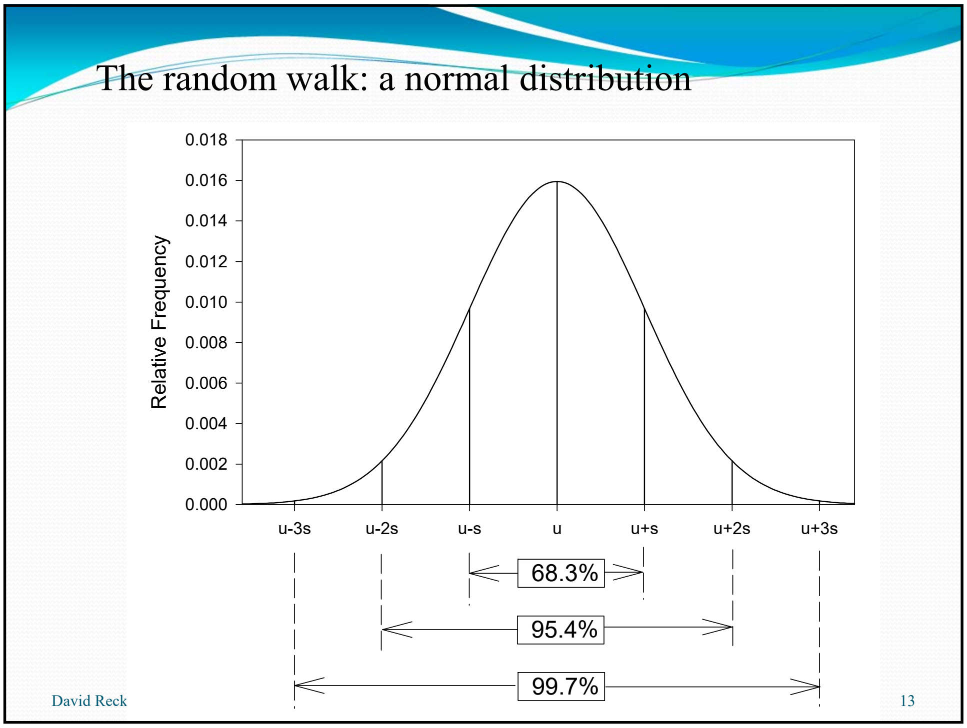
$$c = c_o e^{-kt^*}$$

Where: $t^*=x/U$

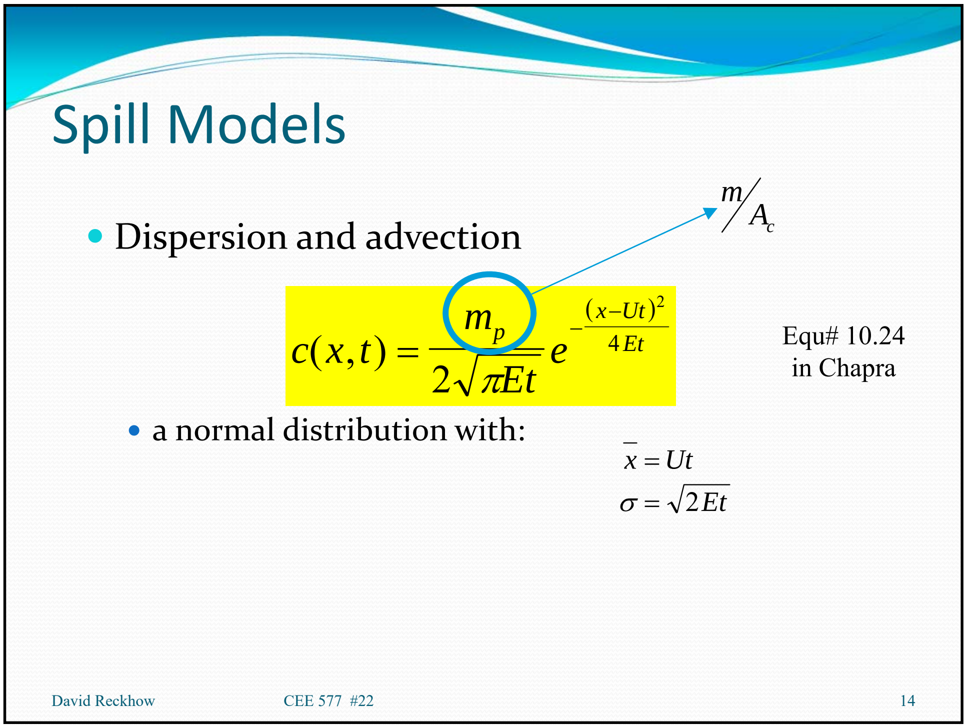
Refer to Example 10.1

Moving vs. Fixed frame of reference





The random walk: a normal distribution



Spill Models

- Dispersion and advection

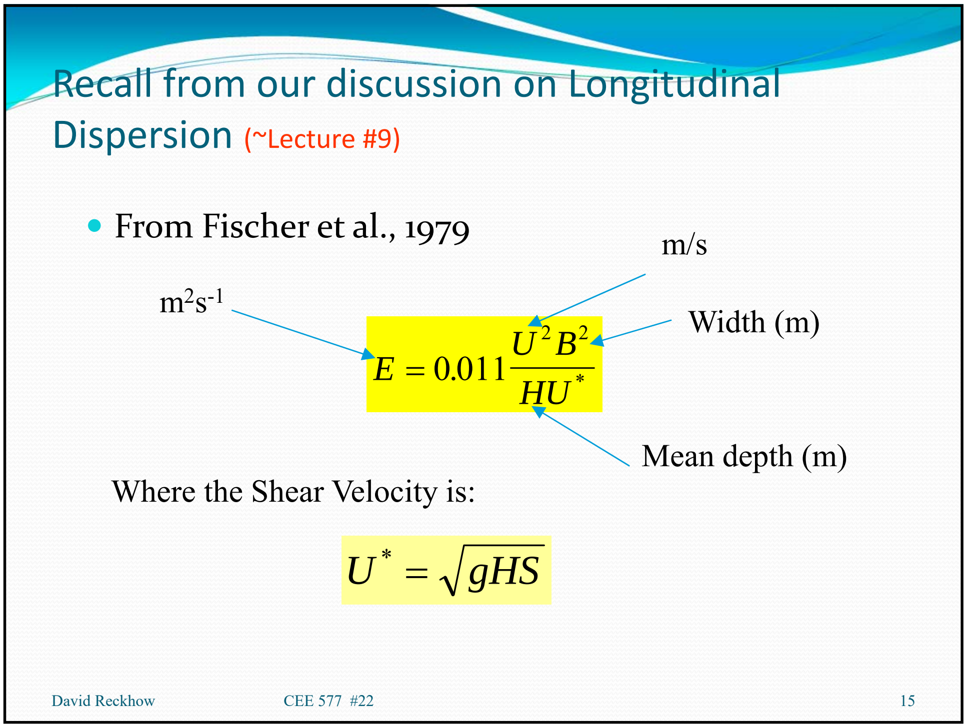
$$c(x,t) = \frac{m_p}{2\sqrt{\pi E t}} e^{-\frac{(x-Ut)^2}{4Et}}$$

m/A_c

Equ# 10.24
in Chapra

  - a normal distribution with:

$$\bar{x} = Ut$$
$$\sigma = \sqrt{2Et}$$



Recall from our discussion on Longitudinal Dispersion (~Lecture #9)

- From Fischer et al., 1979

m^2s^{-1}

m/s

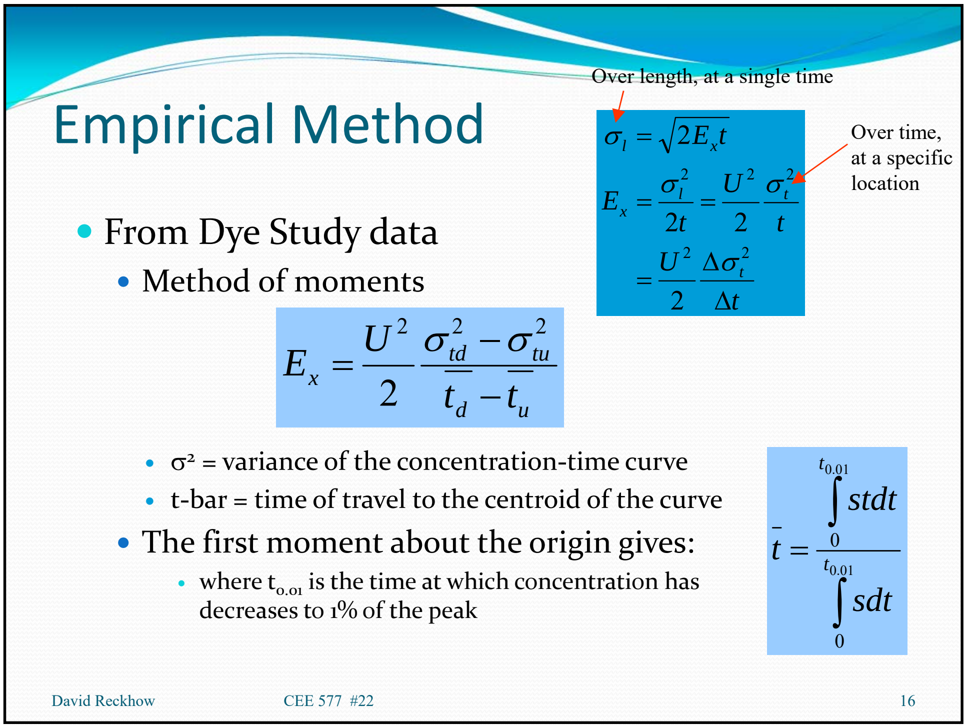
Width (m)

$$E = 0.011\frac{U^2B^2}{HU^*}$$

Mean depth (m)

Where the Shear Velocity is:

$$U^* = \sqrt{gHS}$$



Empirical Method

Over length, at a single time

$$\sigma_l = \sqrt{2E_x t}$$

$$E_x = \frac{\sigma_l^2}{2t} = \frac{U^2}{2}\frac{\sigma_t^2}{t}$$

$$= \frac{U^2}{2}\frac{\Delta\sigma_t^2}{\Delta t}$$

Over time, at a specific location

- From Dye Study data
  - Method of moments

$$E_x = \frac{U^2}{2}\frac{\sigma_{td}^2 - \sigma_{tu}^2}{\bar{t}_d - \bar{t}_u}$$

  - σ^2 = variance of the concentration-time curve
  - t-bar = time of travel to the centroid of the curve
- The first moment about the origin gives:
  - where $t_{0.01}$ is the time at which concentration has decreases to 1% of the peak

$$\bar{t} = \frac{\int_0^{t_{0.01}} stdt}{\int_0^{t_{0.01}} sdt}$$

Empirical Method (cont.)

- And the 2nd moment about the centroid gives:

$$\sigma^2 = \frac{\int_0^{t_{0.01}} st^2 dt}{\int_0^{t_{0.01}} sdt} - (\bar{t})^2$$

- For discrete data these become:

$$\bar{t} = \frac{\sum st^{\Delta}t}{\sum s^{\Delta}t} = \frac{\sum st}{\sum s}$$

Compare to T&M equ 10.32

$$\sigma^2 = \frac{\sum st^{2\,\Delta}t}{\sum s^{\Delta}t} - (\bar{t})^2 = \frac{\sum st^2}{\sum s} - (\bar{t})^2$$

Compare to T&M equ 10.33

Dye Studies (cont.)

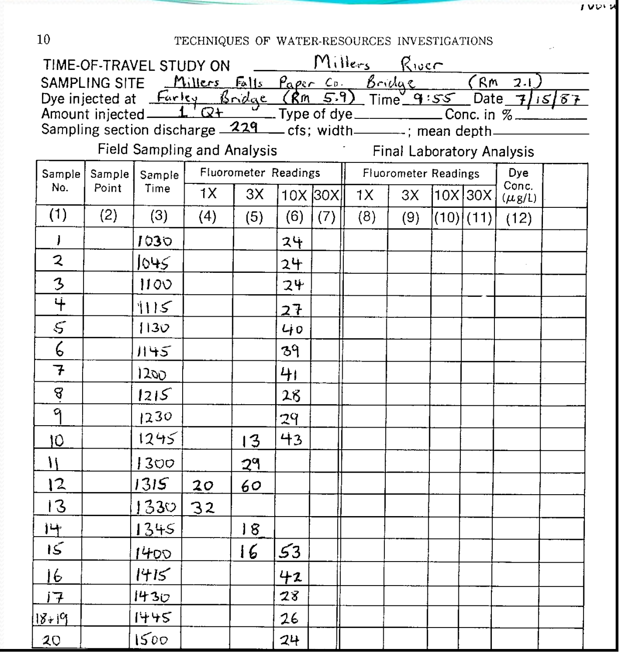
- Single point method
  - Use of peak concentration (c_p) and time to reach peak (t_p)

$$c_p = \frac{M}{2A\sqrt{\pi E_x}}(t_p)^{-0.5}$$

  - plot c_p vs $(t_p)^{-0.5}$ to get a slope that is a function of E_x
  - see sample problem 2.6 in T&M (pg. 78)

Homework #4

- Velocity
- dispersion coefficient



10 TECHNIQUES OF WATER-RESOURCES INVESTIGATIONS

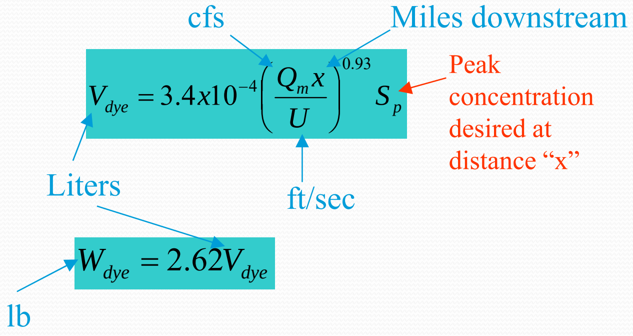
TIME-OF-TRAVEL STUDY ON Millers River
SAMPLING SITE Millers Falls Paper Co. Bridge (RM 2.1)
Dye injected at Farley Bridge (RM 5.9) Time 9:55 Date 7/15/87
Amount injected 1 Qt Type of dye ____ Conc. in % ____
Sampling section discharge 229 cfs; width ____; mean depth ____

Field Sampling and Analysis — Final Laboratory Analysis

Sample No.	Sample Point	Sample Time	Fluorometer Readings				Fluorometer Readings				Dye Conc. (µg/L)	
			1X	3X	10X	30X	1X	3X	10X	30X		
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	
1		1030			24							
2		1045			24							
3		1100			24							
4		1115			27							
5		1130			40							
6		1145			39							
7		1200			41							
8		1215			28							
9		1230			29							
10		1245		13	43							
11		1300		29								
12		1315	20	60								
13		1330	32									
14		1345		18								
15		1400		16	53							
16		1415			42							
17		1430			28							
18+19		1445			26							
20		1500			24							

USGS Guidelines: Input

- The USGS recommends the following volume or mass of Rhodamine WT-20% dye:



cfs

Miles downstream

$$V_{dye} = 3.4x10^{-4}\left(\frac{Q_m x}{U}\right)^{0.93} S_p$$

Peak concentration desired at distance “x”

Liters

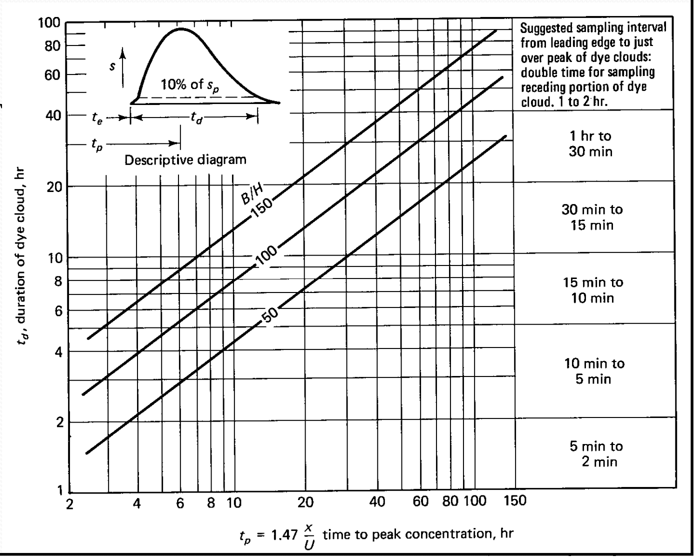
ft/sec

$$W_{dye} = 2.62V_{dye}$$

lb

USGS Guidelines: Sampling

- Duration of dye cloud as a function of travel time to peak, and average channel width-depth ratio (B/H).



- To next lecture