Homework #6

4 points total for entire homework

Toxics Models

1. Problem 40.1

Thomann and DiToro (1983) presented the following data related to the solids budget for the Western Basin of Lake Erie:

Volume = 23x10^9 m^2
Solid loading = 11.4x10^{12} g/yr
Flow = 167x10^9 m^3/yr
Area = 3030x10^6 m^2
Suspended Solids = 20 mg/L

They assumed that the solids settle at a rate of 2.5 m/d (912.5 m/yr), and tha the sediments have ρ=2.4 g/mL and ϕ=0.9. Determine the burial and resuspension velocities.

1 point for #1

Solution:

First, an inflow concentration can be determined as

$$m_{in} = \frac{W_{solids}}{Q} = \frac{Qm_{in}}{Q}$$

$$m_{in} = \frac{11.4x10^{12}}{167x10^{9}} = 68.26\frac{mg}{L}$$

Next, equation 40.25 can be used to calculate:

$$v_b = \frac{Q(m_{in} - m)}{A(1-\phi)\rho}$$

$$v_b = \frac{167x10^9}{3030x10^6}\frac{68.26-20}{(1-0.9)2.4x10^6}$$

$$= 0.0111\frac{m}{yr}$$

$$= 11.1\frac{mm}{yr}$$

This result can be substituted into equation 40.26 to determine:

$$v_r = v_s\frac{m}{(1-\phi)\rho} - v_b$$

$$v_r = 912.5\frac{20}{(1-0.9)24x10^6} - 0.0111$$
$$= 0.06494\frac{m}{yr}$$
$$= 64.94\frac{mm}{yr}$$

2. Problem 40.2

Suppose that a toxic substance that is subject to volatilization (v_v = 100 m/yr) is discharged to Lake Huron with an inflow concentration of 100 µg/L. In the absence of sediment feedback, determine the concentration for three cases: (a) weak sorber (K_d = 0.002 m^3/g), (b) moderate sorber (K_d = 0.1), and (c) strong sorber (K_d = 2). Other necessary information should be taken from Examples 40.1 and 40.2.

Solution:

1.5 points for #2

a. for the weak sorber

The fraction in dissolved form is:

$$F_d = \frac{1}{1+K_d m}$$
$$F_d = \frac{1}{1+0.002(0.531)} = 0.9989$$

and

$$F_p = 1 - F_d = 0.0011$$

Thus, the steady-state concentration can be calculated from equation 40.11 as:

$$c = \frac{Qc_{in}}{Q + kV + v_v AF_d + v_s AF_p}$$
$$= \frac{1.61x10^{11}(100\mu g/L)}{1.61x10^{11} + 100(5.96x10^{10})0.9989 + 191.82(5.96x10^{10})0.0011}$$
$$= 2.63\mu g/L$$

Note that I used the lower value for vs from example problems 40.1 and 40.2. The reason is that this value was determined based on a model without sediment feedback (e.g., resuspension) much like the problem we're doing.

b. for the moderate sorber

The fraction in dissolved form is determined to be 0.9496

Thus, the steady-state concentration can be calculated from equation 40.11 as:

$$c = \frac{Qc_{in}}{Q + kV + v_v AF_d + v_s AF_p}$$
$$= \frac{1.61x10^{11}(100\mu g / L)}{1.61x10^{11} + 100(5.96x10^{10})0.9496 + 191.82(5.96x10^{10})0.0504}$$
$$= 2..52\mu g / L$$

c. for the strong sorber

The fraction in dissolved form is determined to be 0.485

Thus, the steady-state concentration can be calculated from equation 40.11 as:

$$c = \frac{Qc_{in}}{Q + kV + v_v AF_d + v_s AF_p}$$
$$= \frac{1.61x10^{11}(100\mu g / L)}{1.61x10^{11} + 100(5.96x10^{10})0.485 + 191.82(5.96x10^{10})0.515}$$
$$= 1.80\mu g / L$$

3. Problem 40.3a

A substance (K_d = 0.02 m^3/d; M = 300) is discharged into a lake (c_{in} = 100 μg/L) having the following characteristics:

Volume = $1x10^6$ m^3	Mean depth = 5 m
Residence time = 1 year	Suspended solids = 10 mg/L
Settling velocity = 50 m/yr	Sediment deposition = 100 g/m^3/yr
Sediment porosity = 0.85	Sediment density = 2.5 g/cm^3

(a) It the resuspension is negligible, compute the steady-state concentration for three levels of volatilization:
 (i) high soluble (v_v = 0)
 (ii) moderately soluble (v_v = 10 m/yr)
 (iii) nearly insoluble (v_v = 100 m/yr)

Solution

The fractions in the dissolved and particulate form can be determined from:

$$F_d = \frac{1}{1 + 0.02(10)} = 0.8333$$

1.5 points for #2

and

$$F_p = 1 - F_d = 0.1667$$

i. highly soluble

The concentration in the water can be calculated as:

$$c = \frac{Qc_{in}}{Q + kV + v_v AF_d + v_s AF_p}$$
$$= \frac{10^6 (100\mu g / L)}{10^6 + 0\left(\frac{10^6}{5}\right)0.8333 + 50\left(\frac{10^6}{5}\right)0.1667}$$
$$= 37.5\mu g / L$$

ii. moderately soluble

The concentration in the water can be calculated as:

$$c = \frac{Qc_{in}}{Q + kV + v_v AF_d + v_s AF_p}$$
$$= \frac{10^6 (100\mu g / L)}{10^6 + 10\left(\frac{10^6}{5}\right)0.8333 + 50\left(\frac{10^6}{5}\right)0.1667}$$
$$= 23.08\mu g / L$$

iii. insoluble

The concentration in the water can be calculated as:

$$c = \frac{Qc_{in}}{Q + kV + v_v AF_d + v_s AF_p}$$
$$= \frac{10^6 (100\mu g / L)}{10^6 + 100\left(\frac{10^6}{5}\right)0.8333 + 50\left(\frac{10^6}{5}\right)0.1667}$$
$$= 5.17\mu g / L$$