Lecture #13
BOD and Deoxygenation
(Chapra, L20)
BOD Modeling

"L" is modelled as a simple 1st order decay:

\[
\frac{dL}{dt} = -k_1 L
\]

Which leads to:

\[
L = L_o e^{-k_1 t}
\]

And combining with:

\[
BOD_t \equiv y_t = L_o - L_t
\]

We get:

\[
BOD_t \equiv y_t = L_o (1 - e^{-k_1 t})
\]
Temperature Effects

Temperature Dependence

- Chemist's Approach: Arrhenius Equation

\[
\frac{d(\ln k)}{dT_a} = \frac{E_a}{RT_a^2}
\]

\[
k_{T_a} = k_{293^\circ K} e^{E_a (T_a - 293)/RT_a} 293
\]

- Engineer's Approach:

\[
k_T = k_{20^\circ C} \theta^{T-20^\circ C}
\]

Often we use:  
θ = 1.047  
for CBOD
Nitrogenous BOD (NBOD)

\[ \text{Nitrosomonas} \quad NH_3 + 1.5O_2 \rightarrow NO_2^- + H_2O + H^+ \]

\[ \text{Nitroacter} \quad NO_2^- + \frac{1}{2}O_2 \rightarrow NO_3^- \]

2 moles oxygen/1 mole of ammonia
4.57 grams oxygen/gram ammonia-nitrogen

Like CBOD, the NBOD can be modelled as a simple 1st order decay:

\[ \frac{dL^N}{dt} = -k_N L^N \]
NBOD cont.

- The model is then:

\[
NBOD_t = L^N_e \left(1 - e^{-k_N t}\right)
\]

- where:

\[
L^N_e \equiv NBOD_u = 4.57(\text{org} - N + \text{NH}_3 - N)
\]

- Nitrifiers
  - very slow generation time (~1 day)
  - sensitive to low D.O.
- NBOD may be very important for non-nitrified, but otherwise highly treated waters
### Typical Municipal WW Charact.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Typical Wastewater Characteristics, mg/L except pH</th>
<th>U.S. EPA Discharge Standards, mg/L except pH</th>
<th>Typical Concentrations in Lakes or Streams, mg/L except pH</th>
</tr>
</thead>
<tbody>
<tr>
<td>BOD&lt;sub&gt;5&lt;/sub&gt;</td>
<td>150-300</td>
<td>30</td>
<td>2-10</td>
</tr>
<tr>
<td>Total Suspended Solids</td>
<td>150-300</td>
<td>30</td>
<td>2-20</td>
</tr>
<tr>
<td>COD</td>
<td>400-600</td>
<td>N/A</td>
<td>5-50</td>
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<tr>
<td>D.O.</td>
<td>0</td>
<td>4-5</td>
<td>4-Sat.</td>
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<tr>
<td>NH&lt;sub&gt;3&lt;/sub&gt;-N</td>
<td>15-40</td>
<td>*</td>
<td>&lt;1</td>
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<tr>
<td>NO&lt;sub&gt;3&lt;/sub&gt;</td>
<td>0</td>
<td>*</td>
<td>&lt;1</td>
</tr>
</tbody>
</table>
BOD Model

\[ \frac{\partial L}{\partial t} = -U \frac{\partial L}{\partial x} - k_r L \]

Where the total removal rate is:

\[ k_r = k_d + k_s \]

Decomposition rate in the stream

At steady state:

\[ \frac{\partial L}{\partial t} = 0 \]

\[ L = L_0 e^{-k_r x / U} \]

Settling rate

\[ \frac{v_s}{H} \]
Estimating the k’s

\[ \text{Ln } L \]

\[ t^* = \frac{x}{U} \]

- \( k_d \times k_s \)
- \( k_d \)

\[ \theta \approx 1.047 \]

Where: 
- \( C = 0.2 \) for unstable bottoms
- \( C = 0.3 \) for rocky bottoms

\[ k_d = C \left( \frac{H}{8} \right)^{-0.434}, \quad 0 \leq H \leq 8 \text{ ft} \]

\[ k_d = C, \quad H > 8 \text{ ft} \]
Typical DO Sag Curve

Discharge

-2 0 2 4 6 8 10 12

D.O. (mg/L)

Saturation D.O.

Critical Distance

Recovery Zone

Critical Concentration

Decomposition Dominates

Reaeration Dominates

Distance Downstream (miles)
CEE 577: Surface Water Quality Modeling

Lecture #22
(Distributed Systems, time variable. Dye Studies)
Chapra, L10
Plug Flow (time variable)

- Simulating accidental spill, tracer studies

\[ \frac{\partial c}{\partial t} = -U \frac{\partial c}{\partial x} - kc \]

- when a spill causes a concentration of \( c_0 \) at \( x=0 \) and \( t=0 \)

\[ c = c_0 e^{-kt^*} \]

Where: \( t^* = \frac{x}{U} \)

Refer to Example 10.1
Moving vs. Fixed frame of reference
The random walk: a normal distribution

![Graph of a normal distribution with relative frequencies and standard deviations highlighted.]

- 68.3%
- 95.4%
- 99.7%
Spill Models

- Dispersion and advection

\[
c(x, t) = \frac{m_p}{2\sqrt{\pi Et}} e^{-\frac{(x-Ut)^2}{4Et}}
\]

- a normal distribution with:

\[
\begin{align*}
\bar{x} &= Ut \\
\sigma &= \sqrt{2Et}
\end{align*}
\]

Equ# 10.24 in Chapra
Recall from our discussion on Longitudinal Dispersion (~Lecture #9)

- From Fischer et al., 1979

\[ E = 0.011 \frac{U^2 B^2}{H U^*} \]

Where the Shear Velocity is:

\[ U^* = \sqrt{gHS} \]
Empirical Method

- From Dye Study data
  - Method of moments

\[
E_x = \frac{U^2}{2} \frac{\sigma_{td}^2 - \sigma_{tu}^2}{t_d - t_u}
\]

- \(\sigma^2\) = variance of the concentration-time curve
- \(t\)-bar = time of travel to the centroid of the curve
- The first moment about the origin gives:
  - where \(t_{0.01}\) is the time at which concentration has decreases to 1% of the peak
Empirical Method (cont.)

- And the 2nd moment about the centroid gives:

\[
\sigma^2 = \frac{\int_0^{t_{0.01}} st^2 dt - (\bar{t})^2}{\int_0^{t_{0.01}} s dt}
\]

- For discrete data these become:

\[
\eta = \frac{\sum st \Delta t}{\sum s \Delta t} = \frac{\sum st}{\sum s}
\]

\[
\sigma^2 = \frac{\sum st^2 \Delta t}{\sum s^2 \Delta t} - (\bar{t})^2 = \frac{\sum st^2}{\sum s} - (\bar{t})^2
\]

Compare to T&M equ 10.32

Compare to T&M equ 10.33
Dye Studies (cont.)

- Single point method
  - Use of peak concentration \( (c_p) \) and time to reach peak \( (t_p) \)
  
  \[
  c_p = \frac{M}{2A\sqrt{\pi E_x}} \left( t_p \right)^{-0.5}
  \]

- plot \( c_p \) vs \( (t_p)^{-0.5} \) to get a slope that is a function of \( E_x \)
- see sample problem 2.6 in T&M (pg. 78)
Homework #4

- Velocity
- Dispersion coefficient

### Field Sampling and Analysis

<table>
<thead>
<tr>
<th>Sample No.</th>
<th>Sample Point</th>
<th>Sample Time</th>
<th>Fluorometer Readings</th>
<th>Fluorometer Readings</th>
<th>Dye Conc. (µg/L)</th>
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<td></td>
<td></td>
<td></td>
<td>1X</td>
<td>3X</td>
<td>10X</td>
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<tr>
<td>1</td>
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</tbody>
</table>
USGS Guidelines: Input

- The USGS recommends the following volume or mass of Rhodamine WT-20% dye:

\[
V_{dye} = 3.4 \times 10^{-4} \left( \frac{Q_m x}{U} \right)^{0.93} S_p
\]

\[
W_{dye} = 2.62 V_{dye}
\]
USGS Guidelines: Sampling

- Duration of dye cloud as a function of travel time to peak, and average channel width-depth ratio (B/H).

\[ t_p = 1.47 \frac{x}{U} \]  

**time to peak concentration, hr**

Suggested sampling interval from leading edge to just over peak of dye clouds: double time for sampling receding portion of dye cloud. 1 to 2 hr.

<table>
<thead>
<tr>
<th>B/H</th>
<th>Suggested sampling interval</th>
</tr>
</thead>
<tbody>
<tr>
<td>30</td>
<td>1 hr to 30 min</td>
</tr>
<tr>
<td>60</td>
<td>30 min to 15 min</td>
</tr>
<tr>
<td>100</td>
<td>15 min to 10 min</td>
</tr>
<tr>
<td>150</td>
<td>10 min to 5 min</td>
</tr>
<tr>
<td>150</td>
<td>5 min to 2 min</td>
</tr>
</tbody>
</table>
• To next lecture