Hydraulics of Water Systems: Flow in Pipes

Reading: Chapter 4, pp. 93-112

Open Channels

- Los Angeles Aqueduct
  - Owens Lake to LA Aqueduct Plant
- HGL and water surface are coincident
  - Topography has to be right
NYC Tunnel

- Well suited for mountain terrain or river crossings
  - An arch is constructed to prepare the tunnel to be lined with concrete.
Pipelines

- 30 in Steel
  - Transmission main for Sheridan, WY
- Transmission main in Puerto Rico

Pressure

- Units
  - water
  - Hg Manometer
  - Bourdon Gage

- Measurement
  - a) piezometer
  - c) Hg Manometer
  - c) Bourdon Gage

Figs 4-1 to 4-3 in H&H
Basic Hydraulics

- The application presented here is water flowing under pressure in pipes; however, some general principles are also reviewed which apply to other systems. Later, application to sewers or open channel systems is addressed.

- The primary equations and principles that are summarized follow. You should consult your *Fluid Mechanics* book as needed.
  - Continuity Equation
  - Energy Equation
  - Hydraulic Grade Line
  - Energy Grade Line
  - Darcy Weisbach Equation
  - Hazen Williams Equation

Definitions

- **Hydraulic Grade Line (HGL)**
  - It is a measure of the hydraulic head at various points. It is the line connecting points to which the water would rise if piezometer tubes were inserted at these points.
  - Hydraulic Head equals the Pressure Head plus the Elevation (or static) Head

- **Energy Grade Line (EGL)**
  - Line connecting the total energy at various points.
  - Energy Head equals the Pressure Head plus the Velocity Head plus the Elevation Head
Energy & Pipe Flow

- **Hydraulic Grade**
  \[ H_G = Z + \frac{P}{\gamma} \]
  \[ P = (H_G - Z)\gamma \]

- **For const velocity**
  \[ H_{G_2} = H_{G_1} - h_L \]

For constant velocity, the hydraulic grade line is constant.

Lost Energy

Note: specific weight of water
\[ W \equiv \gamma = 62.4 \text{ lb/ft}^3 \]

*Fig 4-5 in H&H*

Darcy Weisbach Equation

- **Fundamental equation describing the head loss due to friction for flow in pipes is the Darcy Weisbach Equation.**

  \[ h_f = f \frac{L}{D} \left( \frac{V^2}{2g} \right) = f \frac{LQ^2}{\pi^2 gD^5} \]

  where:
  - \( f \) is the “Darcy” friction factor (dimensionless)
  - \( L \) is the pipe length (L)
  - \( D \) is the pipe diameter (L)
  - The friction factor depends on the Reynolds number (Re or \( N_R \)) and the relative roughness (\( \varepsilon/D \)).
  
  - Where \( \varepsilon \) is the absolute roughness
  - It therefore depends on the pipe diameter and pipe material.
### Reynolds Number

- The Reynolds number (ratio of inertial to viscous forces) for pipe flow can be calculated from the following expression.

\[ Re = \frac{V D \rho}{\mu} = \frac{V D}{\nu} \]

- where:
  - \( \mu \) is the absolute viscosity (M/L-T)
  - \( \nu \) is the kinematic viscosity (L²/T)

- Note the following:
  - Laminar flow conditions for pipes occur for Re less than about 2000-4000; turbulent flow depends on the relative roughness but begins at Re above 10,000.
  - A Moody diagram (or equation) is used to obtain f, across all Re.
  - f depends on the Re and relative roughness (\( \varepsilon/d \)).

### Friction factor I

- The friction factor, f, is a function of the Reynolds number, Re= VD/\( \nu \), and the roughness ratio, \( \varepsilon/D \) (Moody diagram).

For different flow regimes, equations for "f" are:

- Laminar:
  \[ f = \frac{64}{Re} \]

- Fully turbulent, rough:
  \[ (1/f)^{1/2} = 2 \log (D/2\varepsilon) + 1.74 \]

- Transitional rough:
  \[ (1/f)^{1/2} = - 2.0 \log [(\varepsilon/3.7 D) + (2.51/(Re f^{1/2}))] \]

- Rearranging the D-W equation,
  \[ h_f/L = f = \frac{1}{D} \left( \frac{V^2}{2g} \right) = f \left( \frac{8Q^2}{\pi^2 gD^5} = \left( \frac{8 f}{\pi^2 g} \right) \left[ \frac{Q^2}{D^5} \right] \right) \]
Friction factor III

- Simplified for turbulent flow only
  - $Re > 10,000$

Fig 4-6 in H&H

David Reckhow
Example I

- Determine pipe diameter based on the following:
  - Flow (Q) is 10 ft³/s over a length of 5,000 ft
  - Assume a concrete pipe with $\varepsilon=0.001\text{ft} = 0.3\text{mm}$
  - Headloss cannot exceed 5 ft
- Need to solve the Darcy Weisbach equation for “D”

\[
    h_f = f \frac{L}{D} \left( \frac{V^2}{2g} \right) = f \frac{LQ^2}{\pi^2 gD^5}
\]
- But first we need to determine “f”
- This requires use of the Moody Diagram and an iterative approach, where we assume a “D” and then calculate “$h_f$”

Alternative Approach

- The Darcy Weisbach equation is not typically used in the design of pipes because what you are trying to determine, the pipe size, is needed to determine $f$.
  - Thus a trial and error solution is required.
- The precision that you achieve through its use is not justified for design purposes.
  - For design, $Q$ is a design value chosen by the engineer to represent usually future flow conditions. Also, you are designing for nominal pipe sizes.
- Instead of the Darcy Weisbach equation, the empirical Hazen Williams equation is used.
Hazen Williams Equation

- The Hazen Williams equation gives good results for turbulent flow conditions.

\[ V = 1.318 C R^{0.63} S^{0.54} \]  \hspace{1cm} (6.1a: English Units)

\[ V = 0.85 C R^{0.63} S^{0.54} \]  \hspace{1cm} (6.1b: SI Units)

where:
- \( V \) is the velocity of flow (ft/sec or m/sec)
- \( C \) is the Hazen Williams coefficient of roughness; it depends on the pipe material (see Table 4.2 of text; pg 99)
- \( R \) is the hydraulic radius (ft or m)
  - \( R \) is defined as the cross-sectional area of flow divided by the wetted perimeter. For pipes flowing full, \( R = D/4 \)
- \( S \) is the slope of the Energy Grade line (dimensionless)
  - \( S = h_f/L \)

Useful forms of H-W Equation #1

- for a pipe flowing full,
  - Eq. (6.1) can be expressed in terms of the volumetric flow rate.

\[
Q = 0.432 CD^{2.63} \left( \frac{h_f}{L} \right)^{0.54} \]  \hspace{1cm} \( Q \) in \( \text{ft}^3/\text{s} \); \( D \) in ft

\[
Q = 0.278 CD^{2.63} \left( \frac{h_f}{L} \right)^{0.54} \]  \hspace{1cm} \( Q \) in \( \text{m}^3/\text{s} \); \( D \) in m

\[
Q = 0.279 CD^{2.63} \left( \frac{h_f}{L} \right)^{0.54} \]  \hspace{1cm} \( Q \) in MGD; \( D \) in ft

\[
Q = 0.281 CD^{2.63} \left( \frac{h_f}{L} \right)^{0.54} \]  \hspace{1cm} \( Q \) in gpm; \( D \) in in
Useful forms of H-W Equation #2

- for a pipe flowing full,
  - Eq. (6.1) can be expressed in terms of the headloss.

\[ h_L = 4.73 \frac{Q^{1.85} L}{C^{1.85} D^{4.87}} \begin{array}{c} \text{Q in ft}^3/\text{s}; \ D \text{ in ft} \\ \text{Q in m}^3/\text{s}; \ D \text{ in m} \\ \text{Q in MGD}; \ D \text{ in ft} \\ \text{Q in gpm}; \ D \text{ in in} \end{array} \]

Nomograph

- Hazen-Williams Nomograph
  - C=100

H&H Fig 4-8, pg.102
To next lecture

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