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CEE 577: Surface Water Quality Modeling

Lecture #9
(Diffusion)
Chapra L8

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Forge Pond project

- Continued discussion
  - Update
  - questions

EPA 305(b) listing for Forge Pond

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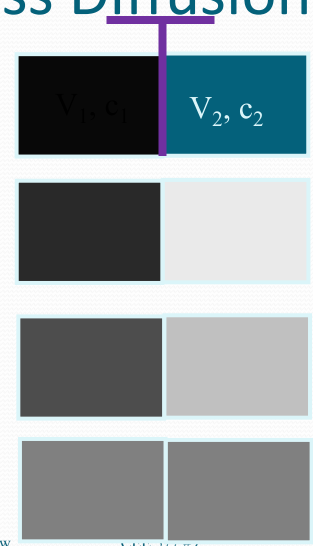
Liquid Water Transport

- Advection: unidirectional flow
- Diffusion: movement of mass that is not unidirectional flow; usually movement in an unorganized fashion
  - Dispersion
  - Eddy Diffusion
  - Molecular Diffusion

Mass Diffusion



T=0

T=1

T=2

T=large

Incorporates molecular movement and interfacial area

Bulk Diffusion (m^2/yr)

$$V_1 \frac{dc_1}{dt} = D'(c_2 - c_1)$$

Concentration Gradient

Fick's First Law

- Mass flux is proportional to the concentration gradient and a diffusion coefficient

$$J_x = -D\frac{dc}{dx}$$

Bulk Diffusion Coefficient

V_1, c_1	V_2, c_2

$$V_1\frac{dc_1}{dt} = -JA_c$$

The mixing length

$$J_x = -D\frac{dc}{dx}$$

$$\frac{dc}{dx} \cong \frac{c_2 - c_1}{\ell}$$

And combining all three:

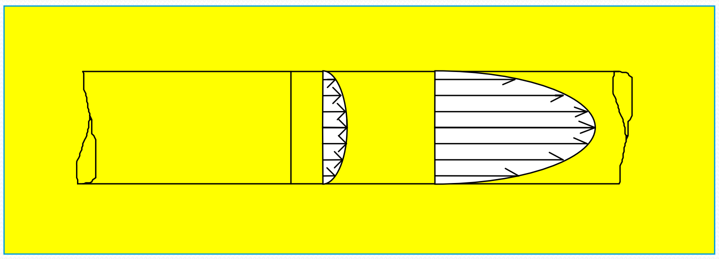
$$V_1\frac{dc_1}{dt} = \underbrace{\frac{DA_c}{\ell}}_{D'}(c_2 - c_1)$$

$$E' = \frac{EA_c}{\ell}$$

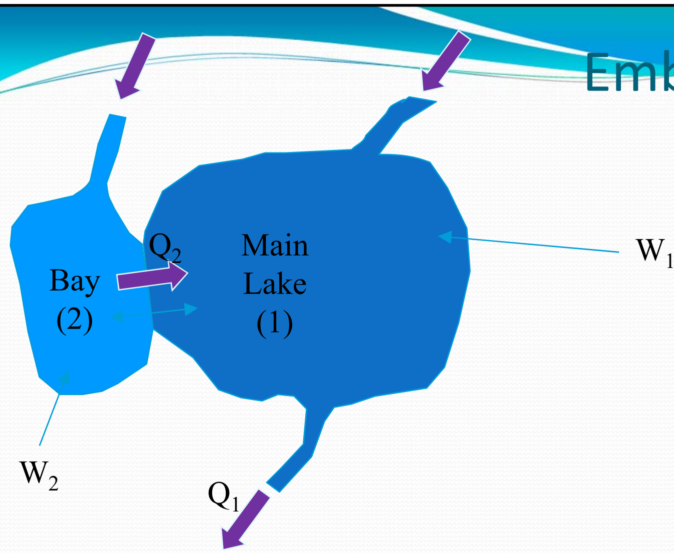
Similar for Eddy Diffusion

Dispersion

- Differences in velocities of parallel flow paths



Embayment Model



$$V_1 \frac{dc_1}{dt} = W_1 - Q_1 c_1 - k_1 V_1 c_1 + Q_2 c_2 + E'(c_2 - c_1)$$

$$V_2 \frac{dc_2}{dt} = W_2 - Q_2 c_2 - k_2 V_2 c_2 + E'(c_1 - c_2)$$

Embayment Model with a Conservative Substance

- Conservative substances (s) are those that do not undergo degradation, thus k=0
- The mass balance on the bay (2), then becomes:

$$V_2 \frac{ds_2}{dt} = W_2 - Q_2 s_2 + E'(s_1 - s_2)$$

And solving for the bulk diffusion coefficient:

$$E' = \frac{W_2 - Q_2 s_2}{s_2 - s_1}$$

Map of Huron/Saginaw System

Parameters for Saginaw Bay

Parameter	Symbol	Value	Units
Volume	V_2	8	10^9 m^3
Depth	H_2	5.81	m
Surface Area	A_2	1,376	10^6 m^2
Outflow	Q_2	7	10^9 m^3/yr
Chloride Conc.	s_2	15.2	mg/L
Chloride Loading	W_{s2}	0.353	10^{12} g/yr
Phosphorus Loading	W_{p2}	1.42	10^{12} mg/yr

Parameters for Lake Huron

Parameter	Symbol	Value	Units
Volume	V_1	3507	10^9 m^3
Depth	H_1	60.3	m
Surface Area	A_1	58,194	10^6 m^2
Outflow	Q_1	161	10^9 m^3/yr
Chloride Conc.	S_1	5.4	mg/L
Chloride Loading	W_{s1}	~0	10^{12} g/yr
Phosphorus Loading	W_{p1}	4.05	10^{12} mg/yr

Chloride Tracer Model

$$E' = \frac{W_2 - Q_2 s_2}{s_2 - s_1}$$

$$E' = \frac{0.353x10^{12} - \left[7x10^9(15.2)\right]}{15.2 - 5.4}$$
$$= 25.2x10^9 m^3 / yr$$

$$E' = \frac{EA_c}{\ell}$$

$$E = \frac{E'\ell}{A_c} = \frac{2.52x10^9\left(10x10^3 m\right)}{0.17x10^6 m^2}$$
$$= 1.48x10^9 m^2 / yr$$
$$= 4.7x10^5 cm^2 / s$$

- To next lecture