Longitudinal Dispersion

- From Fischer et al., 1979

\[ E = 0.011 \frac{U^2 B^2}{H U^*} \]

Where the Shear Velocity is:

\[ U^* = \sqrt{gHS} \]
Lateral Mixing

- Lateral or transverse dispersion coefficient for a stream:
  \[ E_{lat} = 0.6HU \]
  Mean depth
  Shear velocity

- Length required for complete mixing:
  Side discharge:
  \[ L_m = 0.40U \frac{B^2}{E_{lat}} \]
  Center discharge:
  \[ L_m = 0.10U \frac{B^2}{E_{lat}} \]
  Width

General Stream Geometry

- Chapra’s nomenclature for discharge coefficients
  - Velocity
    \[ U = aQ^b \]
  - Depth
    \[ H = aQ^b \]
  - Width
    \[ B = cQ^f \]

  Where: \[ b + \beta + f = 1 \]
  Because \( Q=UHB \)
Sample Problem

The Black River, NY between MP 74.2 and MP 64.7 is to be characterized as a constant flow - constant area reach. Assume the following cross-sectional area ($A_c$) were measured for the given flows:

<table>
<thead>
<tr>
<th>$Q$ (cfs)</th>
<th>500</th>
<th>750</th>
<th>1300</th>
<th>2200</th>
<th>3400</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_c$ (ft²)</td>
<td>680</td>
<td>950</td>
<td>1100</td>
<td>1600</td>
<td>2200</td>
</tr>
</tbody>
</table>

Estimate travel time through this reach for flows of 600 and 3000 cfs

\[
H = \alpha Q^\beta \\
B = c Q^f \\
A = BH = \alpha c Q^{(\beta + f)}
\]

\[
A = 18.3 Q^{0.589}
\]
Manning Equation

- Derived from the momentum balance
- relates velocity to channel characteristics including slope

\[ U = \frac{1.486}{n} R^{2/3} S_e^{1/2} \]

Manning’s roughness coefficient 0.012-0.100
see Table 14.3

Hydraulic Radius (ft) = \( A_c \)/wetted perimeter
≈ \( A_c / (B+2H) \)

Slope of energy grade line = slope of stream bed for constant H & U

Manning Equation adapted to a Trapezoidal section

- Area, perimeter and hydraulic radius can all be expressed as a function of depth
- substitute these into the Manning Equation and calculate “y” from known “Q”

\[ Q = \frac{1.486}{n} A_c R^{2/3} S_e^{1/2} \]

\[ Q = \frac{1.486}{n} \left( \frac{(B_o + sy)y}{(B_o + 2y\sqrt{s^2 + 1})} \right)^{5/3} S_e^{1/2} \]

\[ A_c = \frac{(B_o + sy)y}{(B_o + 2y\sqrt{s^2 + 1})} \]

\[ P = B_o + 2y\sqrt{s^2 + 1} \]

\[ R = \frac{A_c}{P} = \frac{(B_o + sy)y}{B_o + 2y\sqrt{s^2 + 1}} \]
Distributed Systems

- Lecture #9 in Chapra’s book
- systems that have spatial resolution
- Ideal Reactors

\[ \Delta V \frac{\partial c}{\partial t} = J_{in} A_c - J_{out} A_c \pm \text{reaction} \]

Plug-Flow Reactors (PRF)

\[ \frac{\Delta V}{\Delta x} \frac{\partial c}{\partial t} = U c A - U \left( c + \frac{\partial c}{\partial x} \Delta x \right) A - k \Delta V \]

Combining and taking the limit as \( \Delta x \to 0 \)

\[ \frac{\partial c}{\partial t} = -U \frac{\partial c}{\partial x} - kc \]

Which at steady state is:

\[ 0 = -U \frac{\partial c}{\partial x} - kc \]

And for \( c=c_0 \) at \( x=0 \):

\[ C = c_0 e^{-k \sqrt{t}} \]
Plug Flow vs CSTR

- First order reactions
  \[ c = c_0 e^{-kt/u} \]
  \[ c = c_0 \frac{Q}{Q + kV} \]

- Mixed Flow: intermediate
  - read section 9.1.3

Mixed Flow

- Consider mixing in the longitudinal direction

\[ \Delta V \frac{\partial c}{\partial t} = J_{in} A_c - J_{out} A_c + \text{reaction} \]

\[ \Delta V \frac{\partial c}{\partial x} = \left( Uc - E \frac{\partial c}{\partial x} \right) A_c - U \left( c + \frac{\partial c}{\partial x} \Delta x \right) \frac{\partial}{\partial x} - E \frac{\partial^2 c}{\partial x^2} \Delta x \]

Combining and taking the limit as \( \Delta x \to 0 \)

\[ \frac{\partial c}{\partial t} = U \frac{\partial c}{\partial x} + E \frac{\partial^2 c}{\partial x^2} - kc \]

Which at steady state is:

\[ 0 = -U \frac{\partial c}{\partial x} + E \frac{\partial^2 c}{\partial x^2} - kc \]
Mixed Flow

\[ \Delta V \frac{\partial c}{\partial t} = J_{in} A_c - J_{out} A_c \pm \text{reaction} \]

\[ \Delta V \frac{\partial c}{\partial t} = \left[ U_c - E \frac{\partial c}{\partial x} \right] A_c - \left[ U \left( c + \frac{\partial c}{\partial x} \Delta x \right) - E \left( \frac{\partial c}{\partial x} + \frac{\partial^2 c}{\partial x^2} \Delta x \right) \right] A_c - k \Delta V c \]

Consider mixing in the longitudinal direction

- **Peclet Number**
  \[ P_e = \frac{L U}{E} \]
  - rate of advective transport
  - rate of dispersive transport
  \[ P_e > 10, \text{ PFR-like} \]
  \[ P_e < 0.1, \text{ CSTR-like} \]

Application of PRF to streams

- **Point sources**
- **Mass balance:**
  - Water Flow
  - Concentration
  \[ Q = Q_w + Q_r \]
  \[ c_o = \frac{Q_w c_w + Q_r c_r}{Q_w + Q_r} \]

Outfall: \( Q_w c_w \)
Assumptions

- The ideal reactor is:
  - Completely mixed in cross-section
  - Without longitudinal (downstream) mixing
  - Steady State

Chloride Problem

- Determine the required industrial reduction in chlorides to maintain a desired chloride concentration of 250 mg/L at the intake

\[ Q = 25 \text{ cfs} \]
\[ c = 30 \text{ mg/L} \]

\[ Q_w = 6.5 \text{ MGD} \]
\[ c_w = 1500 \text{ mg/L} \]

\[ Q_T = 5 \text{ cfs} \]
\[ c_T = 30 \text{ mg/L} \]
To next lecture