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CEE 577: Surface Water Quality Modeling

Lecture #10
(Rivers & Streams, cont)
Chapra, L14 (cont.)

Longitudinal Dispersion

- From Fischer et al., 1979

$$E = 0.011\frac{U^2B^2}{HU^*}$$

E: m^2s^{-1}; U: m/s; B: Width (m); H: Mean depth (m)

Where the Shear Velocity is:

$$U^* = \sqrt{gHS}$$

Lateral Mixing

- Lateral or transverse dispersion coefficient for a stream:

Mean depth

$$E_{lat} = 0.6HU^*$$

Shear velocity

- Length required for complete mixing:

Side discharge:

$$L_m = 0.40U\frac{B^2}{E_{lat}}$$

Center discharge:

$$L_m = 0.10U\frac{B^2}{E_{lat}}$$

Width

General Stream Geometry

- Chapra's nomenclature for discharge coefficients
  - Velocity $U = aQ^b$
  - Depth $H = \alpha Q^\beta$
  - Width $B = cQ^f$

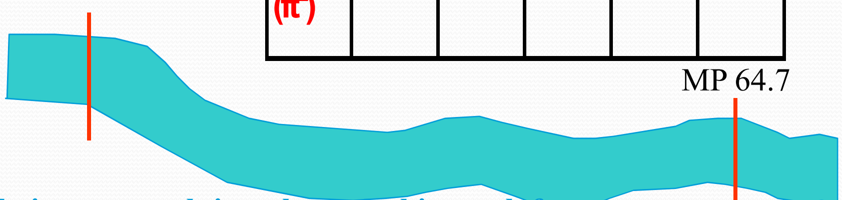
Where: $b + \beta + f = 1$ Because Q=UHB

Thomann & Mueller, problem 2.1

Sample Problem

The Black River, NY between MP 74.2 and MP 64.7 is to be characterized as a constant flow - constant area reach. Assume the following cross-sectional area (A_c) were measured for the given flows:

Q (cfs)	500	750	1300	2200	3400
A_c (ft^2)	680	950	1100	1600	2200

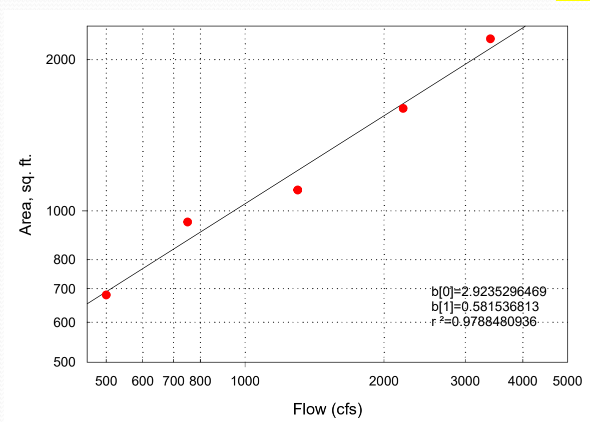


Estimate travel time through this reach for flows of 600 and 3000 cfs

$$H = \alpha Q^{\beta}$$

$$B = cQ^{f}$$

$$A = BH = \alpha c Q^{(\beta + f)}$$



$$A = 18.3Q^{0.589}$$

Manning Equation

- Derived from the momentum balance
  - relates velocity to channel characteristics including slope

$$U = \frac{1.486}{n} R^{2/3} S_e^{1/2}$$

ft/s

Slope of energy grade line = slope of stream bed for constant H & U

Manning's roughness coefficient 0.012-0.100 see Table 14.3

Hydraulic Radius (ft) $=A_c$/wetted perimenter $\approx A_c/(B+2H)$

Manning Equation adapted to a Trapezoidal section

$$Q = \frac{1.486}{n} A_c R^{2/3} S_e^{1/2}$$

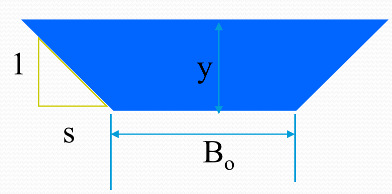
- Area, perimeter and hydraulic radius can all be expressed as a function of depth
- substitute these into the Manning Equation and calculate "y" from known "Q

$$Q = \frac{1.486}{n} \frac{[(B_o + sy)y]^{5/3}}{\left(B_o + 2y\sqrt{s^2+1}\right)^{2/3}} S_e^{1/2}$$

$$A_c = (B_o + sy)y$$

$$P = B_o + 2y\sqrt{s^2+1}$$

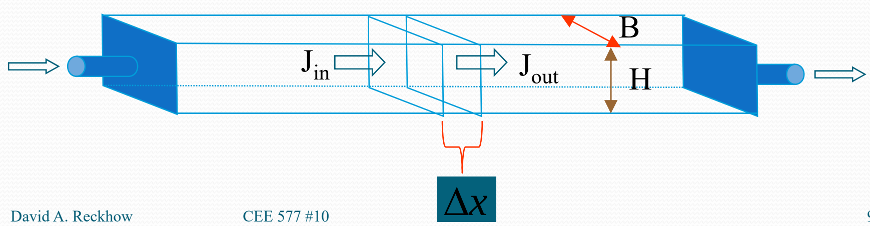
$$R = \frac{A_c}{P} = \frac{(B_o + sy)y}{B_o + 2y\sqrt{s^2+1}}$$



Distributed Systems

- Lecture #9 in Chapra's book
  - systems that have spatial resolution
- Ideal Reactors

$$\Delta V \frac{\partial c}{\partial t} = J_{in} A_c - J_{out} A_c \pm reaction$$



Plug-Flow Reactors (PRF)

$$\Delta V \frac{\partial c}{\partial t} = \boxed{J_{in}} A_c - \boxed{J_{out}} A_c \boxed{\pm reaction}$$

$$\Delta V \frac{\partial c}{\partial t} = \boxed{Uc} A_c - \boxed{U\left(c + \frac{\partial c}{\partial x}\Delta x\right)} A_c \boxed{- k\Delta V \bar{c}}$$

Combining and taking the limit as $\Delta x \rightarrow 0$

$$\frac{\partial c}{\partial t} = -U \frac{\partial c}{\partial x} - kc$$

Which at steady state is:

$$0 = -U \frac{\partial c}{\partial x} - kc$$

And for $c = c_o$ at $x = 0$:

$$c = c_o e^{-k x/u}$$

Plug Flow vs CSTR

- First order reactions

$$c = c_o e^{-k\, x/u}$$

$$c = c_o \frac{Q}{Q + kV}$$

Which is more efficient?

- Mixed Flow: intermediate
  - read section 9.1.3

Mixed Flow

- Consider mixing in the longitudinal direction

$$\Delta V \frac{\partial c}{\partial t} = \boxed{J_{in}} A_c - \boxed{J_{out}} A_c \boxed{\pm\, reaction}$$

$$\Delta V \frac{\partial c}{\partial t} = \boxed{\left[Uc - E\frac{\partial c}{\partial x}\right]} A_c - \boxed{\left[U\left(c + \frac{\partial c}{\partial x}\Delta x\right) - E\left(\frac{\partial c}{\partial x} + \frac{\partial}{\partial x}\left(\frac{\partial c}{\partial x}\right)\Delta x\right)\right]} A_c \boxed{- k\Delta V c}$$

Combining and taking the limit as $\Delta x \rightarrow 0$

$$\frac{\partial c}{\partial t} = -U\frac{\partial c}{\partial x} + E\frac{\partial^2 c}{\partial x^2} - kc$$

Which at steady state is:

$$0 = -U\frac{\partial c}{\partial x} + E\frac{\partial^2 c}{\partial x^2} - kc$$

Mixed Flow

$$\Delta V \frac{\partial c}{\partial t} = \boxed{J_{in}} A_c - \boxed{J_{out}} A_c \boxed{\pm reaction}$$

$$\Delta V \frac{\partial c}{\partial t} = \boxed{\left[Uc - E\frac{\partial c}{\partial x} \right]} A_c - \boxed{\left[U\left(c + \frac{\partial c}{\partial x}\Delta x \right) - E\left(\frac{\partial c}{\partial x} + \frac{\partial}{\partial x}\left(\frac{\partial c}{\partial x} \right)\Delta x \right) \right]} A_c \boxed{- k\Delta V \bar{c}}$$

Consider mixing in the longitudinal direction

- Peclet Number

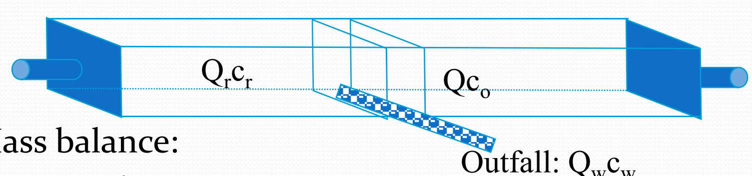
$$P_e = \frac{LU}{E} = \frac{\text{rate of advective transport}}{\text{rate of dispersive transport}}$$

$P_e > 10$, PFR-like

$P_e < 0.1$, CSTR-like

Application of PRF to streams

- Point sources



- Mass balance:
  - Water Flow
  - Concentration

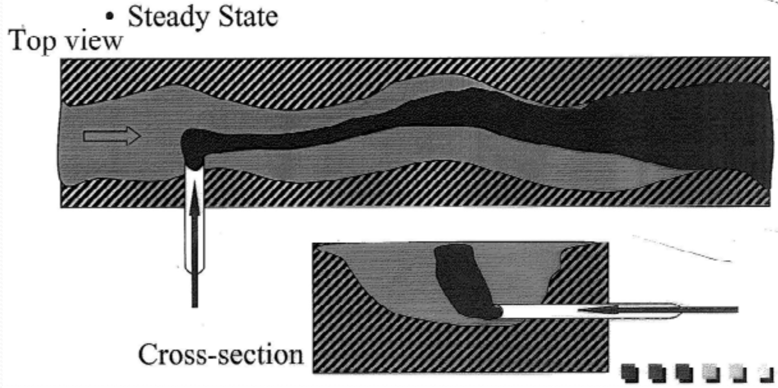
$$Q = Q_w + Q_r$$

$$c_o = \frac{Q_w c_w + Q_r c_r}{Q_w + Q_r}$$

Assumptions

■ The ideal reactor is:

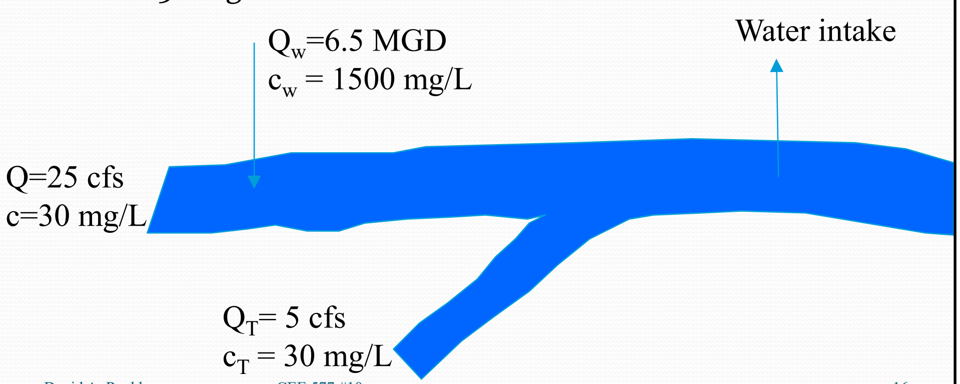
- Completely mixed in cross-section
- Without longitudinal (downstream) mixing
- Steady State

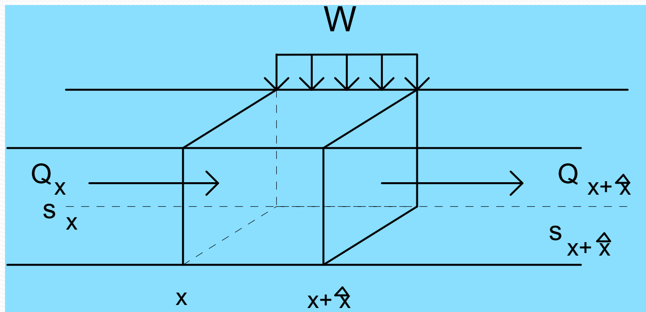


Chloride Problem

1.55 cfs/MGD

- Determine the required industrial reduction in chlorides to maintain a desired chloride concentration of 250 mg/L at the intake





- To next lecture