CEE 371 Fall 2009

Homework #3 Water Distribution Pipe Systems

1. **For the pipe system shown below (Figure 1), determine the length of a single equivalent pipe that has a diameter of 8 inches. Use the Hazen Williams equation and assume that C_{HW} = 120 for all pipes. Solve the problem using the following steps:**

4 points total for entire homework

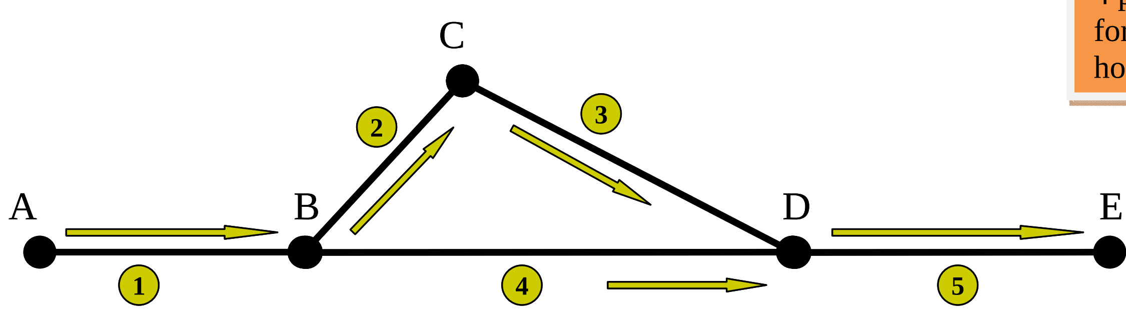


Figure 1. Pipe System for equivalent pipe problem

Table 1. Pipe Data for Figure 1 Pipe System

	Pipe 1	Pipe 2	Pipe 3	Pipe 4	Pipe 5	
Length	500	500	800	1000	700	ft
Diameter	12	6	8	10	12	in

a. **First determine an equivalent pipe (with D=8 in) for pipes #2 and #3 in series. Use a flow of 800 gpm.**

0.5 points for #1a

I used the Hazen Williams equation for Q in gpm and diameter in inches.

$$h_L = 10.5 \frac{Q^{1.85} L}{C^{1.85} D^{4.87}}$$

I used this to calculate the headloss in pipe 2 and pipe 3 (recognizing that the flow in pipe 3 must also be 800 gpm.

	Pipe 2	Pipe 3	
C	120	120	
L	500	800	ft
D	6	8	in
Q	800	800	gpm
h_L	**28.49666**	**11.23213**	ft

The total headloss is then the sum of these two

h_L total = **39.73 ft**

and the equivalent length for a 8 in pipe is calculated by rearranging the H-W formula and solving for L

$$L = \frac{h_L C^{1.85} D^{4.87}}{10.5 Q^{1.85}}$$

$$= 2839\,ft$$

b. <u>Second, determine an equivalent pipe for pipe #4 and the parallel equivalent pipe from part (a). Use the head loss resulting from the flow for part (a) as the basis for determining the equivalent pipe length (use D=8 in). What is the flow split between these two parallel pipes? (i.e., for 800 gpm through the part (a) pipe, what is the flow in the parallel pipe, and the total flow)</u>

Now that we know the headloss from node B to node D is 39.73 feet, we can determine the flow in pipe #4 by the H-W formula, rearranged as follows:

0.5 points for #1b

$$Q = \frac{h_L^{0.54} C D^{2.63}}{3.56 L^{0.54}}$$

$$= 2526\,gpm$$

Now the total flow between nodes B and D is then the sum:

Q_{B-D} = 2526 + 800 = **3326 gpm**

Finally using the H-W equation, you can calculate an equivalent length of an 8 inch pipe that gives the existing headloss with this flow:

$$L = \frac{h_L C^{1.85} D^{4.87}}{10.5 Q^{1.85}}$$

$$= 203\,ft$$

c. <u>Finally, determine a single equivalent pipe (D = 8 in) for the three pipes in series, pipe #1, the pipe from part (b), and pipe #5.</u>

Next you can use the H-W formula to calculate the headloss in pipes #1 and #5, recognizing that the flow in each must be the same as the flow determined for node B to node D (e.g., 3326 gpm):

0.5 points for #1c

	Pipe 1	Pipe 5	
C	120	120	
L	500	700	ft
D	12	12	in
Q	3326	3326	gpm
h_L	13.60007	19.0401	ft

The total headloss is then the sum:

h_L = 39.73 + 13.60 + 19.04 = **72.37 ft** and returning to the H-W equation, we can calculate an equivalent length based on this headloss and to flow:

$$L = \frac{h_L C^{1.85} D^{4.87}}{10.5 Q^{1.85}}$$

$$= 369\, ft$$

d. <u>Show that your pipe is hydraulically equivalent by calculating the head loss for this single pipe and comparing it to the sum of the head losses for pipes in the original system.</u>

Recalculate the headloss in each of the original pipes. Sum the headloss from each node to the next one, recognizing that there are two ways of getting from node B to node D (use either one, but not both).

$$h_L = 10.5 \frac{Q^{1.85} L}{C^{1.85} D^{4.87}}$$

0.5 points for #1d

	Pipe 1	Pipe 2	Pipe 3	Pipe 4	Pipe 5		Total
C	120	120	120	120	120		
L	500	500	800	1000	700	ft	
D	12	6	8	10	12	in	
Q	3325.68	800	800	2525.684	3325.68	gpm	
h_L	13.60007	28.49666	11.23213	39.72879	19.0401	ft	72.36896

2. Use the Hardy Cross method (and the Hazen Williams equation) to solve for the flows in each pipe of the network shown in Figure 2 and described in Table 2. Also, determine the values of the hydraulic grade line (HGL) and pressure at each node (pipe junction) in the system. Assume that C_{HW} = 120 for all pipes. The elevation of water in the tank at node A is 250 ft.

Table 2. Pipe & Node Data for Figure 2 Pipe Network

Pipe #	Nodes	Length (ft)	Diam (in)	Node #	Elev. (ft)	External Demand (gpm)
1	A - B	2000	16	A	180	0
2	B - C	800	12	B	50	0
3	B - D	600	10	C	30	400
4	D - E	800	8	D	80	1200
5	C - E	1200	8	E	40	1400
6	C - F	1300	6	F	60	1000
7	E - F	900	8			

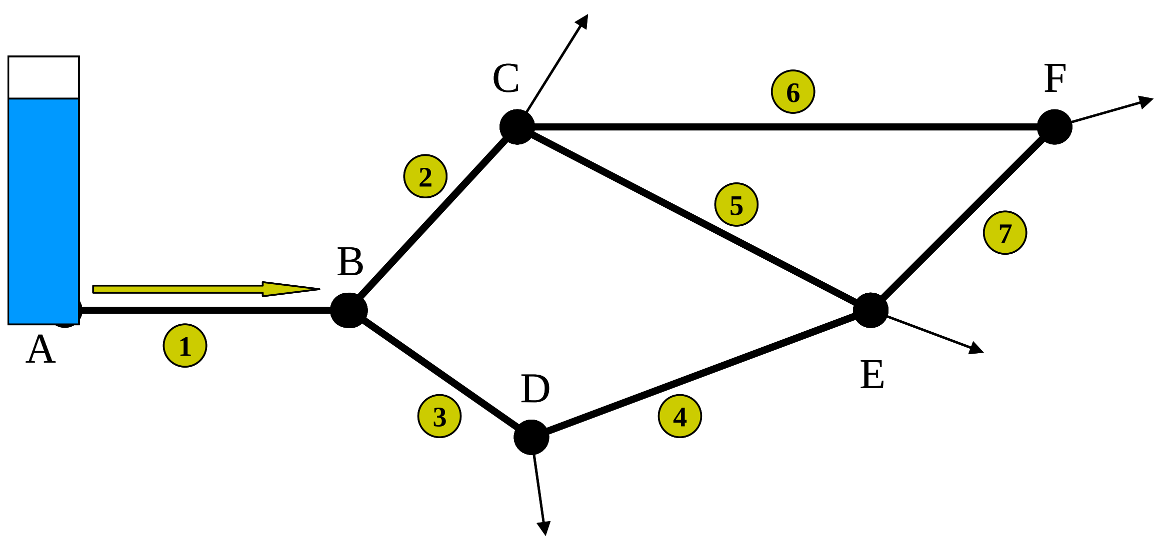


Figure 2. Pipe Network

For the calculation below, I used the following units:

- Q in gpm
- D in inches
- L in feet

This corresponds to the following from of the Hazen-Williams equation

$$h_L = 10.5\frac{Q^{1.85}L}{C^{1.85}D^{4.87}}$$

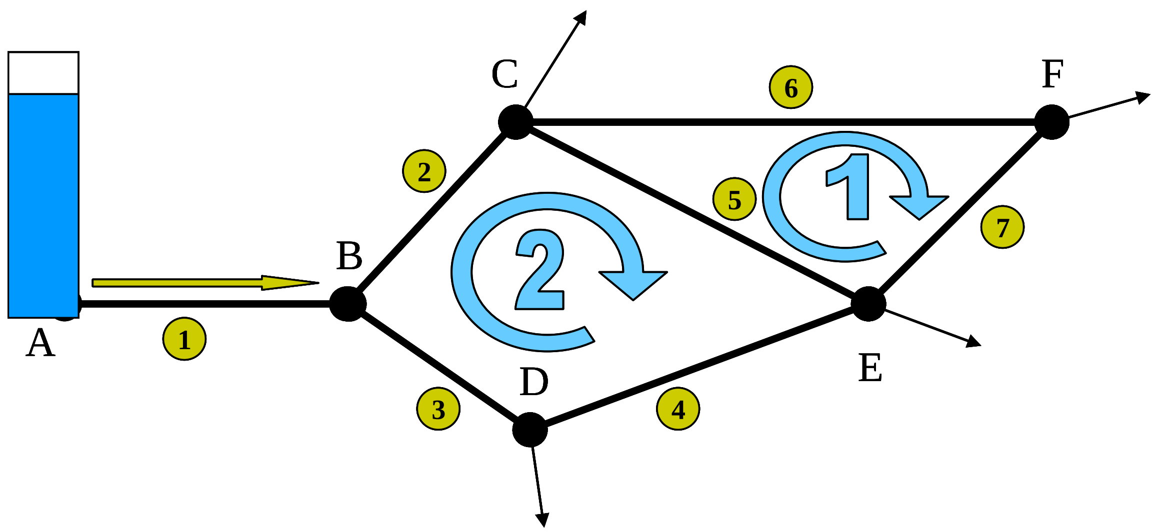
An now I calculate the Hazen-Williams "K" for each pipe from:

$$K = 10.5\frac{L}{C^{1.85}D^{4.87}}$$

Where: L is in ft and D is in inches

	Pipe 7	Pipe 6	Pipe 5	Pipe 4	Pipe 3	Pipe 2		Pipe 1
Length (ft)	900	1300	1200	800	600	800		2000
Diam (in)	8	6	8	8	10	12		16
HW "C"	120	120	120	120	120	120		120
K (HW)	5.38E-05	3.16E-04	7.18E-05	4.78E-05	1.21E-05	6.64E-06		4.09E-06

Then, adopt a pair of loops and sum up headloss around them in clockwise fashion.



Now determine the headloss (in ft) from:

$$h_f = KQ^{1.85}$$

Where Q is in gpm, and K is as calculated before

And the iterative correction Q is:

$$\Delta Q = -\frac{\sum_{loop} h_f}{1.85\sum_{loop}\frac{h_f}{Q}}$$

2 points for #2

First Iteration (start with assumed flows). The assumed flows much be selected based on mass balance considerations for each node (i.e., the flow into each node must be exactly equal to the flow out, including external demands)

	Pipe 7	Pipe 6	Pipe 5	Pipe 4	Pipe 3	Pipe 2	Sum hf	Delta Q	
Q (loop 1)	-200	800	-800						gpm
Q (loop 2)			800	-800	-2000	2000			gpm
head loss 1	-0.97	74.09	-16.85				56.27	-256.60	ft
head loss 2			16.85	-11.23	-15.48	8.49	-1.37	17.29	ft

Second Iteration (adjust Qs by Delta Q; for pipe 5 in both loops, use both delta Qs)

	Pipe 7	Pipe 6	Pipe 5	Pipe 4	Pipe 3	Pipe 2	Sum hf	Delta Q	
Q (loop 1)	-457	543	-1074						gpm
Q (loop 2)			1074	-783	-1983	2017			gpm
head loss 1	-4.48	36.23	-29.05				2.70	-14.10	ft
head loss 2			29.05	-10.79	-15.23	8.63	11.66	-129.89	ft

Third Iteration

	Pipe 7	Pipe 6	Pipe 5	Pipe 4	Pipe 3	Pipe 2	Sum hf	Delta Q	
Q (loop 1)	-471	529	-958						gpm
Q (loop 2)			958	-913	-2113	1887			gpm
head loss 1	-4.74	34.51	-23.52				6.25	-33.84	ft
head loss 2			23.52	-14.33	-17.13	7.63	-0.31	3.47	ft

Fourth Iteration

	Pipe 7	Pipe 6	Pipe 5	Pipe 4	Pipe 3	Pipe 2	Sum hf	Delta Q	
Q (loop 1)	-505	495	-995						gpm
Q (loop 2)			995	-909	-2109	1891			gpm
head loss 1	-5.39	30.54	-25.24				-0.09	0.52	ft
head loss 2			25.24	-14.23	-17.08	7.66	1.59	-17.51	ft

Fifth Iteration

	Pipe 7	Pipe 6	Pipe 5	Pipe 4	Pipe 3	Pipe 2	Sum hf	Delta Q	
Q (loop 1)	-504	496	-977						gpm
Q (loop 2)			977	-927	-2127	1873			gpm
head loss 1	-5.38	30.59	-24.40				0.81	-4.53	ft
head loss 2			24.40	-14.74	-17.34	7.53	-0.15	1.69	ft

Sixth Iteration

	Pipe 7	Pipe 6	Pipe 5	Pipe 4	Pipe 3	Pipe 2	Sum hf	Delta Q	
Q (loop 1)	-509	491	-984						gpm
Q (loop 2)			984	-925	-2125	1875			gpm
head loss 1	-5.47	30.08	-24.69				-0.08	0.43	ft
head loss 2			24.69	-14.69	-17.32	7.54	0.22	-2.45	ft

Looks like we're quite close now. Note that the number of iterations and the rate of convergence will depend on the initial guess for flow in each pipe. The above table shows the final pipe flows in gpm, in accordance with the sign convention. Of course the flow in pipe #1 must be equal to the system demand of 4000 gpm.

for any given pipe:

$$HGL_2 = HGL_1 - h_L$$

Also, the pressure is determined from the difference between the hydraulic grade line and the elevation, then multiplied by the unit weight of water (W) which is 62.4 lb/ft^3 or 0.433 lb/in^2/ft:

$$P = (HGL - Z)W$$

Node	Pipe(s)	hf	HGL	Elev	Pressure (psi)
A (tank)		0.0	250	180.0	30
B	1	18.9	231	50.0	78
C	1,2	7.5	224	30.0	84
D	1,3	17.3	214	80.0	58
E	1,2, 5	32.2	199	40.0	69
E	1,3, 4	32.1	199	40.0	69
F	1,2, 6	37.6	194	60.0	58
F	1,2, 5, 7	37.7	193	60.0	58
F	1,3, 4, 7	37.5	194	60.0	58

So the multiple routes to the same node give identical pressures as they should. Now summarizing with a single unique answer for each node:

Node	hf (ft)	HGL	Elev	Pressure (psi)
A	0.0	250.0	180.0	30.3
B	18.9	231.1	50.0	78.4
C	7.5	223.6	30.0	83.8
D	17.3	213.8	80.0	57.9
E	32.2	199.0	40.0	68.8
F	37.6	193.5	60.0	57.8