Homework #5

Attached is a data sheet from a recent dye study conducted by the Massachusetts DEP on the Millers River. One quart of Rhodamine dye was injected at the Farley Road Bridge (River Mile 5.9) and samples were analyzed for residual dye at the Millers Falls Paper Company Bridge (River Mile 2.1). The dye injection occurred at 9:55 AM on July 15th and the dye slug was followed at the downstream location from 10:30AM to 3:00PM of that same day. Note that most of the fluorometer readings were made with the 10x scale, and as the concentration increased, the scale was reduced to 3x and 1x.

Using the method of moments, estimate:

1. Mean velocity between river miles 5.9 and 2.1 in ft/sec.
2. Mean dispersion coefficient between river miles 5.9 and 2.1 in sq.ft/sec.

Note: In cases where the distance between the point of dye injection and dye detection is large compared to the downstream mixing distance, the ideal initial (t=0) dye slug may be used in place of an upstream dye curve. The attached data may be analyzed in this way.

TIME-OF-TRAVEL STUDY ON Millers River
SAMPLING SITE Millers Falls Paper Co. Bridge (RM 2.1)
Dye injected at Farley Bridge (RM 5.9) Time 9:55 Date 7/15/87
Amount injected 1 Qt Type of dye ______ Conc. in % ______
Sampling section discharge 229 cfs; width ______; mean depth ______

Field Sampling and Analysis — Final Laboratory Analysis

Sample No.	Sample Point	Sample Time	Field Fluorometer Readings 1X	3X	10X	30X	Final Fluorometer Readings 1X	3X	10X	30X	Dye Conc. (μg/L)	
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	
1		1030			24							
2		1045			24							
3		1100			24							
4		1115			27							
5		1130			40							
6		1145			39							
7		1200			41							
8		1215			28							
9		1230			29							
10		1245		13	43							
11		1300		29								
12		1315	20	60								
13		1330	32									
14		1345		18								
15		1400		16	53							
16		1415			42							
17		1430			28							
18+19		1445			26							
20		1500			24							

Column 1. Number on sample bottle.

2. When more than one point in section is sampled, indicate as "A," "B," "C," etc., from left to right bank.

3. Military time.

4-11. Fluorometer dial readings on scales used.

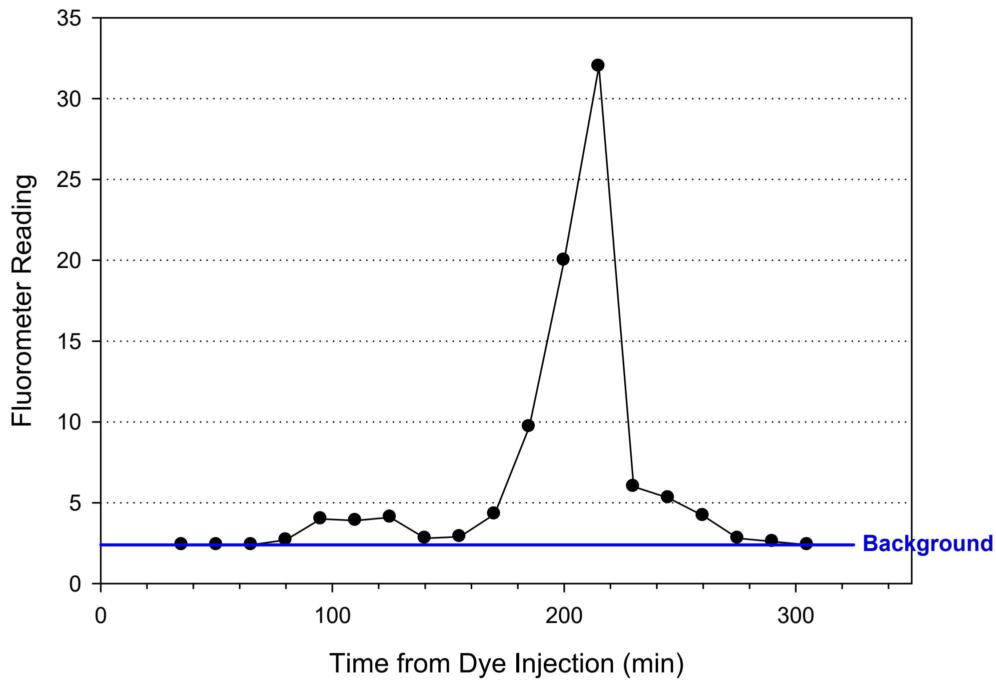
12. Based on fl[illegible]orometer calibration—show dye concentration in microgram[illegible]er liter in stream. If background has not been suppressed o[illegible]e fluorometer, subtract background reading prior to usin[illegible]ation curve.

Figure 6.—Form for recording dye-sample data.

Solution

Preliminary Calculations

The first step is to tablulate the data, and it's helpful (although not necessary) to plot it as well. When you do, it become evident that there is a background reading (about 2.4) that must be subtracted from all raw readings in order to isolate the tracer wave. What is the meaning of this backgroup? It's probably due to natural organic matter that fluoresces to a small degree.



Therefore the "Net" readings show this value removed, and all subsequent calculations should be done on the net reading. The values are shown below:

Using the method presented in class, and in the web notes, you should have the following:

		Start:	9:55 AM		Background=	2.4
		distance	3.80	miles		
Time						
of day	since Injection		Fluorometer Reading			
	(hr)	(min)	Gross	Net	st	st^2
10:30	0:35	35.00	2.4	0	0.00	0
10:45	0:50	50.00	2.4	0	0.00	0
11:00	1:05	65.00	2.4	0	0.00	0
11:15	1:20	80.00	2.7	0.3	24.00	1920
11:30	1:35	95.00	4	1.6	152.00	14440

11:45	1:50	110.00	3.9	1.5	165.00	18150
12:00	2:05	125.00	4.1	1.7	212.50	26562.5
12:15	2:20	140.00	2.8	0.4	56.00	7840
12:30	2:35	155.00	2.9	0.5	77.50	12012.5
12:45	2:50	170.00	4.3	1.9	323.00	54910
13:00	3:05	185.00	9.7	7.3	1350.50	249842.5
13:15	3:20	200.00	20	17.6	3520.00	704000
13:30	3:35	215.00	32	29.6	6364.00	1368260
13:45	3:50	230.00	6	3.6	828.00	190440
14:00	4:05	245.00	5.3	2.9	710.50	174072.5
14:15	4:20	260.00	4.2	1.8	468.00	121680
14:30	4:35	275.00	2.8	0.4	110.00	30250
14:45	4:50	290.00	2.6	0.2	58.00	16820
15:00	5:05	305.00	2.4	0	0.00	0
			SUM =	71.30	14419.00	2991200.00

$$\bar{t} = \frac{\sum st^{\Delta}t}{\sum s^{\Delta}t} = \frac{\sum st}{\sum s}$$
$$= \frac{14419}{71.30}$$
$$= 202.23 \min$$

$$\sigma^2 = \frac{\sum st^{2\,\Delta}t}{\sum s^{\Delta}t} - \left(\bar{t}\right)^2 = \frac{\sum st^2}{\sum s} - \left(\bar{t}\right)^2$$
$$= \frac{2991200}{71.30} - (202.23)^2$$
$$= 1055.336 \min^2$$

a. Determine velocity

$$U = \frac{^{\Delta}x}{\bar{t}}$$
$$= \frac{3.8mi}{202.23 \min}$$
$$= 0.01879 mi / \min$$
$$= 1.654\, ft / \sec$$

=1.815 km/h

b. Determine dispersion coefficient

$$E_x = \frac{U^2}{2}\frac{\sigma_{td}^2 - \sigma_{tu}^2}{\overline{t_d} - \overline{t_u}}$$

$$= \frac{(0.01879)^2}{2}\frac{1055.336}{202.23}$$

$$= 0.000921 mi^2 / \min$$

$$= 428\, ft^2 / s$$

=0.143 km²/h

You can also use Chapra's method (pag 187) which ultimately gets you to the same answer, but requires more calculations. If you followed this approach, your calculations should look like this:

Time, t			c										
of day	since Injection		Fluorometer Reading		a	b	c	ab	d	dc	e	f	fc
	(hr)	(min)	Gross	Net	ct	(ct)i+(ct)i+1	(t)i+1-(t)i		{c}i-{c}i+1		ct^2	{e}l + {e}i+1	
10:30	0:35	35.00	2.4	0	0.00	0.00	15.00	0	0.00	0	0	0	0
10:45	0:50	50.00	2.4	0	0.00	0.00	15.00	0	0.00	0	0	0	0
11:00	1:05	65.00	2.4	0	0.00	24.00	15.00	360	0.30	4.5	0	1920	28800
11:15	1:20	80.00	2.7	0.3	24.00	176.00	15.00	2640	1.90	28.5	1920	16360	245400
11:30	1:35	95.00	4	1.6	152.00	317.00	15.00	4755	3.10	46.5	14440	32590	488850
11:45	1:50	110.00	3.9	1.5	165.00	377.50	15.00	5662.5	3.20	48	18150	44712.5	670687.5
12:00	2:05	125.00	4.1	1.7	212.50	268.50	15.00	4027.5	2.10	31.5	26562.5	34402.5	516037.5
12:15	2:20	140.00	2.8	0.4	56.00	133.50	15.00	2002.5	0.90	13.5	7840	19852.5	297787.5
12:30	2:35	155.00	2.9	0.5	77.50	400.50	15.00	6007.5	2.40	36	12012.5	66922.5	1003837.5
12:45	2:50	170.00	4.3	1.9	323.00	1673.50	15.00	25102.5	9.20	138	54910	304752.5	4571287.5
13:00	3:05	185.00	9.7	7.3	1350.50	4870.50	15.00	73057.5	24.90	373.5	249842.5	953842.5	14307637.5
13:15	3:20	200.00	20	17.6	3520.00	9884.00	15.00	148260	47.20	708	704000	2072260	31083900
13:30	3:35	215.00	32	29.6	6364.00	7192.00	15.00	107880	33.20	498	1368260	1558700	23380500
13:45	3:50	230.00	6	3.6	828.00	1538.50	15.00	23077.5	6.50	97.5	190440	364512.5	5467687.5
14:00	4:05	245.00	5.3	2.9	710.50	1178.50	15.00	17677.5	4.70	70.5	174072.5	295752.5	4436287.5
14:15	4:20	260.00	4.2	1.8	468.00	578.00	15.00	8670	2.20	33	121680	151930	2278950
14:30	4:35	275.00	2.8	0.4	110.00	168.00	15.00	2520	0.60	9	30250	47070	706050
14:45	4:50	290.00	2.6	0.2	58.00	58.00	15.00	870	0.20	3	16820	16820	252300
15:00	5:05	305.00	2.4	0	0.00	14419.00					0		
			SUM =	71.30	14419.00	43257.00	270.00	432570.00	142.60	2139.00	2991200.00	5982400.00	89736000.00

Finally, for the purposes of checking your own results, I have repeated the above calculation, but without subtracting the background values:

Method presented in class, and in the web notes:

	Start:	9:55 AM	Background=	0
	distance	3.80 miles		

Time						
of day	since Injection		Fluorometer Reading			
	(hr)	(min)	Gross	Net	st	st^2
10:30	0:35	35.00	2.4	2.4	84.00	2940
10:45	0:50	50.00	2.4	2.4	120.00	6000
11:00	1:05	65.00	2.4	2.4	156.00	10140
11:15	1:20	80.00	2.7	2.7	216.00	17280
11:30	1:35	95.00	4	4	380.00	36100
11:45	1:50	110.00	3.9	3.9	429.00	47190
12:00	2:05	125.00	4.1	4.1	512.50	64062.5
12:15	2:20	140.00	2.8	2.8	392.00	54880
12:30	2:35	155.00	2.9	2.9	449.50	69672.5
12:45	2:50	170.00	4.3	4.3	731.00	124270
13:00	3:05	185.00	9.7	9.7	1794.50	331982.5
13:15	3:20	200.00	20	20	4000.00	800000
13:30	3:35	215.00	32	32	6880.00	1479200
13:45	3:50	230.00	6	6	1380.00	317400
14:00	4:05	245.00	5.3	5.3	1298.50	318132.5
14:15	4:20	260.00	4.2	4.2	1092.00	283920
14:30	4:35	275.00	2.8	2.8	770.00	211750
14:45	4:50	290.00	2.6	2.6	754.00	218660
15:00	5:05	305.00	2.4	2.4	732.00	223260
			SUM =	116.90	22171.00	4616840.00

Or using Chapra's method:

Time, t			c										
of day	since Injection		Fluorometer Reading		a	b	c	ab	d	dc	e	f	fc
	(hr)	(min)	Gross	Net	ct	(ct)i+(ct)i+1	(t)i+1-(t)i		{c}i-{c}i+1		ct^2	{e}I + {e}i+1	
10:30	0:35	35.00	2.4	2.4	84.00	204.00	15.00	3060	4.80	72	2940	8940	134100
10:45	0:50	50.00	2.4	2.4	120.00	276.00	15.00	4140	4.80	72	6000	16140	242100
11:00	1:05	65.00	2.4	2.4	156.00	372.00	15.00	5580	5.10	76.5	10140	27420	411300
11:15	1:20	80.00	2.7	2.7	216.00	596.00	15.00	8940	6.70	100.5	17280	53380	800700
11:30	1:35	95.00	4	4	380.00	809.00	15.00	12135	7.90	118.5	36100	83290	1249350
11:45	1:50	110.00	3.9	3.9	429.00	941.50	15.00	14122.5	8.00	120	47190	111252.5	1668787.5
12:00	2:05	125.00	4.1	4.1	512.50	904.50	15.00	13567.5	6.90	103.5	64062.5	118942.5	1784137.5
12:15	2:20	140.00	2.8	2.8	392.00	841.50	15.00	12622.5	5.70	85.5	54880	124552.5	1868287.5
12:30	2:35	155.00	2.9	2.9	449.50	1180.50	15.00	17707.5	7.20	108	69672.5	193942.5	2909137.5
12:45	2:50	170.00	4.3	4.3	731.00	2525.50	15.00	37882.5	14.00	210	124270	456252.5	6843787.5
13:00	3:05	185.00	9.7	9.7	1794.50	5794.50	15.00	86917.5	29.70	445.5	331982.5	1131982.5	16979737.5
13:15	3:20	200.00	20	20	4000.00	10880.00	15.00	163200	52.00	780	800000	2279200	34188000
13:30	3:35	215.00	32	32	6880.00	8260.00	15.00	123900	38.00	570	1479200	1796600	26949000
13:45	3:50	230.00	6	6	1380.00	2678.50	15.00	40177.5	11.30	169.5	317400	635532.5	9532987.5
14:00	4:05	245.00	5.3	5.3	1298.50	2390.50	15.00	35857.5	9.50	142.5	318132.5	602052.5	9030787.5
14:15	4:20	260.00	4.2	4.2	1092.00	1862.00	15.00	27930	7.00	105	283920	495670	7435050
14:30	4:35	275.00	2.8	2.8	770.00	1524.00	15.00	22860	5.40	81	211750	430410	6456150
14:45	4:50	290.00	2.6	2.6	754.00	1486.00	15.00	22290	5.00	75	218660	441920	6628800
15:00	5:05	305.00	2.4	2.4	732.00	22903.00					223260		
			SUM =	116.90	22171.00	66429.00	270.00	652890.00	229.00	3435.00	4616840.00	9007480.00	135112200.00

$$\bar{t} = \frac{\sum st^{\Delta}t}{\sum s^{\Delta}t} = \frac{\sum st}{\sum s}$$
$$= \frac{22171}{116.9}$$
$$= 189.66\,\text{min}$$

$$\sigma^2 = \frac{\sum st^{2\,\Delta}t}{\sum s^{\Delta}t} - \left(\bar{t}\right)^2 = \frac{\sum st^2}{\sum s} - \left(\bar{t}\right)^2$$
$$= \frac{4616840}{116.9} - (189.66)^2$$
$$= 3523.835\,\text{min}^2$$

a. Determine velocity

$$U = \frac{^{\Delta}x}{\bar{t}}$$
$$= \frac{3.8mi}{189.66\,\text{min}}$$
$$= 0.020036mi/\text{min}$$
$$= 1.763\,ft/\text{sec}$$

b. Determine dispersion coefficient

$$E_x = \frac{U^2}{2}\frac{\sigma_{td}^2 - \sigma_{tu}^2}{\bar{t_d} - \bar{t_u}}$$
$$= \frac{(0.020035)^2}{2}\frac{3523.835}{189.66}$$
$$= 0.003729mi^2/\text{min}$$
$$= 1732.8\,ft^2/s$$