Mid-Term EXAM

Closed book, 1 sheet of notes allowed.
Answer either question #I or #II. You must also answer #III. Please state any additional assumptions you made, and show all work.

I.   (50%) Hawkeye Creek receives runoff from more than a dozen corn farms in a small area of central Iowa. Each results in a certain dischage of organic nitrogen and contributes to the loading of Hawkeye Creek. Assume that the drainage area for Hawkeye Creek (10,000 ha) to its confluence with the Birdseye River is entirely made up of corn farms. Hawkeye creek receives a baseflow from groundwater that is 1.7 m³/s (at the confluence), and the nitrogen concentration in this baseflow is 0.25 mg/L. On July 7 the flow in Hawkeye creek at the confluence was measured as 3.925 m³/s. Assume the flow on this day was steady and reflective of the daily rainfall that occurred that day and the day before.

a. Determine the basin-wide average runoff coefficient ("C") based on the single flow datum measured on Hawkeye Creek near its confluence with the Birdseye River on July 7 of 3.925 m³/s
b. Determine flows for each of the remaining 9 days using the rainfall data below and the rational formula.
c. Calculate total organic nitrogen concentration for each day using the standard log-log model
d. Using this information, estimate the effective export coefficient for the corn farms during this 10-day period in units of kg-organic-N/ha/yr.
e. Comment on the accuracy and usefulness of this organic nitrogen export coefficient for this watershed. Consider the average rainfall for this watershed is 3.2 inches in July and 27 inches for the entire year.

Date	Avg Daily Rainfall (cm)	Organic Nitrogen Concentration (mg/L)
July 1	1.8	
July 2	3.2	4.2
July 3	0.7	
July 4	0	
July 5	1.1	
July 6	0.9	0.4
July 7	0.9	
July 8	2.1	
July 9	4.4	
July10	0.4	

There are several ways of solving this depending on how you handle the baseflow value. I believe the best way is to separate it from the runoff in all calcuations and separate the loadings of N (baseflow vs

runoff). Below I present my preferred approach (#1), followed by some alternatives that are in decreasing order of rigor and logic (and therefore get somewhat less credit)>

Approach #1: complete separation of baseflow from runoff; both for flow calculations and for determination of Q vs N relationship

Part a.

I subtracted the baseflow from the July 7th Q value to get the flow due to rain-induced runoff (Qrunoff) and then use the rational formula to calcuate "C", which turned out to give a value of 0.2136 or about **0.214**.

However, this presumes that the correct averaging period for the rational formula is a 24 hr day. Depending on the storm hydrograph for this watershed (information you don't have), it might make more sense to use a longer time in which to average rainfall.

Part b.

Now reversing this process and using the daily rainfall, we get the following runoff-based flow and to total flow including baseflow. This is calcuated using the runoff coefficient of 0.214 calculated in Part a.

Day	Qrunoff	Q total
1	4.4500	6.1500
2	7.9111	9.6111
3	1.7306	3.4306
4	0.0000	1.7000
5	2.7194	4.4194
6	2.2250	3.9250
7	2.2250	**3.9250**
8	5.1917	6.8917
9	10.8778	12.5778
10	0.9889	2.6889

Part c.

Here you should recognize that the nitrogen concentrations on July 2nd and July 6th are derived from a combination of base flow nitrogen loading and run-off based loading. So first you need to do a run-off and baseflow mass balance and determine the net runoff nitrogen concentration from those two days (see below). From these data and the runoff flow, you can determine the power function relationship.

		Coefficients from 2 data			
		a	b		Model
	base 10	-0.913769	1.800145		c=10^a(Q)^b
	10^x	0.1219639			log(c) = a + b[log(Q)]

			Gross	Net (from runoff)		Gross	
			Observed	Observed	Calculated	Calculated	
Day	I (cm)	Q(cms)	(mg/L)	(mg/L)	(mg/L)	(mg/L)	W (g/s)
1	1.8	6.1500			1.79	1.365861	8.40
2	3.2	9.6111	4.2	5.0488	5.05	4.20	40.37
3	0.7	3.4306			0.33	0.29	0.99
4	0	1.7000			0.00	0.25	0.43
5	1.1	4.4194			0.74	0.55	2.43
6	0.9	3.9250	0.4	0.5146	0.51	0.40	1.57
7	0.9	3.9250			0.51	0.40	1.57
8	2.1	6.8917			2.37	1.84	12.70
9	4.4	12.5778			8.96	7.78	97.86
10	0.4	2.6889			0.12	0.20	0.54
Average							16.69 g/s = 1441.669 kg/d

Part d.

Average load =		16.69 g/s	=	1,441.67 kg/d	=
				526569.7 kg/yr	
		total area	10000 ha		
				52.7 kg/ha/yr	

Part e.

There's a lot one could say here. Certainly the most notable issue is the variable nature of the export coefficient based on rainfall and runoff.

Also based on this analysis, organic nitrogen export is positively correlated with rainfall. It seems likely that organic nitrogen is associated with particulates (soil from application of fertilizers, plant matter) which is washed into streams at higher rates during wet events. This has the unfortuante affect of changing the export coefficient based on the particular level of rainfall. Over short periods where rainfall can be quite variable, this will add a substantial amount of error. However, over longer time periods, average rainfall becomes more uniform and export coefficients may be more accurate.

Also, it seems likely that nitrate-N washout should be seasonally-dependent. None of the simple models really take this into account.

Approach #2: separate baseflow from runoff to determine C, but not parsing concentrations between runofff and baseflow

	A=	10000	ha =	100000000	m2					baseflow =	1.7	cms		
Q=CIA	C=	0.2136								base N =	0.25	mg/L		
					Coefficients from 2 data									
					a	b		Model				a	b
				base 10	-1.063827	1.667114		c=10^a(Q)^b		base e	-0.30367	0.889883
				10^x	0.0863323			log(c) = a + b[log(Q)]				

			Gross	Net (from runoff)		Gross								
			Observed	Observed	Calculated	Calculated								
Day	I (cm)	Q(cms)	(mg/L)	(mg/L)	(mg/L)	(mg/L)	W(g/s)					Day	Qrunoff	Q total
1	0	1.7000			0.00	0.21	0.36	background		0.25	mg/L	1	0.0000	1.7000
2	0.5	2.9361	0.52	0.8913	0.89	0.52	1.53					2	1.2361	2.9361
3	0.7	3.4306			1.20	0.67	2.31					3	1.7306	3.4306
4	0	1.7000			0.00	0.21	0.36					4	0.0000	1.7000
5	1.1	4.4194			1.80	1.03	4.54					5	2.7194	4.4194
6	3.5	10.3528	4.25	5.0359	5.04	4.25	44.00					6	8.6528	10.3528
7	0.9	3.9250			1.50	0.84	3.31					7	2.2250	3.9250
8	0.2	2.1944			0.39	0.32	0.70					8	0.4944	2.1944
9	4.4	12.5778			6.17	5.88	73.95					9	10.8778	12.5778
10	0.4	2.6889			0.73	0.45	1.21					10	0.9889	2.6889
Sum							132.27	g/s =	11427.73	kg/d				

Average				Average load =	13.23	g/s =	1,142.77	kg/d =
							417397.7	kg/yr
				total area	10000	ha		
							41.7 kg/ha/yr	

Approach #3: ignore baseflow contribution and assume all flow is due to runoff, but subtract the baseflow nitrogen in calculating the loading and export coefficient

	A=	10000 ha =	100000000 m2								baseflow =	1.7 cms		
Q=CIA	C=	0.3768									base N =	0.25 mg/L		
				Coefficients from 2 data										
				a	b		Model				a	b		Model
			base 10	-1.037646	1.385279		c=10^a(Q)^b			base e	-0.30367	0.889883		c=e^a(Q)^b
			10^x	0.0916967			log(c) = a + b[log(Q)]							ln(c) = a +

			Gross	Net (from runoff)		Gross								
			Observed	Observed	Calculated	Calculated								
Day	I (cm)	Q(cms)	(mg/L)	(mg/L)	(mg/L)	(mg/L)	W(g/s)				Day	Qrunoff	Q total	C
1	0	0.0000			-0.25	0.00	0.00	background		0.25 mg/L	1	-1.7000	0.0000	
2	0.5	2.1806	0.52	1.4751	0.27	0.27	0.59				2	0.4806	2.1806	0.0830
3	0.7	3.0528			0.18	0.43	1.31				3	1.3528	3.0528	0.1670
4	0	0.0000			-0.25	0.00	0.00				4	-1.7000	0.0000	
5	1.1	4.7972			0.55	0.80	3.86				5	3.0972	4.7972	0.2433
6	3.5	15.2639	4.25	4.7513	4.00	4.00	61.06				6	13.5639	15.2639	0.3348
7	0.9	3.9250			0.36	0.61	2.39				7	2.2250	3.9250	0.2136
8	0.2	0.8722			-0.17	0.08	0.07				8	-0.8278	0.8722	-0.3576
9	4.4	19.1889			5.24	5.49	105.39				9	17.4889	19.1889	0.3434
10	0.4	1.7444			-0.05	0.20	0.35				10	0.0444	1.7444	0.0096
Sum							175.01 g/s =	15120.85 kg/d						

Average					Average load =	17.50 g/s =	1,512.09 kg/d =	
							552289.2 kg/yr	
					total area	10000 ha		
							55.2 kg/ha/yr	

Approach #4: ignore baseflow contribution and assume all flow is due to runoff and ignore baseflow N content

	A=	10000 ha =	100000000 m2					baseflow =	1.7 cms	
Q=CIA	C=	0.3768						base N =	0.25 mg/L	

Coefficients from 2 data

			a	b		Model			a	b
		base 10	-0.649521	1.079621		c=10^a(Q)^b		base e	-0.30367	0.889883
		10^x	0.2241192			log(c) = a + b[log(Q)]				

Day	I (cm)	Q(cms)	Gross Observed (mg/L)	Net (from runoff) Observed (mg/L)	Net (from runoff) Calculated (mg/L)	Gross Calculated (mg/L)	W(g/s)			Day	Qrunoff
1	0	0.0000			#NUM!	0.00	0.00	background	0.25 mg/L	1	-1.7000
2	0.5	2.1806	0.52	1.4751	0.38	0.52	1.13			2	0.4806
3	0.7	3.0528			0.97	0.75	2.28			3	1.3528
4	0	0.0000			#NUM!	0.00	0.00			4	-1.7000
5	1.1	4.7972			2.02	1.22	5.84			5	3.0972
6	3.5	15.2639	4.25	4.7513	7.51	4.25	64.87			6	13.5639
7	0.9	3.9250			1.50	0.98	3.85			7	2.2250
8	0.2	0.8722			#NUM!	0.19	0.17			8	-0.8278
9	4.4	19.1889			9.42	5.44	104.41			9	17.4889
10	0.4	1.7444			0.05	0.41	0.71			10	0.0444
Sum							183.27 g/s =	15834.68 kg/d			

Average				Average load =	18.33 g/s =	1,583.47 kg/d =		
						578361.9 kg/yr		
				total area	10000 ha			
					57.8 kg/ha/yr			

II.   (50%) On a separate sheet of paper, answer any five (5) of the following questions.

A.   Calculate the % loss of CBOD as water moves 2 kilometers downstream in a river flowing at 0.01 m/s. Assume the CBOD deoxygenation rate is 0.12 d^{-1}, and the CBOD settling rate is 0.10 d^{-1}.

B.   Describe the steps involved in a wasteload allocation process

C.   What is the steady state concentration of dioxane in a lake that receives wastewater from Acme Chemical Company (ACC). The lake has an area of 100,000 m^2, an average depth of 1 m, and an outflow of 1000 m^3/day. Dioxane decays at a 1st order rate of 0.5 yr^{-1}, and assume ACC discharges 11 kg/yr to the lake. Assume there are no other sources of dioxane loading to the lake.

D.   Describe what happens when a wastewater with ammonia is discharged into a flowing river. Be specific on the chemical changes and microbial ecology.

E.   Is it common to add an inhibitor to the BOD test? What does an inhibitor do and why or why not is it added?

F.   Explain the difference between CBOD and NBOD.

Most of the answers shown below are not complete; instead they make reference to sources of the correct information.

Part A.

10 points

Simple stream BOD model

		0.01	m/s		
		2000	m		
	travel time =	200000	s =	2.314815	days
	kd =	0.12	d-1		
	ks =	0.1	d-1		
	kr =	0.22	d-1		
	c/c0 =	60.1%	remaining		
	1-c/co =	**39.9%**	**lost**		

Part B.

10 points

Points to make here are:
- Beneficial use for the water body is specified

7

- This is translated into a <u>ambient water quality standards</u> for parameters of critical concern
- A <u>model</u> is developed for the water body
- Design conditions are established (usually 7Q10 flow)
- Using this model under the design condition, reductions waste loading are simulated and a set of feasible options are determined that are predicted to lead to compliance with the ambient standard
- At various points there will be model validation tests and sensitivity tests for key parameters

Part C.

<div align="right">10 points</div>

This is a simple steady state lake model.

Lake Data										
	Q =	1.00E+03	m3/day		Co =	0	mg/L		tau =	0.27	yr
	A =	1.00E+05	m2						depth =	1.00E+00	m
	V =	1.00E+05	m3		k =	0.0013689	/day =	0.5	"/yr	
SS Loading										
	W =	1.10E+01	kg/y							

Solution							
		$$\lambda = \frac{Q}{V} + k$$		lambda =	0.011369 /d	=	4.1525 /yr
				assimilation =	1.14E+03 m3/d	=	415250 m3/yr
SS Solution							
		W/a =	2.65E-05 kg/m3 =		**26.5 ug/L**		

Part D.

<div align="right">10 points</div>

Key points:
- Overall oxidation of ammonia is called nitrification
- Oxidation of ammonia to nitrite by nitrosomonas
- Oxidation of nitrite to nitrate by nitrobacter.
- This creates a nitrogenous oxygen demand of 2 moles of oxygen per mole of ammonia (~4.6 mg O_2/mg-ammonia-N)

Part E.

<div align="right">10 points</div>

Yes it is common. An inhibitor is used to stop nitrification. The BOD test is intended to measure carbonaceous BOD only. NBOD can be easily measured using the TKN analysis.

Part F.

<div align="right">10 points</div>

Key points
- CBOD is carbonaceous, resulting from oxidation of compounds containing carbon, but not from oxidation of any nitrogen – this must be assessed using a biological test (the BOD test)

- NBOD is nitrogenous, resulting from oxidation of any nitrogen, whether inorganic (ammonia, nitrite) or organic-N (amines, nucleic acids, etc) – this can be assessed by measuring Total Kjeldahl Nitrogen (TKN).

III. (50%) The Green River emerges from pristine headwaters and runs through an agricultural region. Four miles into this agricultural region is a municipal wastewater discharge (from the city of Millbury). The $CBOD_u$ of the headwaters is 2.8 mg/L, and the dissolved oxygen is 7.2 mg/L (saturation is 9.1 mg/L for 20°C). Starting at mile point zero, there is a significant non-point agricultural runoff of $CBOD_u$ amounting to 45 kg/mile/day. At mile point 4 is the Millbury WWTP outfall. Here a fully nitrified WW flow of 10 cfs is discharged with a $CBOD_u$ of 30 mg/L and a DO of 4 mg/L. Immediately past this outfall is 3 more miles of agricultural land.

A) Calculate the dissolved oxygen concentration immediately above the WWTP outfall (i.e. at MP 3.999999)
B) Calculate the dissolved oxygen concentration immediately below the WWTP outfall (i.e. at MP 4.000001)
C) Calculate the dissolved oxygen concentration at the end of the agricultural land (i.e. at MP 7

Assume complete mixing across the stream in cross section. Assume also a temperature of 20°C, a flow of 40 cfs from the headwaters and no incremental flow downstream except for flow from the Millbury WWTP. Many years of discharge has resulted in a very high SOD immediately downstream of the WWTP outfall of 5 $g/m^2/d$ for the first mile (i.e., up to MP 5). After this, you can assume that it decreases to a fixed value of 0.5 $g/m^2/d$.
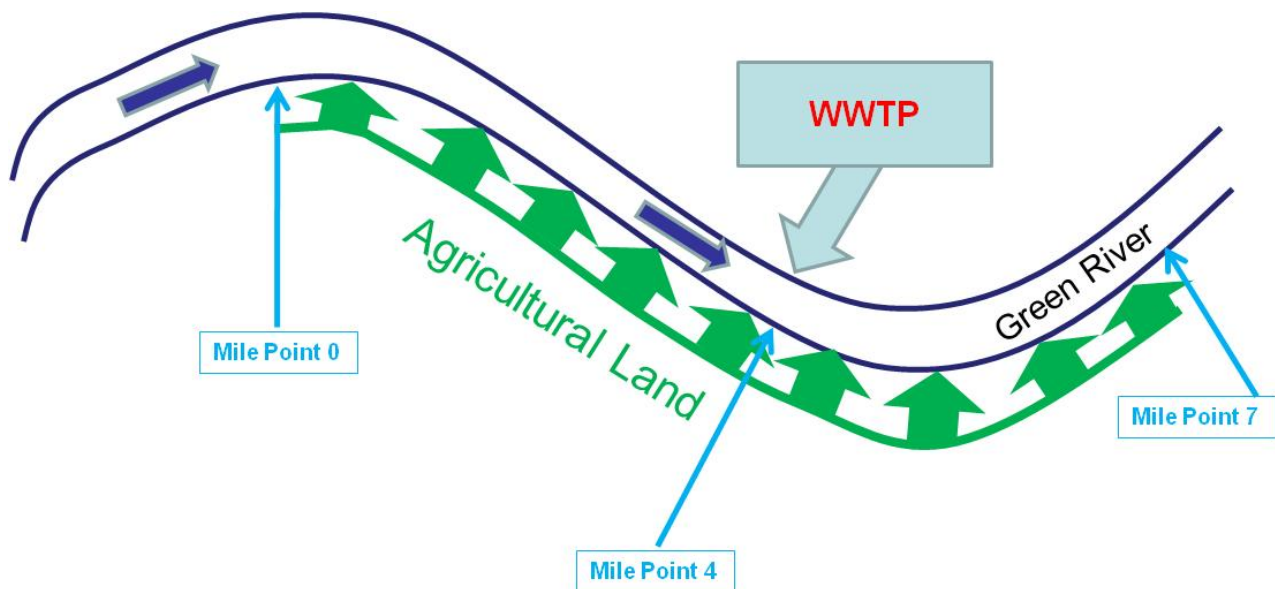
Additional Information:

U = 0.200 ft/sec = 3.27 miles/day	T = 20°C
DO_{sat} or C_S = 9.1 mg/L (at 20°C)	H = 4 ft = 1.22 m
BOD deoxygenation rate ($k_N = k_d$) = 0.8 day^{-1} (at 20°C)	for k_N and k_d, θ=1.047
CBOD settling rate (k_S) = 0.080 day^{-1} (at 20°C)	for reareation, θ=1.024

A) Calculate the dissolved oxygen concentration immediately above the WWTP outfall (i.e. at MP 3.999999)

Now the general form of the extended Streeter-Phelps equation is:

$$D = D_o e^{-k_a t} + \frac{k_d L_o}{k_a - k_r}(e^{-k_r t} - e^{-k_a t}) + \frac{k_d L_{No}}{k_a - k_n}(e^{-k_n t} - e^{-k_a t})$$
$$+ \frac{R - P + (S'_B/H)}{k_a}(1 - e^{-k_a t}) + \frac{k_d S_d}{k_r k_a}(1 - e^{-k_a t}) - \frac{k_d S_d}{k_r(k_a - k_r)}(e^{-k_r t} - e^{-k_a t})$$
$$+ \frac{k_n S_{Nd}}{k_n k_a}(1 - e^{-k_a t}) - \frac{k_{nd} S_{Nd}}{k_n(k_a - k_n)}(e^{-k_n t} - e^{-k_a t})$$

And in this case without any NBOD or algal activity, this reduces to:

$$D = D_o e^{-k_a t} + \frac{k_d L_o}{k_a - k_r}(e^{-k_r t} - e^{-k_a t}) + \frac{(S'_B/H)}{k_a}(1 - e^{-k_a t}) + \frac{k_d S_d}{k_r k_a}(1 - e^{-k_a t})$$
$$- \frac{k_d S_d}{k_r(k_a - k_r)}(e^{-k_r t} - e^{-k_a t})$$

We are given the following:

k_d = 0.8 d^{-1} @ 20°C
k_s = 0.08 d^{-1} @ 20°C
H = 4 ft = 1.22 m
C_s = 9.1 mg/L
U = 3.27 miles/day

And since our design temperature is 20°C, there is no need for any temperature corrections

Now, begins the determination of the missing parameters

Rearation Constant:. Based on depth and velocity, it is appropriate to use the O'Connor Dobbins formula.

O'Connor–Dobbins / Churchill,et.al. / Owens,et.al.

DEPTH, ft. — VELOCITY, ft./sec.
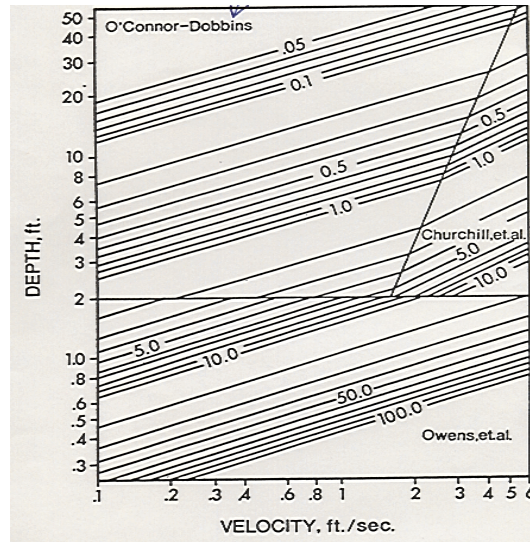
$$k_a = 12.9 \frac{(0.2)^{0.5}}{(4)^{1.5}} = 0.721 d^{-1}$$

$$D = D_o e^{-0.721 d^{-1} t} + \frac{k_d L_o}{0.721 d^{-1} - k_r}\left(e^{-k_r t} - e^{-0.721 d^{-1} t}\right) + \frac{(S_B'/H)}{0.721 d^{-1}}\left(1 - e^{-0.721 d^{-1} t}\right)$$
$$+ \frac{k_d S_d}{k_r 0.721 d^{-1}}\left(1 - e^{-0.721 d^{-1} t}\right) - \frac{k_d S_d}{k_r(0.721 d^{-1} - k_r)}\left(e^{-k_r t} - e^{-0.721 d^{-1} t}\right)$$

And we might as well insert the given parameters too at this point

$$D = D_o e^{-0.721 d^{-1} t} + \frac{0.8 d^{-1} L_o}{0.721 d^{-1} - k_r}\left(e^{-k_r t} - e^{-0.721 d^{-1} t}\right) + \frac{(S_B'/1.22m)}{0.721 d^{-1}}\left(1 - e^{-0.721 d^{-1} t}\right)$$
$$+ \frac{0.8 d^{-1} S_d}{k_r 0.721 d^{-1}}\left(1 - e^{-0.721 d^{-1} t}\right) - \frac{0.8 d^{-1} S_d}{k_r(0.721 d^{-1} - k_r)}\left(e^{-k_r t} - e^{-0.721 d^{-1} t}\right)$$

And now the <u>overall CBOD loss rate</u> is the sum of the deoxygenation rate and the settling rate, so:

$$k_r = k_d + k_s = 0.8 \text{ d}^{-1} + 0.08 \text{ d}^{-1} = \textbf{0.88 d}^{-1} \quad @ \text{ 20°C}$$

The <u>time of travel</u> is equal to x/U or

$$t = x/U = 4 \text{ miles} / 3.27 \text{ miles/d} = \textbf{1.222 d}$$

The <u>areal non-point loading</u> term is calculated from:

Cross sectional area (Acs = Q/U = 40 cfs / 3.27 mi/d)

$$A_{cs} = \frac{40 ft^3 s^{-1}}{0.2 fts^{-1}}\left(\frac{m}{3.2808 ft}\right)^2 = 18.58 m^2$$

Now S_d is:

$$S_d = \frac{S}{A_{cs}} = \frac{45\frac{Kg}{mi-d}}{18.58m^2}\left(\frac{mi}{5280ft}\right)\left(\frac{3.2808ft}{m}\right)\frac{1000g}{Kg} = \mathbf{1.5048}\frac{\boldsymbol{mg}}{\boldsymbol{L-d}}$$

$$D = D_o e^{-0.721d^{-1}1.222d} + \frac{0.8d^{-1}L_o}{0.721d^{-1} - 0.88d^{-1}}\left(e^{-0.88d^{-1}1.222d} - e^{-0.721d^{-1}1.222d}\right)$$
$$+ \frac{(S_B'/1.22m)}{0.721d^{-1}}\left(1 - e^{-0.721d^{-1}1.222d}\right) + \frac{0.8d^{-1}1.505\frac{mg}{L-d}}{0.88d^{-1}0.721d^{-1}}\left(1 - e^{-0.721d^{-1}1.222d}\right)$$
$$- \frac{0.8d^{-1}1.505\frac{mg}{L-d}}{0.88d^{-1}(0.721d^{-1} - 0.88d^{-1})}\left(e^{-0.88d^{-1}1.222d} - e^{-0.721d^{-1}1.222d}\right)$$

Now all we need are the initial values for DO deficit and CBOD to solve. These are normally obtained from simple mass balances at the head of the reach, but since there was no upstream point souce, these values are simple the upstream values as cited in the problem statement.

$L_o = L_u = 2.8$ mg/L   (note, all CBOD values provided are "ultimate")
$C_o = C_u = 7.2$ mg/L

And therefore the deficit is:

$D_o = C_s - C_o = 9.1$mg/L – 7.2 mg/L = 1.9 mg/L

Now substituting in:

$$D = 1.9\frac{mg}{L}e^{-0.721d^{-1}1.222d} + \frac{0.8d^{-1}2.8\frac{mg}{L}}{0.721d^{-1} - 0.88d^{-1}}\left(e^{-0.88d^{-1}1.222d} - e^{-0.721d^{-1}1.222d}\right)$$
$$+ \frac{(S_B'/1.22m)}{0.721d^{-1}}\left(1 - e^{-0.721d^{-1}1.222d}\right) + \frac{0.8d^{-1}1.505\frac{mg}{L-d}}{0.88d^{-1}0.721d^{-1}}\left(1 - e^{-0.721d^{-1}1.222d}\right)$$
$$- \frac{0.8d^{-1}1.505\frac{mg}{L-d}}{0.88d^{-1}(0.721d^{-1} - 0.88d^{-1})}\left(e^{-0.88d^{-1}1.222d} - e^{-0.721d^{-1}1.222d}\right)$$

And recognizing that we don't have any SOD in this upstream region:

$$D = 1.9\frac{mg}{L}e^{-0.721d^{-1}1.222d} + \frac{0.8d^{-1}2.8\frac{mg}{L}}{0.721d^{-1} - 0.88d^{-1}}\left(e^{-0.88d^{-1}1.222d} - e^{-0.721d^{-1}1.222d}\right)$$
$$+ \frac{0.8d^{-1}1.505\frac{mg}{L-d}}{0.88d^{-1}0.721d^{-1}}\left(1 - e^{-0.721d^{-1}1.222d}\right)$$
$$- \frac{0.8d^{-1}1.505\frac{mg}{L-d}}{0.88d^{-1}(0.721d^{-1} - 0.88d^{-1})}\left(e^{-0.88d^{-1}1.222d} - e^{-0.721d^{-1}1.222d}\right)$$

$$D = 0.7870\frac{mg}{L} + 1.0307\frac{mg}{L} + 1.1113\frac{mg}{L} - 0.6295\frac{mg}{L} = 2.2995\frac{mg}{L}$$

$$C = C_s\text{-}D = 9.1\text{-}2.30 = 6.80 \text{ mg/L}$$

B) Calculate the dissolved oxygen concentration immediately below the WWTP outfall (i.e. at MP 4.000001)

To calculate this value, we simply need a mass balance around the mixing point at the WWTP outfall.

$$C = \frac{Q_u C_u + Q_{ww} C_{ww}}{Q_u + Q_{ww}} = \frac{40\frac{ft^3}{s}6.80\frac{mg}{L} + 10\frac{ft^3}{s}4.0\frac{mg}{L} +}{40\frac{ft^3}{s} + 10\frac{ft^3}{s}} = 6.24\frac{mg}{L}$$

So:

$$C = 6.24 \text{ mg/L}$$

C) Calculate the dissolved oxygen concentration at the end of the agricultural land (i.e. at MP 7

Now we use the same equations as in part "A", but this time with new initial values and a non-zero SOD. In addition, we must break this reach into two separate ones as the SOD is higher between MP 4 and 5 than it is between MP 5 and 7.

SOD: Recall that S'$_B$ is given as 5 g/m^2/d for the first mile downstream of the WWTP

And the initial deficit is simply from the saturation value and the answer in part B

$$D_o = C_s - C_o = 9.1 \text{mg/L} - 6.24 \text{ mg/L} = 2.86 \text{ mg/L}$$

And finally, the initial CBOD is from a similar mass balance, but we first have to calculate the BOD at the end of the first reach. This is done with the BOD model including non-point sources:

$$L = L_o e^{-k_r t} + \frac{S_d}{k_r}(1 - e^{-k_r t})$$

$$L = 2.8\frac{mg}{L}e^{-0.88d^{-1}1.222d} + \frac{1.505\frac{mg}{L-d}}{0.88d^{-1}}(1 - e^{-0.88d^{-1}1.222d}) = 0.9551\frac{mg}{L} + 1.1267\frac{mg}{L} = 2.082\frac{mg}{L}$$

And the mass balance is:

$$L = \frac{Q_u L_u + Q_{ww} L_{ww}}{Q_u + Q_{ww}} = \frac{40\frac{ft^3}{s}2.082\frac{mg}{L} + 10\frac{ft^3}{s}30\frac{mg}{L} +}{40\frac{ft^3}{s} + 10\frac{ft^3}{s}} = 7.67\frac{mg}{L}$$

Likewise for the deficit

13

$$D = \frac{Q_u D_u + Q_{ww} D_{ww}}{Q_u + Q_{ww}} = \frac{40\frac{ft^3}{s}1.41\frac{mg}{L} + 10\frac{ft^3}{s}5.1\frac{mg}{L}+}{40\frac{ft^3}{s} + 10\frac{ft^3}{s}} = 2.86\frac{mg}{L}$$

With the higher flow, and a fixed depth and velocity, we have a larger width and need to re-determine S_d :

$$S_d = \frac{S}{A_{cs}} = \frac{45\frac{Kg}{mi-d}}{23.22m^2}\left(\frac{mi}{5280ft}\right)\left(\frac{3.2808ft}{m}\right)\frac{1000g}{Kg} = \mathbf{1.2039}\frac{mg}{L-d}$$

The new <u>time of travel</u> for the 1-mile reach with elevated SOD is:

t = x/U = 1 mile / 3.27 miles/d = **0.30555 d**

So now we can plug these values into the Streeter-Phelps equation:

$$D = 2.86\frac{mg}{L}e^{-0.721d^{-1}0.3056d} + \frac{0.8d^{-1}7.67\frac{mg}{L}}{0.721d^{-1} - 0.88d^{-1}}\left(e^{-0.88d^{-1}0.3056d} - e^{-0.721d^{-1}0.3056d}\right)$$
$$+ \frac{\left(5\frac{g}{m^2-d}/1.22m\right)}{0.721d^{-1}}\left(1 - e^{-0.721d^{-1}0.3056d}\right)$$
$$+ \frac{0.8d^{-1}1.204\frac{mg}{L-d}}{0.88d^{-1}0.721d^{-1}}\left(1 - e^{-0.721d^{-1}0.3056d}\right)$$
$$- \frac{0.8d^{-1}1.204\frac{mg}{L-d}}{0.88d^{-1}(0.721d^{-1} - 0.88d^{-1})}\left(e^{-0.88d^{-1}0.3056d} - e^{-0.721d^{-1}0.3056d}\right)$$

$$D = 2.29\frac{mg}{L} + 1.47\frac{mg}{L} + 1.125\frac{mg}{L} + 0.30\frac{mg}{L} - 0.26\frac{mg}{L} = 4.92\frac{mg}{L}$$

C = C$_s$-D = 9.1-4.92 = 4.18 mg/L

Now recalculate initial conditions and do the same for the reach with the lower SOD

<u>SOD:</u> Recall that S'$_B$ is given as 0.5 g/m^2/d for the second and third mile downstream of the WWTP

The new <u>initial deficit</u> is simply from the answer above

D$_o$ = C$_s$ – C$_o$ = 9.1mg/L – 4.18 mg/L = 4.92 mg/L

And finally, the <u>initial CBOD</u> is the BOD at the end of the previous reach. This is done with the BOD model including non-point sources:

$$L = L_o e^{-k_r t} + \frac{S_d}{k_r}(1 - e^{-k_r t})$$

$$L = 7.67\frac{mg}{L}e^{-0.88d^{-1}0.3056d} + \frac{1.204\frac{mg}{L-d}}{0.88d^{-1}}\left(1 - e^{-0.88d^{-1}0.3056d}\right) = 5.86\frac{mg}{L} + 0.32\frac{mg}{L} = 6.18\frac{mg}{L}$$

Note that there is no change in the S_d value

$$S_d = \frac{S}{A_{cs}} = \frac{45\frac{Kg}{mi-d}}{20.90m^2}\left(\frac{mi}{5280ft}\right)\left(\frac{3.2808ft}{m}\right)\frac{1000g}{Kg} = \mathbf{1.204}\frac{mg}{L-d}$$

The new <u>time of travel</u> for the 2-mile reach with lower SOD is:

t = x/U = 2 mile / 3.27 miles/d = **0.6111 d**

So now we can plug these values into the Streeter-Phelps equation:

$$D = 4.92\frac{mg}{L}e^{-0.721d^{-1}0.6111d} + \frac{0.8d^{-1}6.18\frac{mg}{L}}{0.721d^{-1} - 0.88d^{-1}}\left(e^{-0.88d^{-1}0.6111d} - e^{-0.721d^{-1}0.6111d}\right)$$
$$+ \frac{\left(0.5\frac{g}{m^2-d}/1.22m\right)}{0.721d^{-1}}\left(1 - e^{-0.721d^{-1}0.6111d}\right)$$
$$+ \frac{0.8d^{-1}1.204\frac{mg}{L-d}}{0.88d^{-1}0.721d^{-1}}\left(1 - e^{-0.721d^{-1}0.6111d}\right)$$
$$- \frac{0.8d^{-1}1.204\frac{mg}{L-d}}{0.88d^{-1}(0.721d^{-1} - 0.88d^{-1})}\left(e^{-0.88d^{-1}0.6111d} - e^{-0.721d^{-1}0.6111d}\right)$$

$$D = 3.17\frac{mg}{L} + 1.85\frac{mg}{L} + 0.203\frac{mg}{L} + 0.54\frac{mg}{L} - 0.41\frac{mg}{L} = 5.36\frac{mg}{L}$$

C = C$_s$-D = 9.1-5.36 = 3.74 mg/L

Summary table for problem III:

MP	C (mg/L)	D (mg/L)	L (mg/L)	Δt (d)
0	7.2	1.9	2.8	
3.99999	6.8	2.3	2.082	1.222
4.00001	6.24	2.86	7.67	
5	4.18	4.92	6.18	0.3056
7	3.74	5.36		0.611

Useful conversions:

1 mile = 1609 meters
1 ha = 10,000 m^2 = 2.471 acres
1 inch = 2.54 cm