Homework #2

All problems are from the course textbook (Chapra).

1. ***Problem 1.2   (1 point)***
2. ***Problem 1.3   (1point)***
3. ***Problem 1.4   (1 point)***
4. ***Problem 1.6   (1 point)***
5. ***Problem 1.13   (1 point)***
6. ***Problem 1.14   (1 point)***
7. ***Problem 2.1   (1 point)***
8. ***Problem 2.4   (1 point)***
9. ***Problem 2.13   (1 point)***
10. ***Problem 2.21   (1 point)***

Total: 10 points

SOLUTIONS:

1.2

Problem

Each individual in a city of 100,000 people contributes about 650 L/capita/d of wastewater and 135 g/capita/d of BOD.

(a)   Determine the flow rate (m^3s^{-1}) and mass loading rate of BOD (mta) generated by such a population.

(b)   Determine the BOD concentration of the wastewater (mg/L)

Answer

(a)

$$Q = 100,000 capita(650L/capita/d)\frac{1m^3}{10^3 L}\frac{1d}{86,400s}$$

$$= 0.7523\frac{m^3}{s}$$

$$\boxed{\approx 0.75\frac{m^3}{s}}$$

$$W = 100,000\,capita\left(135\,g\,/\,capita\,/\,d\right)\frac{365d}{yr}\frac{1kg}{10^3\,g}\frac{1mta}{10^3\,kg}$$

$$= 4927.5\,mta$$

$$\boxed{\approx 4.9x10^3\,mta}$$

(b)

$$c = \frac{W}{Q} = \frac{135\,g\,/\,capita\,/\,d}{650L\,/\,capita\,/\,d}\frac{10^3\,L}{1m^3}$$

$$= 207.7\frac{g}{m^3}\,or\,\frac{mg}{L}$$

$$\boxed{\approx 208\,mg\,/\,L}$$

1.3

Problem

You are studying a 3-km stretch of stream that is about 35 m wide. A gaging station on the stream provides you with an estimate that the average flow rate during your study was 3 cubic meters per second (cms). You toss a float into the stream and observe that it takes about 2 hr to traverse the stretch. Calculate the average velocity (mps), cross-sectional area (m^2), and depth (m), for the stretch. To make these estimates, assume that the stretch can be idealized as a rectangular channel (constant L, B & H).

Answer

$$U = \frac{L}{t} = \frac{3km}{2hr}\frac{1000m}{km}\frac{1hr}{3600s}$$

$$= 0.4167\frac{m}{s}$$

$$\boxed{\approx 0.4\frac{m}{s}}$$

$$A = \frac{Q}{U} = \frac{3cms}{0.4167mps}$$

$$\boxed{= 7.2m^2}$$

$$H = \frac{A}{B} = \frac{7.2m^2}{35m}$$

$$= 0.206m$$

$$\boxed{\approx 0.21m}$$

1.4

Problem

In the early 1970s Lake Michigan had a total phosphorus loading of 6950 mta and an in-lake concentration of 8 ug/L (Chapran and Sonzogni 1979).

(a)   Determine the lake's assimilation factor (km^3yr^{-1}).
(b)   What loading rate would be required to bring in-lake levels down to approximately 5 ug/L?
(c)   Express the results of(b) as a percent reduction.

Answer

(a)

$$a = \frac{W}{c} = \frac{6950mta}{8\frac{mg}{m^3}} \frac{10^9 mg}{tonne} \frac{km^3}{10^9 m^3}$$

$$= 868.75 \frac{km^3}{yr}$$

$$\boxed{\approx 8.7x10^2 \frac{km^3}{yr}}$$

(b)

$$W = ac = 868.75 \frac{mta}{\mu g/L} 5\mu g/L$$

$$= 4343.5 mta$$

$$\boxed{\approx 4.3x10^3 mta}$$

(c)

$$\% reduction = \frac{W_{present} - W_{future}}{W_{present}}(100\%) = \frac{6950 - 4343.75}{6950}100\%$$

$$\boxed{= 37.5\%}$$

1.6

Problem

You mix two volumes of water having the following characteristics:

	Volume 1	Volume 2
Volume	1 gal	2 L
Concentration	250 ppb	2000 mg/m^3

a.   Calculate the concentration (mg/L) for the mixture
b.   Determine the mass in each volume and in the final mixture. Express your result in grams.

Answer

(a)

$$V_1 = 1gal \frac{1L}{0.264172\,gal} = 3.7854L$$

$$c = \frac{V_1 c_1 + V_2 c_2}{V_1 + V_2} = \frac{3.7854L(0.250mg/L) + 2L(2mg/L)}{3.7854L + 2L}$$

$$= 0.855\frac{mg}{L}$$

$$\boxed{\approx 0.86\frac{mg}{L}}$$

(b)

$$m_1 = c_1 V_1 = 0.250\frac{mg}{L}(3.7854L)\tfrac{g}{10^3 mg} = 0.9464x10^{-3}\,g \boxed{\approx 0.95x10^{-3}\,g}$$

$$m_2 = c_2 V_2 = 2\frac{mg}{L}(2L)\tfrac{g}{10^3 mg} = \boxed{4x10^{-3}\,g}$$

$$m = cV = 0.855\frac{mg}{L}(5.7854L)\tfrac{g}{10^3 mg} = 4.9464x10^{-3}\,g \boxed{\approx 4.9x10^{-3}\,g}$$

1.13

Problem

The Boulder, Colorado, wastewater treatment plant enters Boulder Creek just above a USGS flow gaging station.
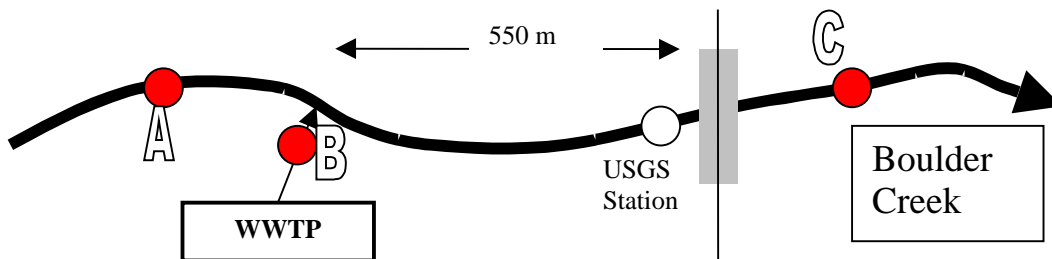
At 8:00 A.M. on December 29, 1994, conductivities of 170, 820, and 639 µmho/cm were measured at A, B and C respectively. (Note: Conductivity provides an estimate of the total dissolved solids (TDS) in a solution by measuring its capability to carry an electrical current.) If the flow at the gaging station was 0.494 cms, estimate the flows for the treatment plant and the creek.

Answer

The flow in the creek and the treatment plant should be equal to the flow below the mixing point,

$$Q_A + Q_B = 0.494$$

Because conductivity and TDS are conservative substances, the following mass balance should hold

$$639 = \frac{Q_A(170) + Q_B(820)}{0.494}$$

These two equations can be combined to yield,

$$639 = \frac{Q_A(170) + (0.494 - Q_A)(820)}{0.494}$$

which can be solved for:

$$Q_A = \frac{0.494(639 - 820)}{170 - 820} = 0.13756 cms \approx 0.138 cms \quad \text{The Creek}$$

$$Q_B 0.494 - 0.13756 = 0.356 cms \quad \text{The Treatment Plant}$$

1.14

Problem

Sediment traps are small collecting devices that are suspended in the water column to measure the downward flux of settling solids. Suppose that you suspend a rectangular trap (1m x 1m) at the bottom of a layer of water. After 10 d, you remove the trap and determine that 20 g of organic carbon has collected on its surface.
(a)   Determine the downward flux of organic carbon
(b)   If the concentration of organic carbon in the water layer is 1 mg/L, determine the downward velocity of organic carbon
(c)   If the surface area at the bottom of the layer is 10^5 m^2, calculate how many kilograms of carbon are transported across the area over a 1-month period.

Answer

(a)

$$J = \frac{20g}{1m^2(10d)} = 2\frac{g}{m^2 d}$$

(b)

$$v = \frac{J}{c} = \frac{2g/m^2/d}{1g/m^3} = 2\frac{m}{d}$$

(c)

$$M = 2\frac{g}{m^2 d}(1month)10^5 m^2 \frac{30.42d}{month}\frac{kg}{10^3 g} = 6,084kg \approx 6,100kg$$

2.1

Problem

You perform a series of batch experiments and come up with the following data:

t (hr)	0	2	4	6	8	10

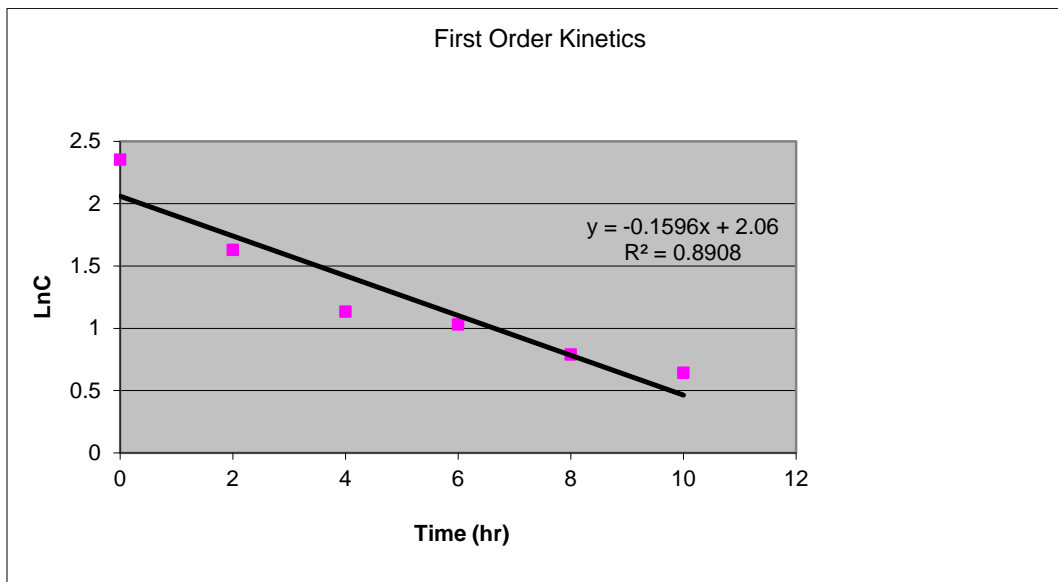| **C (µg/L)**	10.5	5.1	3.1	2.8	2.2	1.9

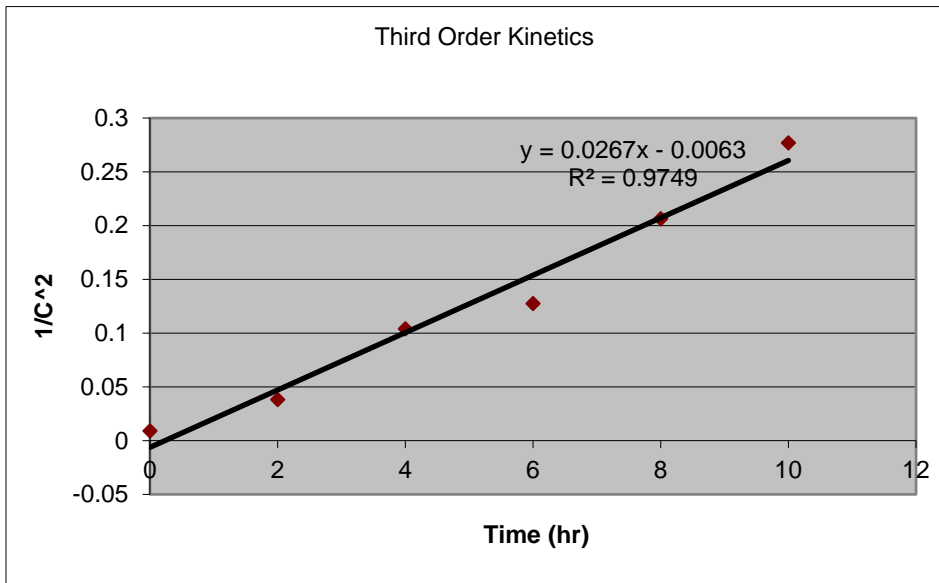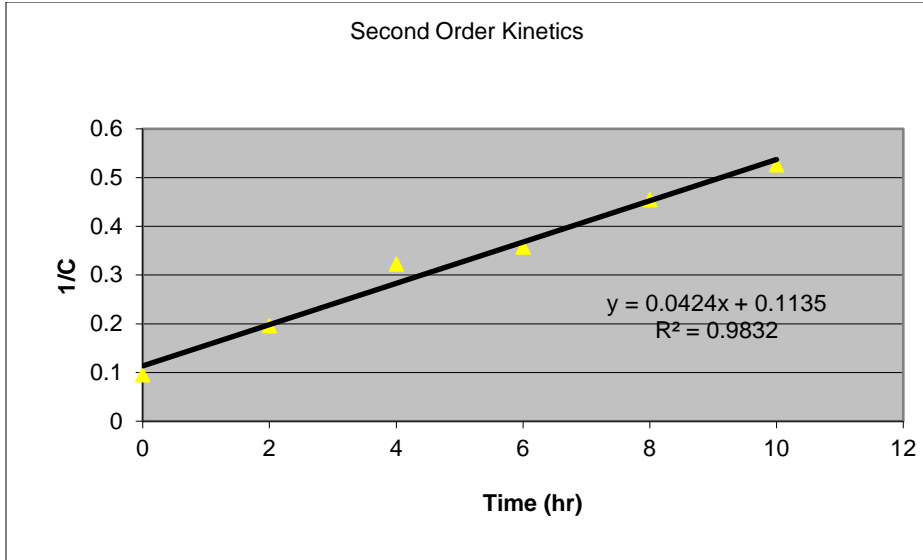Determine the order (n) and the rate constant (k) for the underlying reaction.

Solution

Consider both integral and differential approaches. The following table includes calculations for both.

t (hr)	C (ug/L)	lnC	1/C	1/(C^2)	dc/dt	t-midpoint	C-midpoint	$\Delta C/\Delta t$
0	10.5	2.351375	0.095238	0.00907	4	1	7.8	2.7
2	5.1	1.629241	0.196078	0.038447	1.7	3	4.1	1
4	3.1	1.131402	0.322581	0.104058	0.5	5	2.95	0.15
6	2.8	1.029619	0.357143	0.127551	0.27	7	2.5	0.3
8	2.2	0.788457	0.454545	0.206612	0.12	9	2.05	0.15
10	1.9	0.641854	0.526316	0.277008	0.08			

Prepare integral plots for 1st, 2nd, and 3rd order reactions:

First Order Kinetics

$y = -0.1596x + 2.06$
$R^2 = 0.8908$

Second Order Kinetics

y = 0.0424x + 0.1135
R² = 0.9832

(1/C vs. Time (hr))

Third Order Kinetics

y = 0.0267x - 0.0063
R² = 0.9749

(1/C^2 vs. Time (hr))

Of the three integral plots, the one presuming <u>second order</u> appears to fit slightly better than the third order plot. You could end the problem here.

Or you could continue testing via a more "advanced" differential approach. The first, I will call "direct differential". This comes directly from the midpoint slopes ($\Delta C/\Delta t$) and the midpoint concentrations. A log-log plot gives a slope indicative of the reaction order, and an intercept which yields the reaction rate.

Direct Differential

$$y = 2.2605x - 1.5615$$
$$R^2 = 0.8702$$

(y-axis: log(dc/dt), x-axis: Log (t))
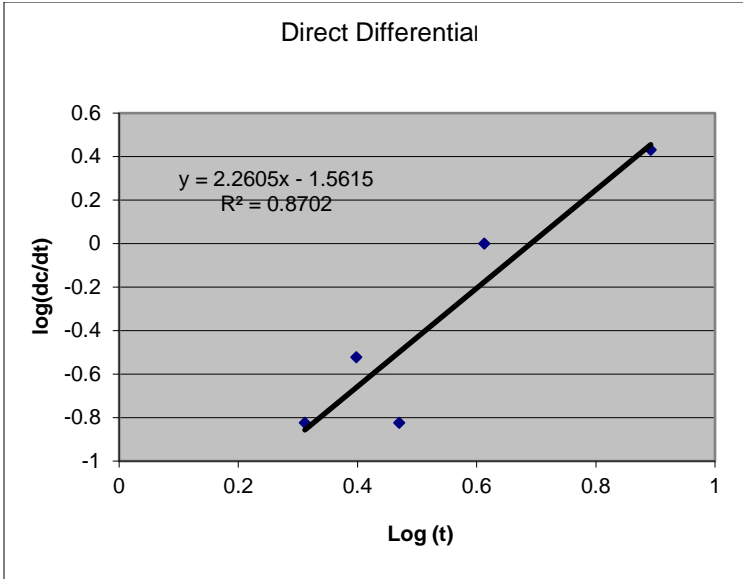
A second differential method involves some graphical smoothing. Refer to Chapra's example 2.2 on page 33 for a description as to how this can be done. The graph below presents the results of this analysis.

Smoothed Differential

$$y = 2.3334x - 1.6213$$
$$R^2 = 0.9441$$

(y-axis: Log (dc/dt), x-axis: Log (t))
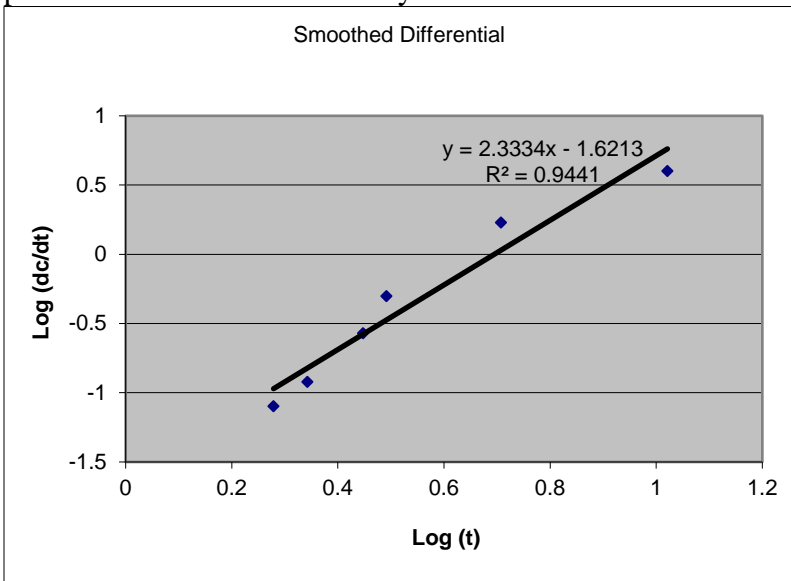
One last approach is the use of non-linear least squares analysis. This is a powerful technique that is now easily done with modern numerical software. Many statistical packages have routines that will do this.

	Integral	Direct Differential	Smoothed Differential	Non-linear Least Squares
Order	2	2.26	2.33	2.23
Rate constant	0.0424	0.0274	0.0239	0.0326

Units on the various rates constants are "$hr^{-1}(\mu g/L)^{n-1}$".

2.4

Problem

At a later date the laboratory informs you that they have a more complete data set than the two measurements used in Example 2.5:

T (C)	4	8	12	16	20	24	28
k (d^{-1})	0.120	0.135	0.170	0.200	0.250	0.310	0.360

Use this data to estimate θ and k at 20 C/

Solution

Taking the logarithm of equation 2.44 yields:
$$\log(k(t)) = \log(k(20)) + (T - 20)\log\theta$$
Thus, a plot of Log[k(t)] versus (T-20) should yield a straight line with a slope of $\log\theta$ and an intercept of log[k(20). Linear regression can be used to do this yielding:
$$\log(k(t)) = -0.60474 + 0.020722(T - 20)$$
$$\theta = 10^{0.020722} = 1.0489 \approx 1.049$$
$$k_{20} = 10^{-0.60474} = 0.248d^{-1}$$

2.13

Problem

Estimate the age of the fossil remains of a skeleton with 2.5% or its original carbon-14 content. Note that carbon-14 has a half-life of 5730yr.

Solution

The first-order decay model can be expressed in terms of half-life as:
$$c = c_o e^{-0.693t/t_{1/2}}$$
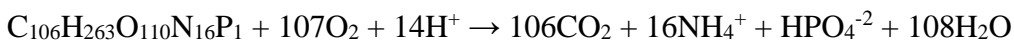which can be solved for:
$$t = \frac{t_{1/2}}{0.693}\ln\left(\frac{c_o}{c}\right) = \frac{5730}{0.693}\ln\left(\frac{1}{0.025}\right) = 30,500\,yrs$$

2.21

Problem

A more complete representation (as compared to equation 2.3) of the decomposition reaction in provided by:

$$C_{106}H_{263}O_{110}N_{16}P_1 + 107O_2 + 14H^+ \rightarrow 106CO_2 + 16NH_4^+ + HPO_4^{-2} + 108H_2O$$

On the basis of this equation, given that 10 g-C m^{-3} of organic matter is decomposed, calculate (a) the stoichiometric ratio for the amount of oxygen consumed per carbon decomposed, roc, (b) the amount of oxygen consumed in g-O/m^3, and (c) the amount of ammonium released in mg-N/m^3.

Solution

(a)

$$r_{oc} = \frac{107x32}{106x12} = 2.69 \frac{g-O}{g-C}$$

(b)

$$10 \frac{g-C}{m^3} x 2.69 \frac{g-O}{g-C} = 26.9 \frac{g-O}{m^3}$$

(c)

$$10 \frac{g-C}{m^3} x \frac{16x14 g-N}{106x12 g-C} \frac{10^3 mg-N}{g-N} = 1761 \frac{mg-N}{m^3}$$