Homework #4

4 points total for entire homework

All problems are from the course textbook (Chapra) unless otherwise indicated.

1. Chapra problem 3.2

2 points for #1

A lake with a single inflow stream has the following characteristics:

- Mean Depth = 5 m
- Surface Area = 11×10^6 m^2
- Residence time = 4.6 yr

An industrial plant presently discharges malathion (W = 2000 x 10^6 g/yr) to the lake. In addition, the inflowing stream also contains malathion (c_{in} = 15 mg/L). Note that the volumetric rate of inflow and outflow are equal. Assuming that a first-order decay reaction can be used to characterize malathion decay (k = 0.1 yr^{-1}),

(a) Write a mass balance equation for malathion for this system

$$\boxed{V\frac{dc}{dt} = W(t) + Qc_{in} - QC - kVc}$$

(b) If the lake is at SS, compute the malathion concentration

$$V = AH = 11x10^6(5) = 55x10^6 m^3$$

$$Q = \frac{V}{\tau_w} = \frac{55x10^6 m^3}{4.6 yr} = 11.96x10^6 \frac{m^3}{yr}$$

$$c = \frac{W + Qc_{in}}{Q + kV}$$

$$= \frac{2000x10^6 \frac{g}{yr} + 11.96x10^6 \frac{m^3}{yr}\left(15\frac{g}{m^3}\right)}{11.96x10^6 \frac{m^3}{yr} + 0.1yr^{-1}\left(55x10^6\right)}$$

$$\boxed{= 124.8\frac{mg}{L}}$$

(c) If the lake is at SS, what percent reduction in industrial loading is needed to drop the lake concentration to 30 ppm?

$$W = Qc + kVc - Qc_{in}$$

$$= 11.96x10^6 \frac{m^3}{yr}\left(30\frac{g}{m^3}\right) + 0.1yr^{-1}\left(55x10^6 m^3\right)30\frac{g}{m^3} - 11.96x10^6 \frac{m^3}{yr}\left(15\frac{g}{m^3}\right)$$

$$= 344.4x10^6 \frac{g}{yr}$$

$$\%reduction = \frac{2000 - 344.4}{2000}(100\%)$$

$$\boxed{= 82.78\%}$$

(d)Evaluate each of the following options

(i) build a WWTP that reduces the industrial discharge of Malathion by 50%

$$c = \frac{0.5x2000x10^6 \frac{g}{yr} + 11.96x10^6 \frac{m^3}{yr}\left(15\frac{g}{m^3}\right)}{11.96x10^6 \frac{m^3}{yr} + 0.1yr^{-1}\left(55x10^6 m^3\right)} = \boxed{67.24\frac{mg}{L}}$$

(ii) double the lake's depth by dredging

$$c = \frac{2000x10^6 \frac{g}{yr} + 11.96x10^6 \frac{m^3}{yr}\left(15\frac{g}{m^3}\right)}{11.96x10^6 \frac{m^3}{yr} + 0.1yr^{-1}\left(2x55x10^6 m^3\right)} = \boxed{94.92\frac{mg}{L}}$$

(iii) double the lake's outflow by diverting an unpolluted nearby stream

$$c = \frac{2000x10^6 \frac{g}{yr} + 11.96x10^6 \frac{m^3}{yr}\left(15\frac{g}{m^3}\right)}{2x11.96x10^6 \frac{m^3}{yr} + 0.1yr^{-1}\left(55x10^6 m^3\right)} = \boxed{74.07\frac{mg}{L}}$$

(e) What other factors would need to be considered when making a decision in "d"?

- economics (e.g,. cost of each option).
- impacts on the biota (e.g., dredging might impact bottom organisms which can served as a source of food for fish).
- impacts on existing uses (e.g., dredging could decrease water clarity and interfere with swimming. Increasing depth could lower long term lake temperature).
- secondary benefits (e.g., doubling inflow could improve water clarity).

2, 3, 4.Chapra problem 4.5 to 4.7

A well-mixed lake has a steady-state concentration of 5 μg/L of total phosphorus. At the beginning of 1994 it received an additional loading of 500 kg/yr from a fertilizer processing plant. The lake has the following characteristics:

Inflow = outflow = 5x10^5 m^3/yr
Volume = 4x10^7 m^3
Surface are = 5x10^6 m^2

If the total phosphorus settles at a rate of 8 m/yr, compute the concentration of the lake from 1994 to 2010, **and show a graph of total P (in μg/L) vs year from 1990 to 2010**

1 point for #2, 3 & 4

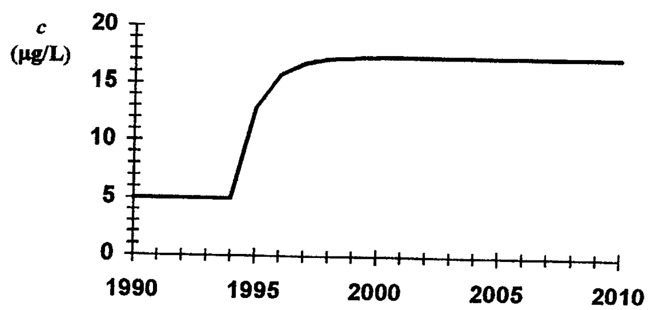
4.5 The lake's eigenvalue can be computed as

$$\lambda = \frac{Q}{V} + \frac{v}{H} = \frac{5\times10^5}{4\times10^7} + \frac{8}{8} = \frac{1.0125}{\text{yr}}$$

The solution for the step loading is

$$c = \frac{W}{\lambda V}\left(1-e^{-\lambda t}\right) = \frac{5\times10^8}{1.0125(4\times10^7)}\left(1-e^{-1.0125(t-1994)}\right) = 12.346\left(1-e^{-1.0125(t-1994)}\right)$$

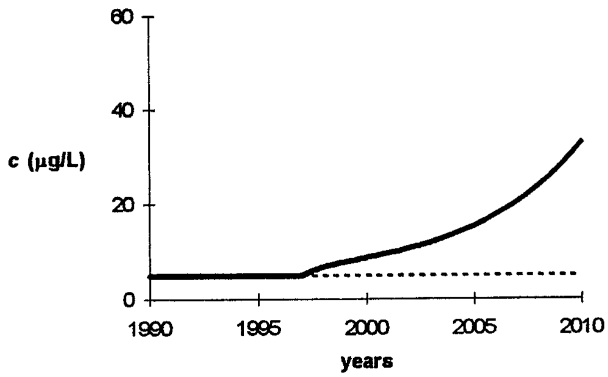
This equation can be used to compute the values for the lake over time. These values, along with the base concentration of 5 μg/L are shown in the following plot



4.6 The lake's eigenvalue can be computed as $\lambda = 1.0125$/yr. The solution for the exponential loading is

$$c = \frac{200(0.5\times10^6)}{4\times10^7(1.0125+0.2)}\left(e^{0.2(t-1997)} - e^{-1.0125(t-1997)}\right) = 2.062\left(e^{0.2(t-1997)} - e^{-1.0125(t-1997)}\right)$$

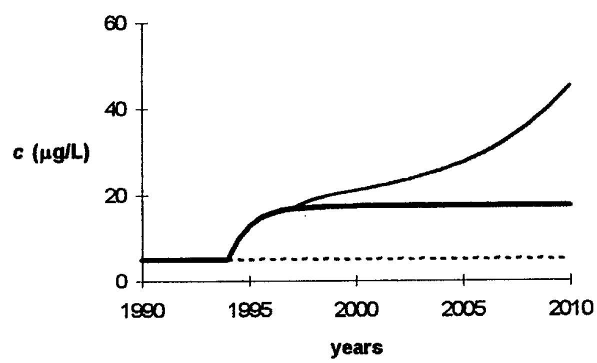
This equation can be used to compute the values for the lake over time. These values, along with the base concentration of 5 μg/L are shown in the following plot,



4.7 The lake's eigenvalue can be computed as $\lambda = 1.0125$/yr. The solution for background concentration, along with the step and exponential loadings are

$$c = 5 + 12.346\left(1 - e^{-1.0125(t-1994)}\right) + 2.062\left(e^{0.2(t-1997)} - e^{-1.0125(t-1997)}\right)$$

This equation can be used to compute the values in the following plot



5. Problem 1.3 a&b from Thomann & Mueller (reprinted below):

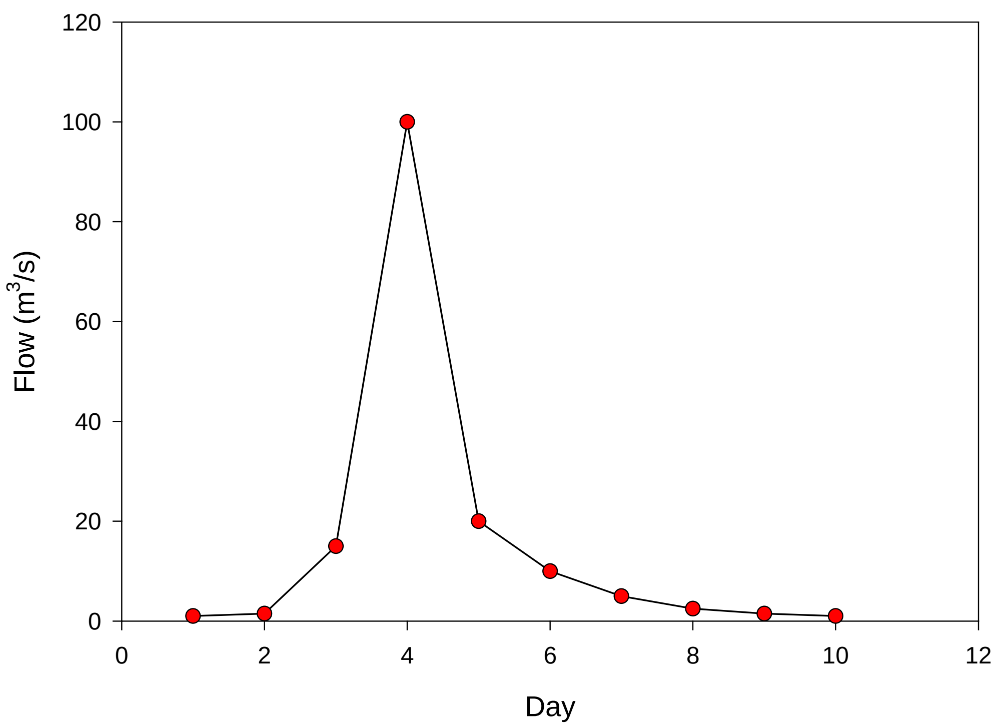
A particular river had a significant effect on a reservoir through the discharge of suspended solids from the river to the reservoir. It is important to estimate the mass loading of solids to the reservoir to determine the extent of long-term sedimentation of solids in the reservoir, reducing its available volume. A brief survey over a 10-day period is shown below.

a. plot the flow versus time in days.
b. Estimate the mean mass loading of solids (in kg/d) into the reservoir, using a log-log plot of flow vs. solids.

1 point for #5

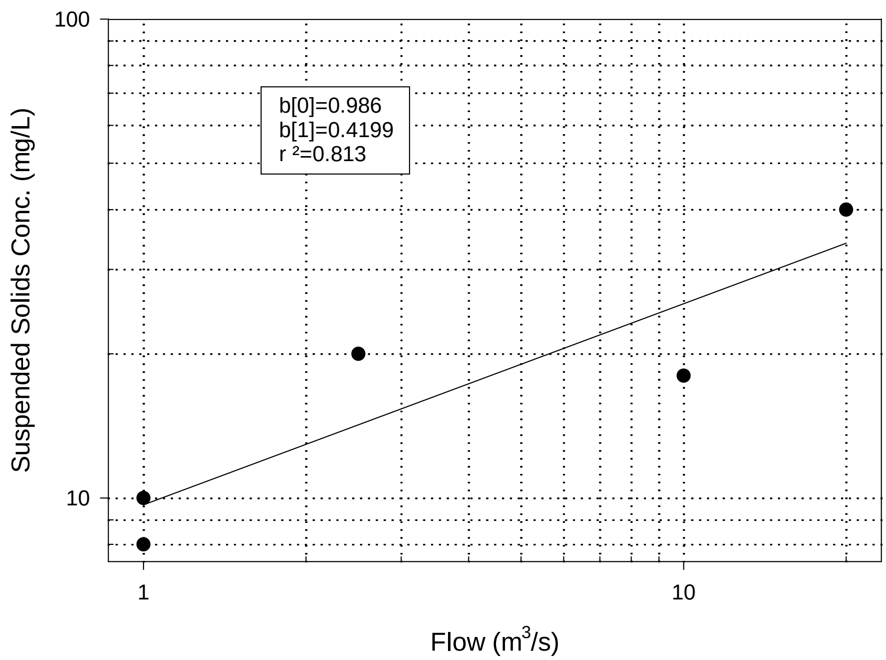
Day	Flow (m^3/s)	Suspended Solids Concentration (mg/L)
1	1.0	10
2	1.5	
3	15.0	
4	100.0	
5	20.0	40
6	10.0	18
7	5.0	
8	2.5	20
9	1.5	
10	1.0	8

(a)



(b)

Plot known suspended solids concentration vs flow on a log-log scale to develop power function model. Be careful to plot the independent variable (flow) on the x-axid and the dependent variable (SS conc) on the y-axis.



A least squares linear regression produces a suspended solids power function model of the following

$$logTSS = 0.9856 + 0.420 * logQ$$

Which corresponds to

$$TSS = 9.67 * Q^{0.420}$$

Now, use that equation to fill in the missing TSS values. Once this is done, a loading can be calculated for each date, and the average loading determined from the sum of these individual loadings divided by the number of days of observation (j).

$$\bar{W} = \frac{1}{j}\sum_{i-1}^{j} Q_i c_i$$

The following presents calculations for all 10 days as well as the sum and average loadings.

Day	Flow(cms)	Conc(mg/L)	W(g/s)		
1	1	9.6745996	9.6746		
2	1.5	11.470249	17.20537		
3	15	30.162535	452.438		
4	100	66.89952	6689.952		
5	20	34.035254	680.7051		
6	10	25.440638	254.4064		
7	5	19.016343	95.08172		
8	2.5	14.214318	35.53579		
9	1.5	11.470249	17.20537		
10	1	9.6745996	9.6746		
Sum			8261.879 g/s	=	713826.3 kg/d
Average			826.1879 g/s	=	71,383 kg/d

So the final estimate is:

$$\boxed{\overline{W} = 71{,}400 kg/d}$$