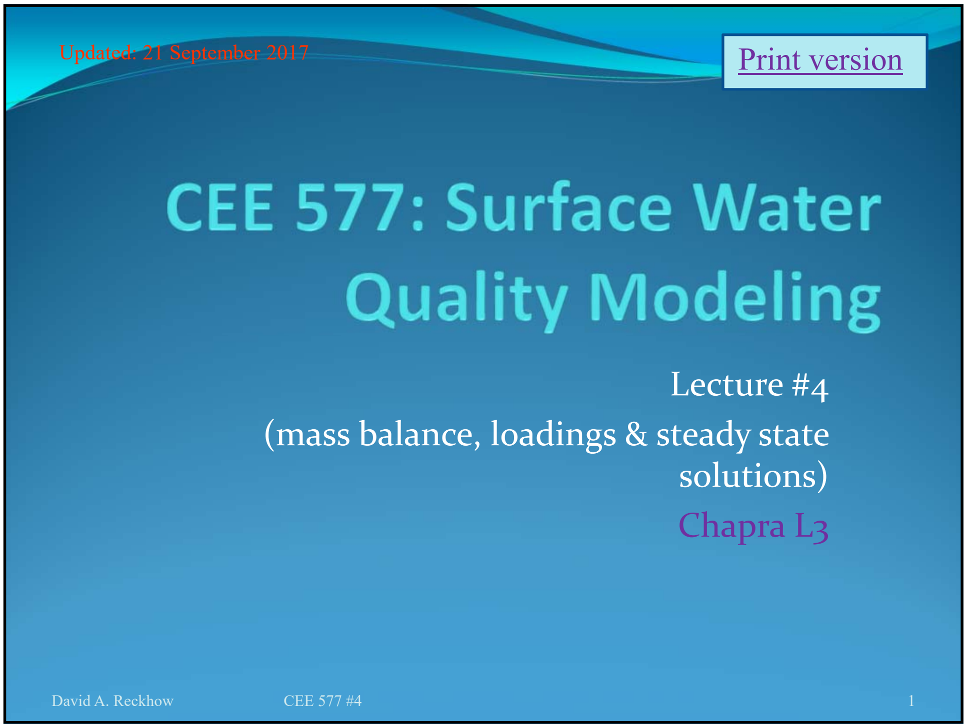

Updated: 21 September 2017

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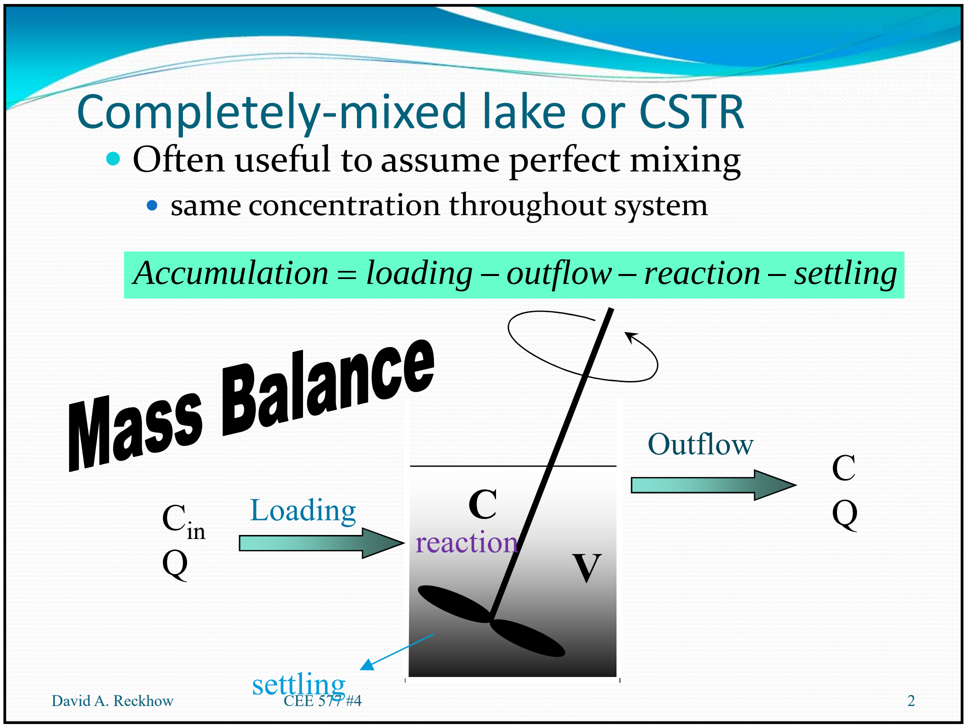
CEE 577: Surface Water Quality Modeling

Lecture #4

(mass balance, loadings & steady state solutions)

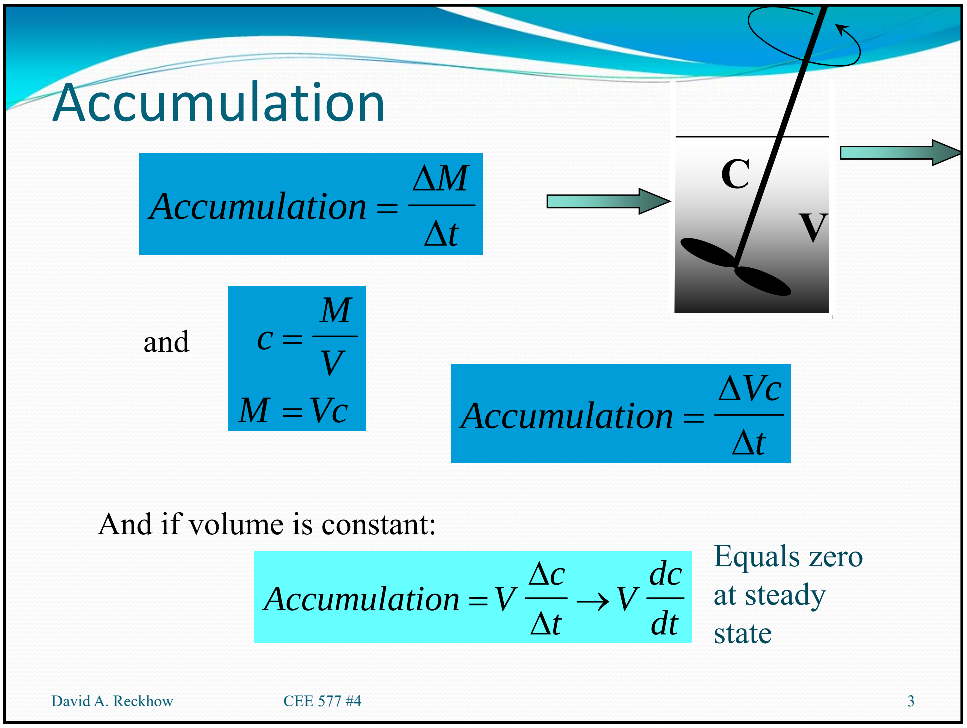
Chapra L3



Completely-mixed lake or CSTR

- Often useful to assume perfect mixing
  - same concentration throughout system

Accumulation = loading – outflow – reaction – settling



Accumulation

$$Accumulation = \frac{\Delta M}{\Delta t}$$

and $c = \frac{M}{V}$

$M = Vc$

$$Accumulation = \frac{\Delta Vc}{\Delta t}$$

And if volume is constant:

$$Accumulation = V\frac{\Delta c}{\Delta t} \rightarrow V\frac{dc}{dt}$$

Equals zero at steady state

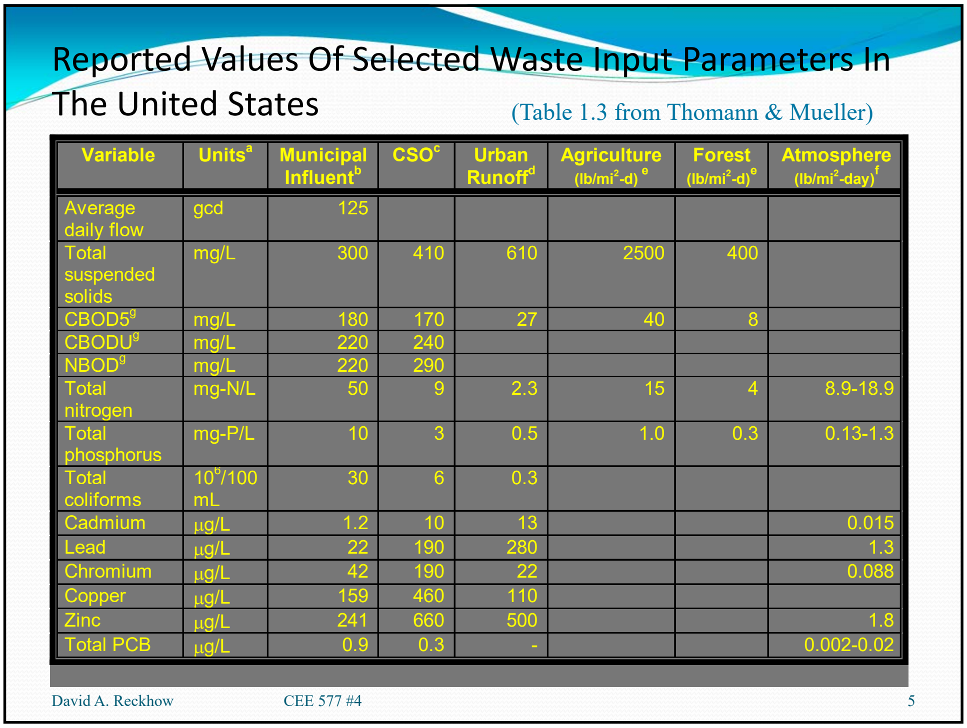
Loading

$$Loading = W(t) = Qc_{in}(t)$$

- Point Sources
  - Municipal Wastewater
  - Industrial Wastewater
  - Tributaries
- Non-point sources
  - agricultural
  - silvicultural
  - atmospheric
  - urban & suburban runoff
  - groundwater

Well defined origin
easily measured
more constant

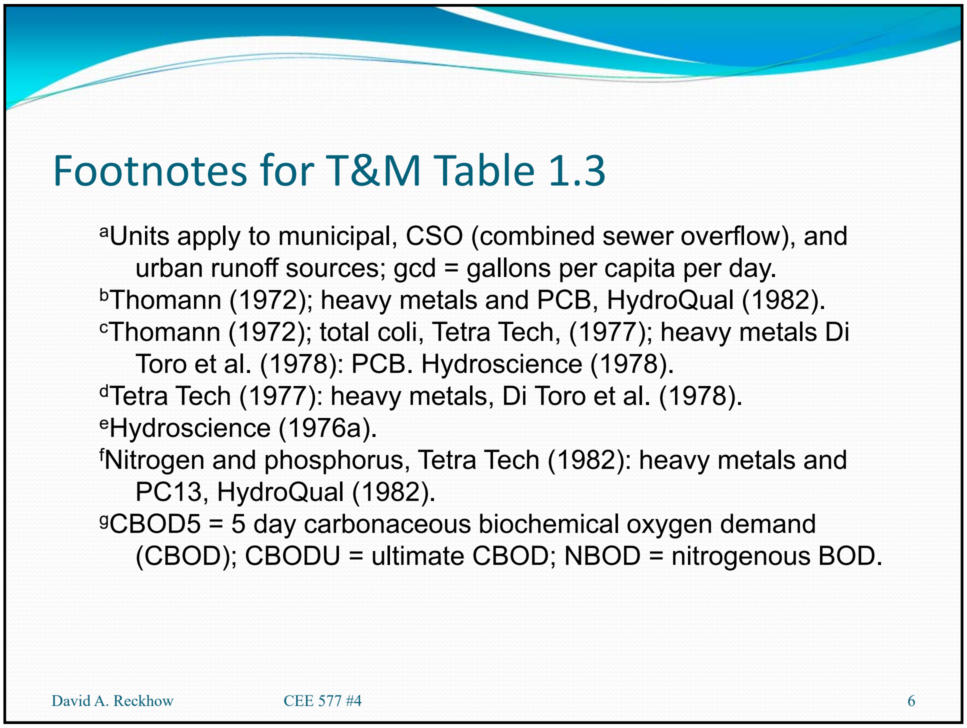
Diffuse origin
more transient
often dependent on precipitation



Reported Values Of Selected Waste Input Parameters In The United States

(Table 1.3 from Thomann & Mueller)

Variable	Units[a]	Municipal Influent[b]	CSO[c]	Urban Runoff[d]	Agriculture (lb/mi^2-d)[e]	Forest (lb/mi^2-d)[e]	Atmosphere (lb/mi^2-day)[f]
Average daily flow	gcd	125					
Total suspended solids	mg/L	300	410	610	2500	400	
CBOD5[g]	mg/L	180	170	27	40	8	
CBODU[g]	mg/L	220	240				
NBOD[g]	mg/L	220	290				
Total nitrogen	mg-N/L	50	9	2.3	15	4	8.9-18.9
Total phosphorus	mg-P/L	10	3	0.5	1.0	0.3	0.13-1.3
Total coliforms	10^6/100 mL	30	6	0.3			
Cadmium	µg/L	1.2	10	13			0.015
Lead	µg/L	22	190	280			1.3
Chromium	µg/L	42	190	22			0.088
Copper	µg/L	159	460	110			
Zinc	µg/L	241	660	500			1.8
Total PCB	µg/L	0.9	0.3	-			0.002-0.02



Footnotes for T&M Table 1.3

[a] Units apply to municipal, CSO (combined sewer overflow), and urban runoff sources; gcd = gallons per capita per day.

[b] Thomann (1972); heavy metals and PCB, HydroQual (1982).

[c] Thomann (1972); total coli, Tetra Tech, (1977); heavy metals Di Toro et al. (1978): PCB. Hydroscience (1978).

[d] Tetra Tech (1977): heavy metals, Di Toro et al. (1978).

[e] Hydroscience (1976a).

[f] Nitrogen and phosphorus, Tetra Tech (1982): heavy metals and PC13, HydroQual (1982).

[g] CBOD5 = 5 day carbonaceous biochemical oxygen demand (CBOD); CBODU = ultimate CBOD; NBOD = nitrogenous BOD.

Loading: Flow as a function of precipitation

- Non point sources are difficult to characterize
  - Empirical approach: export coefficients (see Table 3.1 in T&M)
  - Mechanistic approach: relate to meteorology, topology, etc.
    - Flow: use the rational formula: $Q_R = cIA$

Drainage Area [L^2]

Runoff flow [L^3/T]

Rainfall Intensity [L/T]

Runoff coefficient

0.1-0.3 for rural areas (1 person/acre)

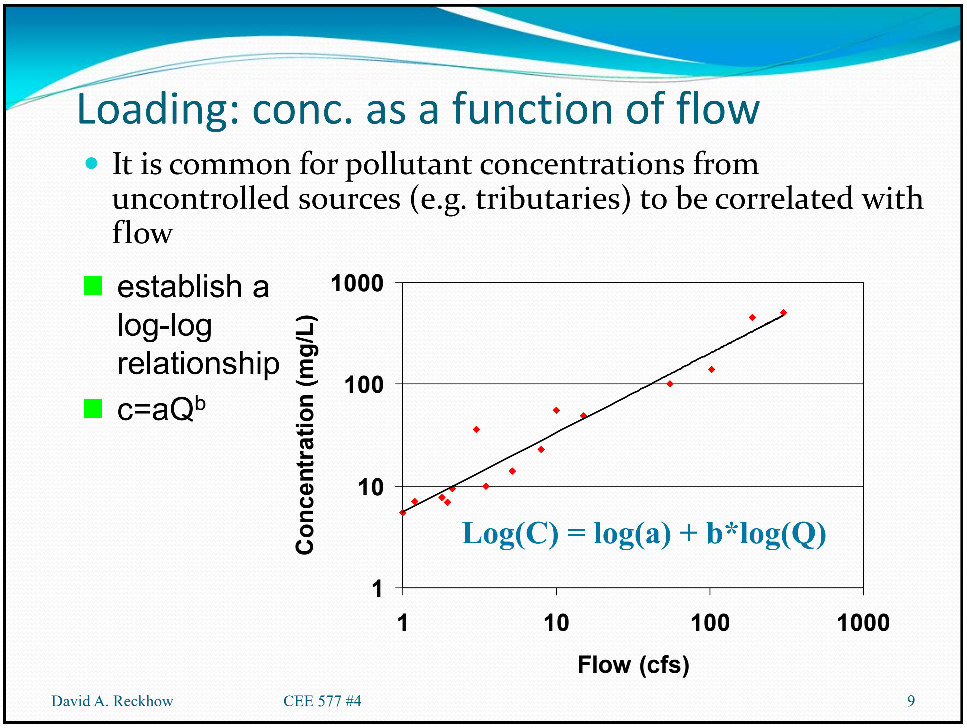
0.7-0.9 for heavy commercial areas

Note:

1 acre-in/hr ≈1 cfs

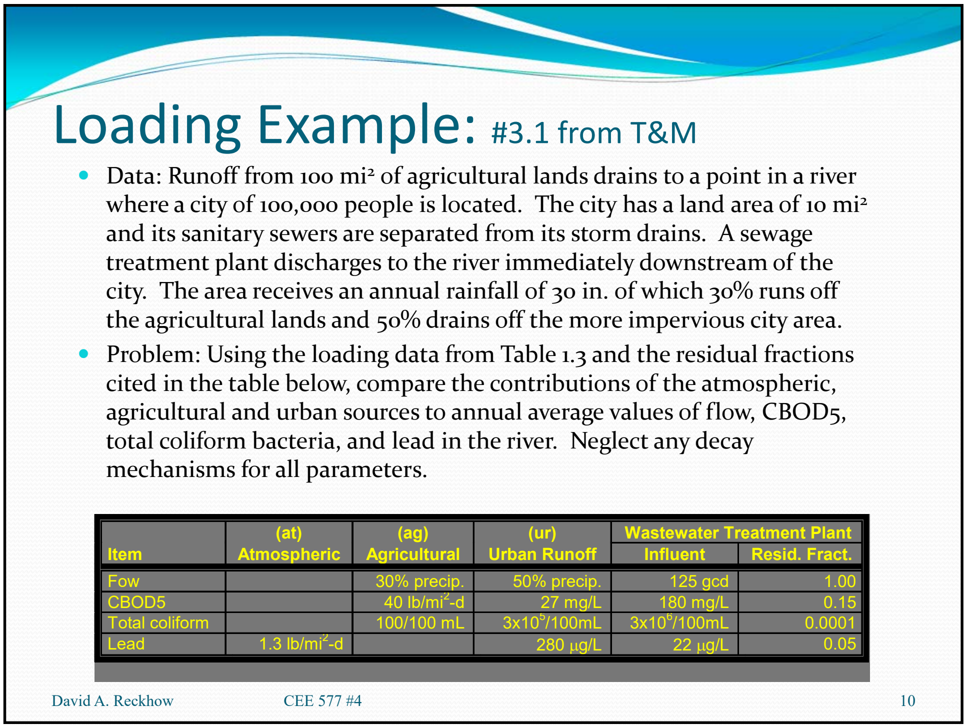
Runoff: Contrasting approaches

- Lumped model
  - Empirical
  - Built on a single rainfall intensity from rain gage data
- Distributed model
  - Mechanistic
  - Built on radar data for rainfall
    - Spatial & temporal resolution
  - Combine with overland flow models
    - Many computer codes
      - CASC2D, CUHP, CUHP/SWMM, DR3M, HEC-1, HSPF, PSRM, SWMM, TR20



Loading: conc. as a function of flow

- It is common for pollutant concentrations from uncontrolled sources (e.g. tributaries) to be correlated with flow
  - 🟩 establish a log-log relationship
  - 🟩 $c=aQ^b$



Loading Example: #3.1 from T&M

- Data: Runoff from 100 mi² of agricultural lands drains to a point in a river where a city of 100,000 people is located. The city has a land area of 10 mi² and its sanitary sewers are separated from its storm drains. A sewage treatment plant discharges to the river immediately downstream of the city. The area receives an annual rainfall of 30 in. of which 30% runs off the agricultural lands and 50% drains off the more impervious city area.
- Problem: Using the loading data from Table 1.3 and the residual fractions cited in the table below, compare the contributions of the atmospheric, agricultural and urban sources to annual average values of flow, CBOD5, total coliform bacteria, and lead in the river. Neglect any decay mechanisms for all parameters.

Item	(at) Atmospheric	(ag) Agricultural	(ur) Urban Runoff	Wastewater Treatment Plant Influent	Wastewater Treatment Plant Resid. Fract.
Fow		30% precip.	50% precip.	125 gcd	1.00
CBOD5		40 lb/mi^2-d	27 mg/L	180 mg/L	0.15
Total coliform		100/100 mL	3x10^5/100mL	3x10^6/100mL	0.0001
Lead	1.3 lb/mi^2-d		280 μg/L	22 μg/L	0.05

Solution to loading problem

- Flow contributions

$$Q(ag) = 100mi^2 (30in/yr) 0.3 \left(\frac{5280\,ft}{mi}\right)^2 \frac{1\,ft}{12in}\,\frac{1yr}{365d}\,\frac{1d}{86{,}400s}$$

$$= 66.3cfs$$

$$Q(ur) = 10mi^2 (30in/yr) 0.5 \left(\frac{5280\,ft}{mi}\right)^2 \frac{1\,ft}{12in}\,\frac{1yr}{365d}\,\frac{1d}{86{,}400s}$$

$$= 11.1cfs$$

$$Q(wwtp) = 100{,}000cap \frac{125gal}{cap-d}\,\frac{1MG}{10^6\,gal}$$

$$= 12.5MGD \left(\frac{1.548cfs}{MGD}\right)$$

$$= 19.4cfs$$

Solution to loading problem (cont.)

- CBOD5 loading

$$W(ag) = 100mi^2 \left(40\frac{lb}{mi^2 d}\right)$$

$$= 4000\frac{lb}{d}$$

$$W(ur) = 11.1cfs (27mg/L) 5.4 \frac{lb/d}{cfs-mg/L}$$

$$= 1620\frac{lb}{d}$$

$$W(wwtp) = 12.5MGD (180mg/L) 0.15 \left(\frac{8.34lb/d}{MGD*mg/L}\right)$$

$$= 2810\frac{lb}{d}$$

Solution to loading problem (cont.)

- Lead loading

$$W(atm) = 100mi^2\left(1.3\frac{lb}{mi^2 d}\right)0.1$$
$$= 13\frac{lb}{d}$$

$$W(ur) = 11.1cfs(280\mu g/L)5.4\frac{lb/d}{cfs-mg/L}\left(\frac{10^{-3}mg}{\mu g}\right)$$
$$= 16.8\frac{lb}{d}$$

$$W(wwtp) = 12.5MGD(22\mu g/L)0.05\left(\frac{8.34 lb/d}{MGD^{*}mg/L}\right)\frac{10^{-3}mg}{\mu g}$$
$$= 0.11\frac{lb}{d}$$

Other Terms in the Mass Balance

- Outflow

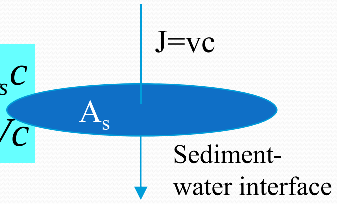
$$Outflow = Qc$$

- Reaction

$$Reaction = kM = kVc$$

- Settling

$$Settling = vA_s c$$
$$= k_s Vc$$



Since: $k_s = {}^{v}\!/\!_{H}$ $V = A_s H$

Combining all terms:

$$V\frac{dc}{dt} = W(t) - Qc - kVc - vA_s c$$

- Dependent variable: c
- Independent variable: t
- Forcing function: W(t), the way in which the external world “forces” the system
- Parameters: V, Q, k, v, A_s

Steady State Case

- Mass Balance $$\overbrace{V\frac{dc}{dt}} = 0 = \underbrace{W(t)}_{\rightarrow W} - Qc - kVc - vA_s c$$
- Solution $$c = \frac{W}{Q + kV + vA_s} \quad \text{or} \quad c = \frac{W}{a}$$
- Assimilation factor
  - Where $$a = Q + kV + vA_s$$
  - The assimilation or “cleansing” factor

Steady State Example

#3.1 from Chapra (pg.52)

A lake has the following characteristics:

$$\begin{aligned} \mathit{Volume} &= 50{,}000\ m^3 \\ \text{Mean Depth} &= 2\ \text{m} \\ \text{Inflow} &= \text{Outflow} = 7500\ \text{m}^3 d^{-1} \\ \text{Temperature} &= 25^{\circ} C \end{aligned}$$

The lake receives the input of a pollutant from three sources: a factory discharge of 50 kg d^{-1}, a flux from the atmosphere of 0.6 g m^{-2} d^{-1}, and the inflow stream that has a concentration of 10 mg/L. If the pollutant decays at the rate of 0.25/d at 20°C (note: ϴ=1.05).

a. compute the assimilation factor
b. steady state concentration
c. show breakdown for each term

Example 3.1: Solution

First correct the decay rate for temperature

$$\begin{aligned} k &= 0.25\theta^{25-20} = 0.25(1.05)^{25-20} \\ &= 0.319 d^{-1} \end{aligned}$$

Now the assimilation factor

$$\begin{aligned} a &= Q + kV \\ &= 7500 + 0.319(50{,}000) \\ &= 23{,}454 m^3 d^{-1} \end{aligned}$$

Example 3.1: Solution (cont.)

The surface area of the lake is:

$$A_s = \frac{V}{H} = \frac{50{,}000}{2} = 25{,}000 m^2$$

The atmospheric and inflow load is then:

$$W_{atmosphere} = JA_s = 0.6(25{,}000) = 15{,}000 g/d$$

$$W_{\inf low} = 7500(10) = 75{,}000 g/d$$

Combining all loads:

$$\begin{aligned} W &= W_{factory} + W_{atmosphere} + W_{\inf low} \\ &= 50{,}000 + 15{,}000 + 75{,}000 \\ &= 140{,}000 g/d \end{aligned}$$

Example 3.1: Solution (cont.)

And finally, the concentration:

$$\begin{aligned} c &= \frac{W}{a} \\ &= \frac{140{,}000 g/d}{23{,}454 m^3/d} \\ &= 5.97 mg/L \end{aligned}$$

Transfer function & residence time

$$c = \frac{W}{Q + kV + vA_s} = \frac{Qc_{in}}{Q + kV + vA_s}$$

$$\frac{c}{c_{in}} = \beta \equiv \frac{Q}{Q + kV + vA_s}$$

Transfer function

$$\tau_E = \frac{E}{dE/dt}$$

generic

Residence times

$$\tau_w = \frac{V}{Q}$$

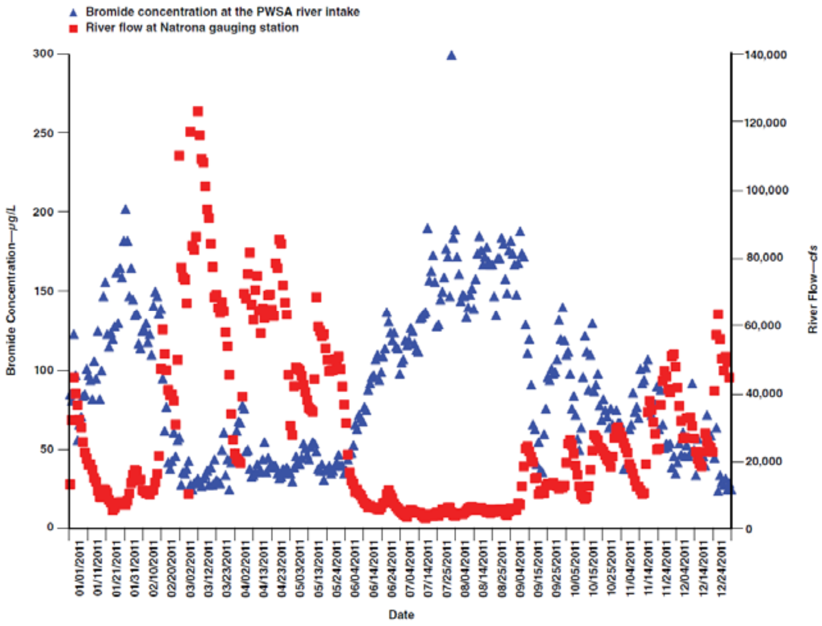
water

$$\tau_c = \frac{Vc}{Qc + kVc + vA_s c} = \frac{V}{Q + kV + vA_s}$$

contaminant

- P

FIGURE 3 Bromide concentration and river flow



PWSA—Pittsburgh Water and Sewer Authority

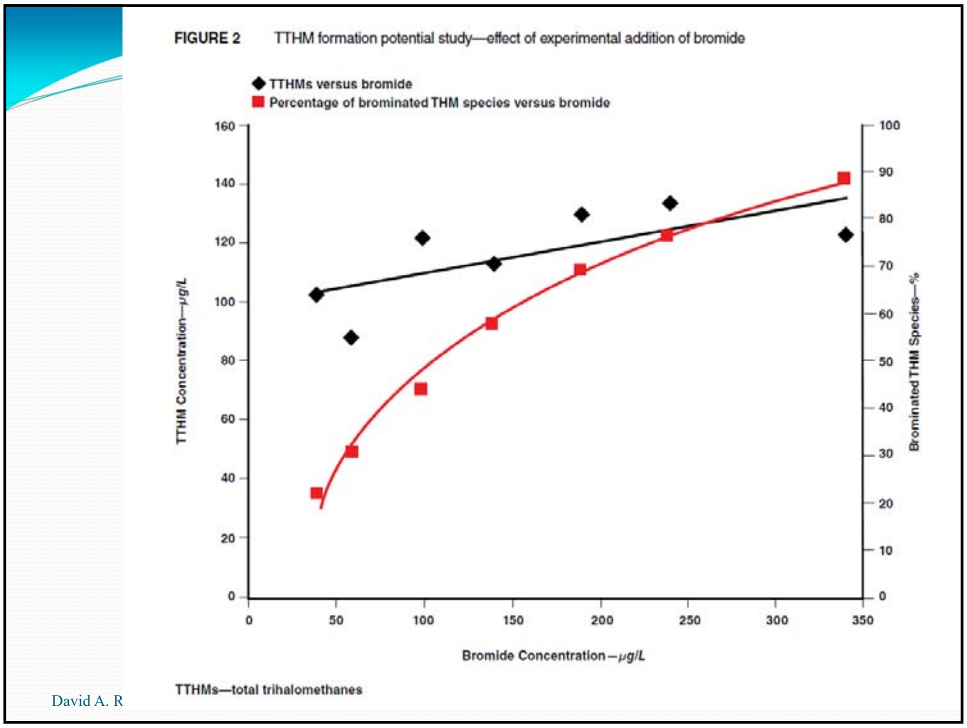


FIGURE 2 TTHM formation potential study—effect of experimental addition of bromide

TTHMs—total trihalomethanes

David A. R

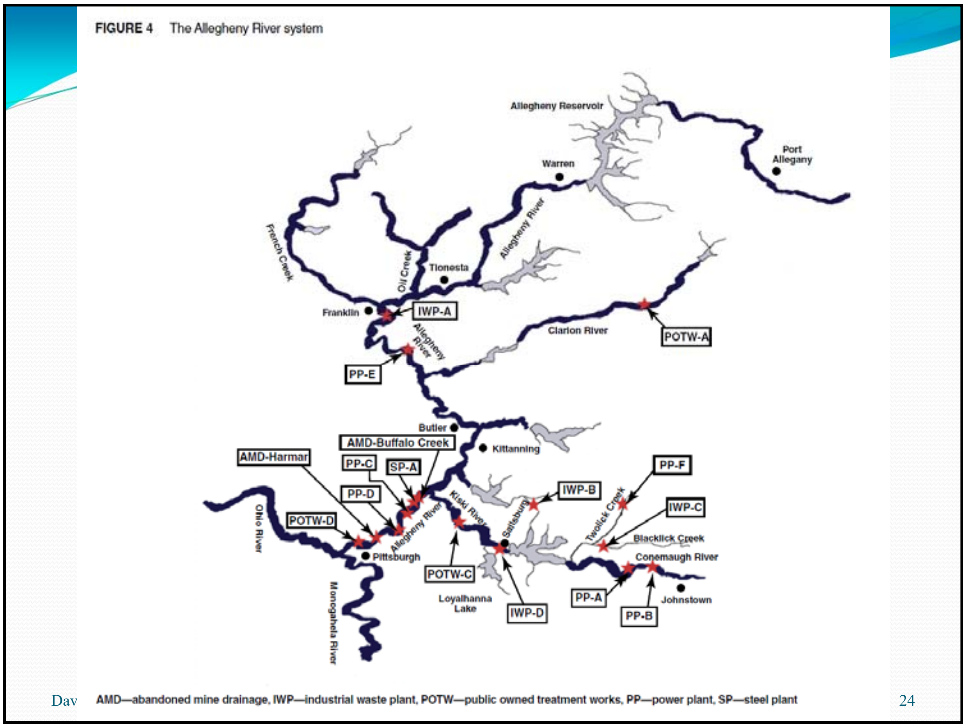


FIGURE 4 The Allegheny River system

AMD—abandoned mine drainage, IWP—industrial waste plant, POTW—public owned treatment works, PP—power plant, SP—steel plant

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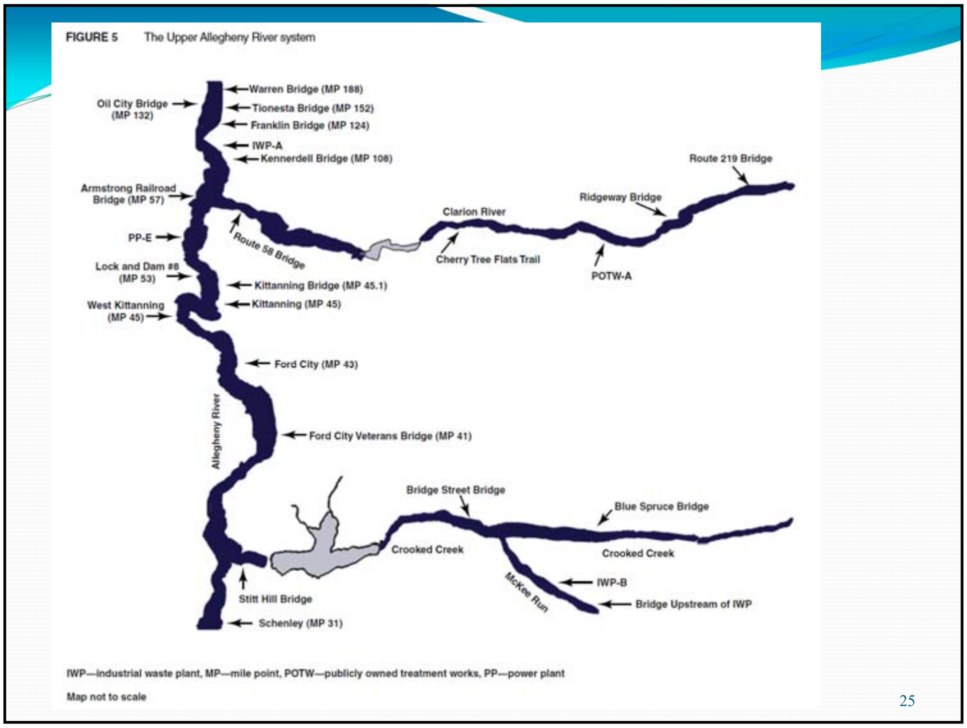


FIGURE 5 The Upper Allegheny River system

IWP—industrial waste plant, MP—mile point, POTW—publicly owned treatment works, PP—power plant

Map not to scale

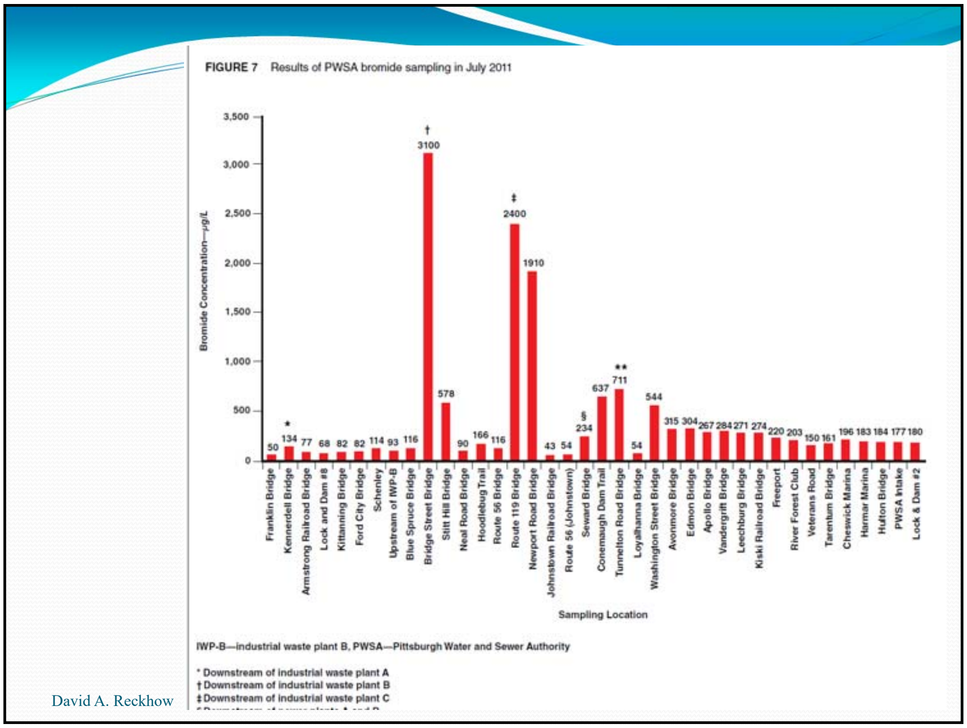


FIGURE 7 Results of PWSA bromide sampling in July 2011

IWP-B—industrial waste plant B, PWSA—Pittsburgh Water and Sewer Authority

* Downstream of industrial waste plant A
† Downstream of industrial waste plant B
‡ Downstream of industrial waste plant C

David A. Reckhow

Bromide in PA

Kelly D. Good and Jeanne M. VanBriesen, 2016 “Current and Potential Future Bromide Loads from Coal-Fired Power Plants in the Allegheny River Basin and Their Effects on Downstream Concentrations”, ES&T 50, 9078

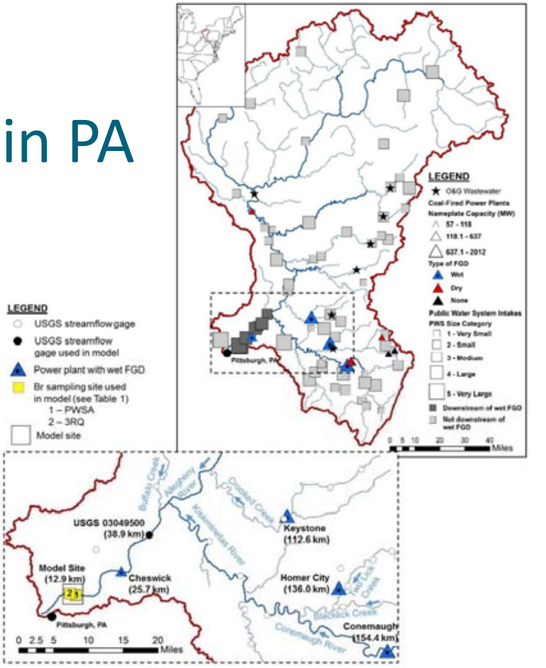
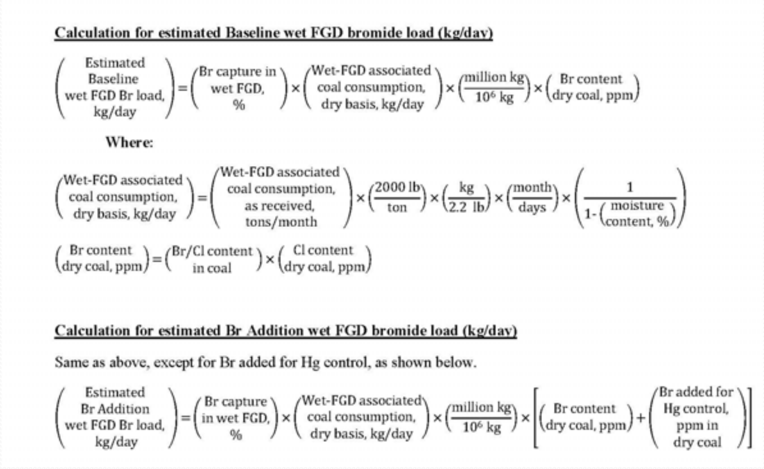


Figure 1. Map of the Allegheny River Basin showing coal-fired power plants and public drinking water systems. Inset map shows wet FGD power plants (blue triangles), USGS streamflow gage used in the model (black circle), sampling sites for bromide (yellow squares), and the model site at river kilometer (RKM) 12.9. Distances provided are RKM measured from the confluence in Pittsburgh, PA.



Calculation for estimated Baseline wet FGD bromide load (kg/day)

$$\begin{pmatrix}\text{Estimated}\\\text{Baseline}\\\text{wet FGD Br load,}\\\text{kg/day}\end{pmatrix}=\begin{pmatrix}\text{Br capture in}\\\text{wet FGD,}\\\%\end{pmatrix}\times\begin{pmatrix}\text{Wet-FGD associated}\\\text{coal consumption,}\\\text{dry basis, kg/day}\end{pmatrix}\times\left(\frac{\text{million kg}}{10^6\text{ kg}}\right)\times\begin{pmatrix}\text{Br content}\\\text{dry coal, ppm}\end{pmatrix}$$

Where:

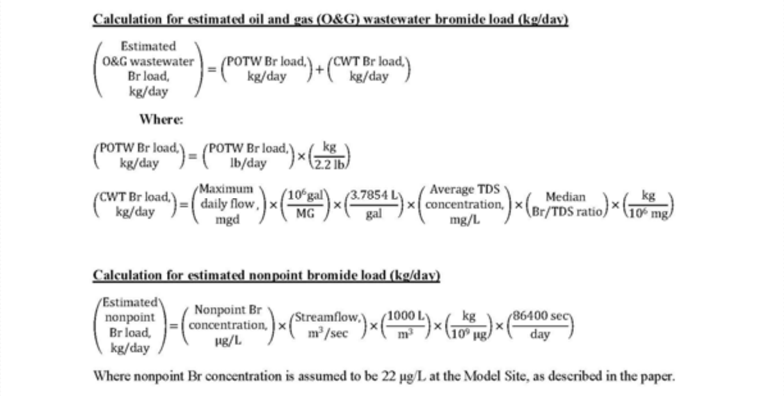
$$\begin{pmatrix}\text{Wet-FGD associated}\\\text{coal consumption,}\\\text{dry basis, kg/day}\end{pmatrix}=\begin{pmatrix}\text{Wet-FGD associated}\\\text{coal consumption,}\\\text{as received,}\\\text{tons/month}\end{pmatrix}\times\left(\frac{2000\text{ lb}}{\text{ton}}\right)\times\left(\frac{\text{kg}}{2.2\text{ lb}}\right)\times\left(\frac{\text{month}}{\text{days}}\right)\times\left(\frac{1}{1-\begin{pmatrix}\text{moisture}\\\text{content, \%}\end{pmatrix}}\right)$$

$$\begin{pmatrix}\text{Br content}\\\text{dry coal, ppm}\end{pmatrix}=\begin{pmatrix}\text{Br/Cl content}\\\text{in coal}\end{pmatrix}\times\begin{pmatrix}\text{Cl content}\\\text{dry coal, ppm}\end{pmatrix}$$

Calculation for estimated Br Addition wet FGD bromide load (kg/day)

Same as above, except for Br added for Hg control, as shown below.

$$\begin{pmatrix}\text{Estimated}\\\text{Br Addition}\\\text{wet FGD Br load,}\\\text{kg/day}\end{pmatrix}=\begin{pmatrix}\text{Br capture}\\\text{in wet FGD,}\\\%\end{pmatrix}\times\begin{pmatrix}\text{Wet-FGD associated}\\\text{coal consumption,}\\\text{dry basis, kg/day}\end{pmatrix}\times\left(\frac{\text{million kg}}{10^6\text{ kg}}\right)\times\left[\begin{pmatrix}\text{Br content}\\\text{dry coal, ppm}\end{pmatrix}+\begin{pmatrix}\text{Br added for}\\\text{Hg control,}\\\text{ppm in}\\\text{dry coal}\end{pmatrix}\right]$$



Calculation for estimated oil and gas (O&G) wastewater bromide load (kg/day)

$$\begin{pmatrix}\text{Estimated}\\ \text{O\&G wastewater}\\ \text{Br load,}\\ \text{kg/day}\end{pmatrix} = \begin{pmatrix}\text{POTW Br load,}\\ \text{kg/day}\end{pmatrix} + \begin{pmatrix}\text{CWT Br load,}\\ \text{kg/day}\end{pmatrix}$$

Where:

$$\begin{pmatrix}\text{POTW Br load,}\\ \text{kg/day}\end{pmatrix} = \begin{pmatrix}\text{POTW Br load,}\\ \text{lb/day}\end{pmatrix} \times \left(\frac{\text{kg}}{2.2\ \text{lb}}\right)$$

$$\begin{pmatrix}\text{CWT Br load,}\\ \text{kg/day}\end{pmatrix} = \begin{pmatrix}\text{Maximum}\\ \text{daily flow,}\\ \text{mgd}\end{pmatrix} \times \left(\frac{10^6\text{gal}}{\text{MG}}\right) \times \left(\frac{3.7854\ \text{L}}{\text{gal}}\right) \times \begin{pmatrix}\text{Average TDS}\\ \text{concentration,}\\ \text{mg/L}\end{pmatrix} \times \begin{pmatrix}\text{Median}\\ \text{Br/TDS ratio}\end{pmatrix} \times \left(\frac{\text{kg}}{10^6\ \text{mg}}\right)$$

Calculation for estimated nonpoint bromide load (kg/day)

$$\begin{pmatrix}\text{Estimated}\\ \text{nonpoint}\\ \text{Br load,}\\ \text{kg/day}\end{pmatrix} = \begin{pmatrix}\text{Nonpoint Br}\\ \text{concentration,}\\ \mu\text{g/L}\end{pmatrix} \times \begin{pmatrix}\text{Streamflow,}\\ \text{m}^3\text{/sec}\end{pmatrix} \times \left(\frac{1000\ \text{L}}{\text{m}^3}\right) \times \left(\frac{\text{kg}}{10^9\ \mu\text{g}}\right) \times \left(\frac{86400\ \text{sec}}{\text{day}}\right)$$

Where nonpoint Br concentration is assumed to be 22 μg/L at the Model Site, as described in the paper.

- To next lecture