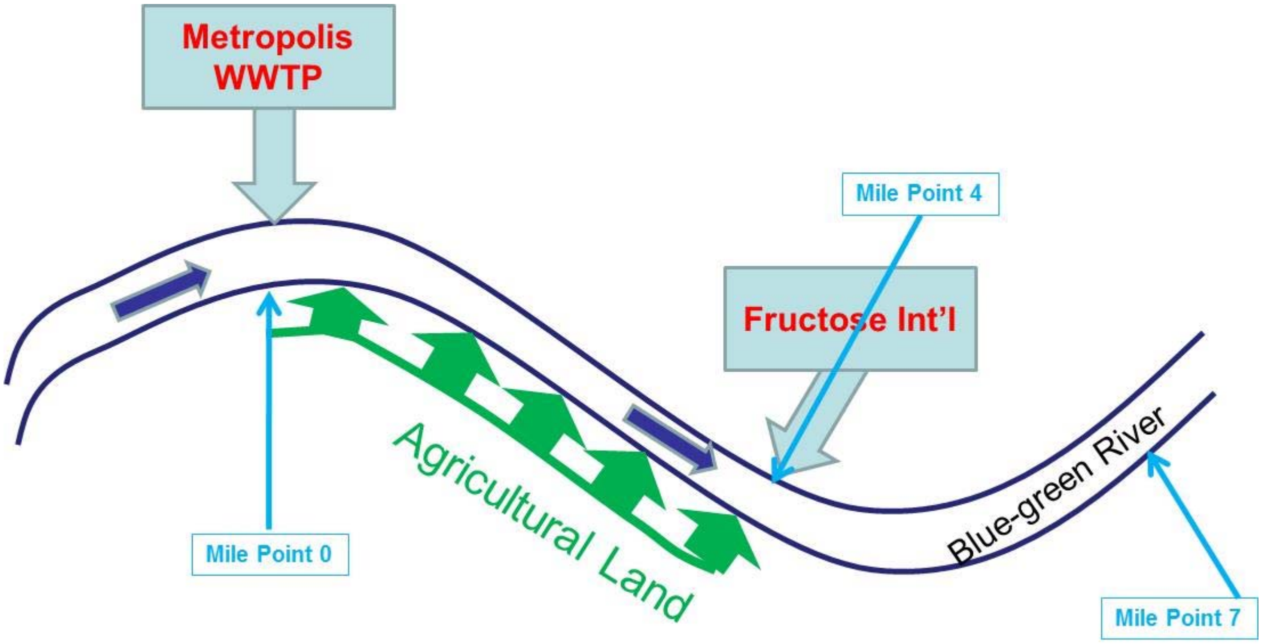
CEE 577 Fall 2017

Homework #6

The Blue-green River emerges from pristine headwaters and runs by a city (Metropolis with its WWTP) and then through an agricultural region that stretches for 4 miles. At milepoint 4, there is an industrial wastewater discharge (from Fructose International Corporation, FIC). The $CBOD_u$ of the headwaters is 2.8 mg/L, and the dissolved oxygen is 7.2 mg/L (saturation is 9.1 mg/L for 20ºC). At mile point 0 is the Metropolis wastewater outfall. Here a fully nitrified WW flow of 10 cfs is discharged with a $CBOD_u$ of 20 mg/L and a DO of 4 mg/L. Also starting at mile point zero, there is a significant non-point agricultural runoff of $CBOD_u$ amounting to 45 kg/mile/day. At milepoint 4 is the FIC outfall with its 200 mg/L of $CBOD_u$ in the form of high fructose corn syrup. This flow is 2 cfs and the DO is zero. Immediately past this outfall is 3 more miles of undeveloped land.

A) Calculate the dissolved oxygen concentration immediately above the FIC wastewater outfall (i.e. at MP 3.9999)
B) Calculate the dissolved oxygen concentration immediately below the FIC outfall (i.e. at MP 4.0001)
C) Calculate the dissolved oxygen concentration at the end of the undeveloped land (i.e. at MP 7)

Assume complete mixing across the stream in cross section. Assume also a temperature of 20ºC, a flow of 40 cfs from the headwaters and no incremental flow downstream except for flow from the city and FIC industrial outfall. Many years of discharge has resulted in a very high SOD immediately downstream of the municipal WWTP outfall of 2.2 $g/m^2/d$ for the first four miles (i.e., up to MP 4). After this, you can assume that it decreases to a fixed value of 0.5 $g/m^2/d$. Assume the undeveloped land does not introduce any additional flow or BOD loading.



Additional Information:

U = 0.200 ft/sec = 3.27 miles/day	T = 20ºC
DO_{sat} or C_s = 9.1 mg/L (at 20ºC)	H = 4 ft = 1.22 m
BOD deoxygenation rate ($k_N = k_d$) = 0.8 day^{-1} (at 20ºC)	for k_N and k_d, θ=1.047
CBOD settling rate (k_s) = 0.080 day^{-1} (at 20ºC)	for reareation, θ=1.024

A) <u>Calculate the dissolved oxygen concentration immediately above the FIC outfall (i.e. at MP 3.999999)</u>

Now the general form of the extended Streeter-Phelps equation is:

$$\begin{aligned} D = D_o e^{-k_a t} &+ \frac{k_d L_o}{k_a - k_r}\left(e^{-k_r t} - e^{-k_a t}\right) + \frac{k_d L_{No}}{k_a - k_n}\left(e^{-k_n t} - e^{-k_a t}\right) \\ &+ \frac{R - P + (S'_B/H)}{k_a}\left(1 - e^{-k_a t}\right) + \frac{k_d S_d}{k_r k_a}\left(1 - e^{-k_a t}\right) \\ &- \frac{k_d S_d}{k_r(k_a - k_r)}\left(e^{-k_r t} - e^{-k_a t}\right) + \frac{k_n S_{Nd}}{k_n k_a}\left(1 - e^{-k_a t}\right) \\ &- \frac{k_{nd} S_{Nd}}{k_n(k_a - k_n)}\left(e^{-k_n t} - e^{-k_a t}\right) \end{aligned}$$

And in this case without any NBOD or algal activity, this reduces to:

$$D = D_o e^{-k_a t} + \frac{k_d L_o}{k_a - k_r}\left(e^{-k_r t} - e^{-k_a t}\right) + \frac{(S_B'/H)}{k_a}\left(1 - e^{-k_a t}\right) + \frac{k_d S_d}{k_r k_a}\left(1 - e^{-k_a t}\right) - \frac{k_d S_d}{k_r(k_a - k_r)}\left(e^{-k_r t} - e^{-k_a t}\right)$$

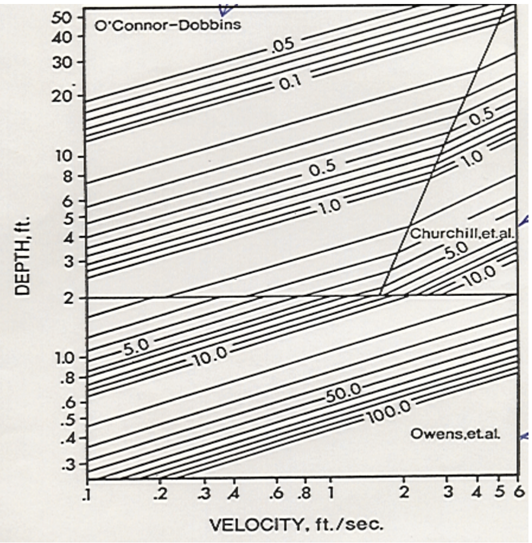
We are given the following:

$k_d = 0.8\ d^{-1}$ @ 20°C
$k_s = 0.08\ d^{-1}$ @ 20°C
H = 4 ft = 1.22 m
C_s = 9.1 mg/L
U = 3.27 miles/day

And since our design temperature is 20°C, there is no need for any temperature corrections

Now, begins the determination of the missing parameters

<u>Rearation Constant</u>:. Based on depth and velocity, it is appropriate to use the O'Connor Dobbins formula.



$$k_a = 12.9\frac{(0.2)^{0.5}}{(4)^{1.5}} = 0.721 d^{-1}$$

$$D = D_o e^{-0.721 d^{-1} t} + \frac{k_d L_o}{0.721 d^{-1} - k_r}\left(e^{-k_r t} - e^{-0.721 d^{-1} t}\right) + \frac{(S_B'/H)}{0.721 d^{-1}}\left(1 - e^{-0.721 d^{-1} t}\right) + \frac{k_d S_d}{k_r 0.721 d^{-1}}\left(1 - e^{-0.721 d^{-1} t}\right) - \frac{k_d S_d}{k_r(0.721 d^{-1} - k_r)}\left(e^{-k_r t} - e^{-0.721 d^{-1} t}\right)$$

And we might as well insert the given parameters too at this point

$$D = D_o e^{-0.721d^{-1}t} + \frac{0.8d^{-1}L_o}{0.721d^{-1} - k_r}\left(e^{-k_r t} - e^{-0.721d^{-1}t}\right) + \frac{(S_B'/1.22m)}{0.721d^{-1}}\left(1 - e^{-0.721d^{-1}t}\right) + \frac{0.8d^{-1}S_d}{k_r 0.721d^{-1}}\left(1 - e^{-0.721d^{-1}t}\right) - \frac{0.8d^{-1}S_d}{k_r(0.721d^{-1} - k_r)}\left(e^{-k_r t} - e^{-0.721d^{-1}t}\right)$$

And now the overall CBOD loss rate is the sum of the deoxygenation rate and the settling rate, so:

$k_r = k_d + k_s = 0.8\ d^{-1} + 0.08\ d^{-1} = \mathbf{0.88\ d^{-1}}$ @ 20°C

The time of travel is equal to x/U or

$t = x/U = 4$ miles / 3.27 miles/d $= \mathbf{1.222\ d}$

The areal non-point loading term is calculated from:

Cross sectional area (Acs = Q/U = 40 cfs / 3.27 mi/d)

$$A_{cs} = \frac{40ft^3s^{-1} + 10ft^3s^{-1}}{0.2fts^{-1}}\left(\frac{m}{3.2808ft}\right)^2 = 23.226m^2$$

Now S_d is:

$$S_d = \frac{S}{A_{cs}} = \frac{45\frac{Kg}{mi-d}}{23.226m^2}\left(\frac{mi}{5280ft}\right)\left(\frac{3.2808ft}{m}\right)\frac{1000g}{Kg} = \mathbf{1.2039\frac{mg}{L-d}}$$

$$D = D_o e^{-0.721d^{-1}1.222d} + \frac{0.8d^{-1}L_o}{0.721d^{-1} - 0.88d^{-1}}\left(e^{-0.88d^{-1}1.222d} - e^{-0.721d^{-1}1.222d}\right) + \frac{(S_B'/1.22m)}{0.721d^{-1}}\left(1 - e^{-0.721d^{-1}1.222d}\right) + \frac{0.8d^{-1}1.2039\frac{mg}{L-d}}{0.88d^{-1}0.721d^{-1}}\left(1 - e^{-0.721d^{-1}1.222d}\right) - \frac{0.8d^{-1}1.2039\frac{mg}{L-d}}{0.88d^{-1}(0.721d^{-1} - 0.88d^{-1})}\left(e^{-0.88d^{-1}1.222d} - e^{-0.721d^{-1}1.222d}\right)$$

Now all we need are the initial values for DO deficit and CBOD to solve. These are normally obtained from simple mass balances at the head of the reach, but since there was no upstream point source, these values are simple the upstream values as cited in the problem statement.

$$L_0 = \frac{Q_u L_u + Q_w L_w}{Q_u + Q_w}$$

$$L_0 = \frac{40cfs(\frac{2.8mg}{L}) + 10cfs(\frac{20mg}{L})}{40cfs + 10\ cfs} = 6.24\frac{mg}{L}$$

(note, all CBOD values provided are "ultimate")

And now the initial dissolved oxygen

$$C_0 = \frac{Q_u C_u + Q_w C_w}{Q_u + Q_w}$$

$$C_0 = \frac{40cfs(\frac{7.2mg}{L}) + 10cfs(\frac{4.0mg}{L})}{40cfs + 10\ cfs} = \mathbf{6.56\frac{mg}{L}}$$

And therefore the deficit is:

$D_o = C_s - C_o = 9.1mg/L - 6.56\ mg/L =$ **2.54 mg/L**

Now substituting in:

$$\begin{aligned} D = {} & 2.54\frac{mg}{L}e^{-0.721d^{-1}1.222d} + \frac{0.8d^{-1}6.24\frac{mg}{L}}{0.721d^{-1} - 0.88d^{-1}}\left(e^{-0.88d^{-1}1.222d} - e^{-0.721d^{-1}1.222d}\right) \\ & + \frac{\left(2.2\frac{g}{m^2d}/1.22m\right)}{0.721d^{-1}}\left(1 - e^{-0.721d^{-1}1.222d}\right) \\ & + \frac{0.8d^{-1}1.2039\frac{mg}{L-d}}{0.88d^{-1}0.721d^{-1}}\left(1 - e^{-0.721d^{-1}1.222d}\right) \\ & - \frac{0.8d^{-1}1.2039\frac{mg}{L-d}}{0.88d^{-1}(0.721d^{-1} - 0.88d^{-1})}\left(e^{-0.88d^{-1}1.222d} - e^{-0.721d^{-1}1.222d}\right) \end{aligned}$$

$$D = 1.0521\frac{mg}{L} + 2.2970\frac{mg}{L} + 1.4658\frac{mg}{L} + 0.3854\frac{mg}{L} = 5.2003\frac{mg}{L}$$

$C = C_s$-D = 9.1-5.2003 = 3.8997 mg/L

B) <u>Calculate the dissolved oxygen concentration immediately below the FIC outfall (i.e. at MP 4.000001)</u>

To calculate this value, we simply need a mass balance around the mixing point at the WWTP outfall.

$$C = \frac{Q_u C_u + Q_{ww} C_{ww}}{Q_u + Q_{ww}} = \frac{50\frac{ft^3}{s}3.8897\frac{mg}{L} + 2\frac{ft^3}{s}0\frac{mg}{L} +}{50\frac{ft^3}{s} + 2\frac{ft^3}{s}} = 3.7497\frac{mg}{L}$$

So:

C = 3.75 mg/L

C) Calculate the dissolved oxygen concentration at the end of the agricultural land (i.e. at MP 7

Now we use the same equations as in part "A", but this time with new initial values and a different non-zero SOD.

SOD: Recall that S'_B is given as 2.2 g/m^2/d for the first four miles downstream of the WWTP, and then it drops to fixed value of 0.5 g/m^2/d

And the initial deficit for this second reach is simply from the saturation value and the final DO from the answer in part B

$D_o = C_s – C_o$ = 9.1mg/L – 3.75 mg/L = 5.35 mg/L

And finally, the initial CBOD is from a similar mass balance, but we first have to calculate the BOD at the end of the first reach. This is done with the BOD model including non-point sources:

$$L = L_o e^{-k_r t} + \frac{S_d}{k_r}\left(1 - e^{-k_r t}\right)$$

$$L = 6.24\frac{mg}{L} e^{-0.88d^{-1}1.222d} + \frac{1.204\frac{mg}{L-d}}{0.88d^{-1}}\left(1 - e^{-0.88d^{-1}1.222d}\right) = 2.129\frac{mg}{L} + 0.914\frac{mg}{L}$$
$$= 3.03\frac{mg}{L}$$

And the mass balance is:

$$L = \frac{Q_u L_u + Q_{ww} L_{ww}}{Q_u + Q_{ww}} = \frac{50\frac{ft^3}{s}3.03\frac{mg}{L} + 2\frac{ft^3}{s}200\frac{mg}{L} +}{50\frac{ft^3}{s} + 2\frac{ft^3}{s}} = 10.61\frac{mg}{L}$$

The new time of travel for the 3-mile reach with is:

t = x/U = 3 mile / 3.27 miles/d = **0.917 d**

So now we can plug these values into the Streeter-Phelps equation:

$$D = 5.35\frac{mg}{L}e^{-0.721d^{-1}0.917d} + \frac{0.8d^{-1}10.61\frac{mg}{L}}{0.721d^{-1} - 0.88d^{-1}}\left(e^{-0.88d^{-1}0.917} - e^{-0.721d^{-1}0.917d}\right)$$
$$+ \frac{\left(0.5\frac{g}{m^2 - d}/1.22m\right)}{0.721d^{-1}}\left(1 - e^{-0.721d^{-1}0.917d}\right)$$
$$+ \frac{0.8d^{-1}0\frac{mg}{L-d}}{0.88d^{-1}0.721d^{-1}}\left(1 - e^{-0.721d^{-1}0.917d}\right)$$
$$- \frac{0.8d^{-1}0\frac{mg}{L-d}}{0.88d^{-1}(0.721d^{-1} - 0.88d^{-1})}\left(e^{-0.88d^{-1}0.917d} - e^{-0.721d^{-1}0.917d}\right)$$

$$D = 2.76\frac{mg}{L} + 3.74\frac{mg}{L} + 0.275\frac{mg}{L} + 0.0\frac{mg}{L} - 0.0\frac{mg}{L} = 6.77\frac{mg}{L}$$

$C = C_s$-D = 9.1-6.77 = 2.33 mg/L