

Updated: 26 November 2019

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CEE 370

Environmental Engineering Principles

Lecture #32

Wastewater Treatment III: Process Modeling & Residuals

Reading M&Z: Chapter 9

[Reading: Davis & Cornwall, Chapt 6-1 to 6-8](#)

[Reading: Davis & Masten, Chapter 11-11 to 11-12](#)

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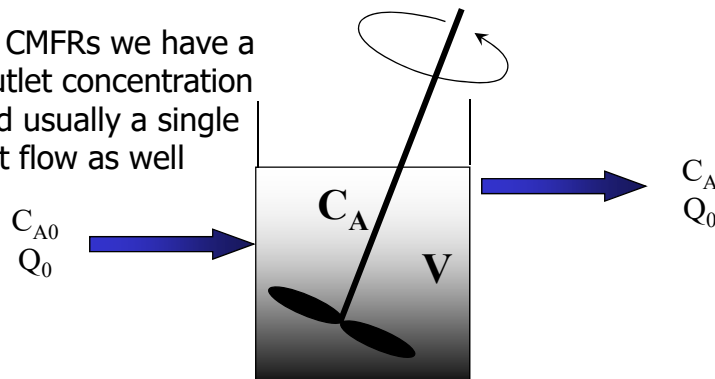
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Microbial Biomass in a CMFR

General Reactor
mass balance

$$\frac{dm_A}{dt} = \sum_{i=1}^n (C_{Ai} Q_i)_{in} - \sum_{j=1}^n (C_{Aj} Q_j)_{out} - r_A V$$

But with CMFRs we have a
single outlet concentration
(C_A) and usually a single
inlet flow as well



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Batch Microbial Growth

General Reactor mass balance

$$\frac{dM_A}{dt} = \sum_{i=1}^n (C_{Ai} Q_i)_{in} - \sum_{j=1}^n (C_{Aj} Q_j)_{out} - r_A V$$

Batch reactors are usually filled, allowed to react, then emptied for the next batch

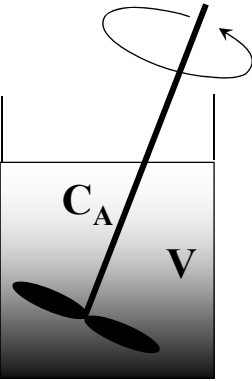
Because there isn't any flow in a batch reactor:

$$\frac{1}{V} \frac{dM_A}{dt} = -r_A$$

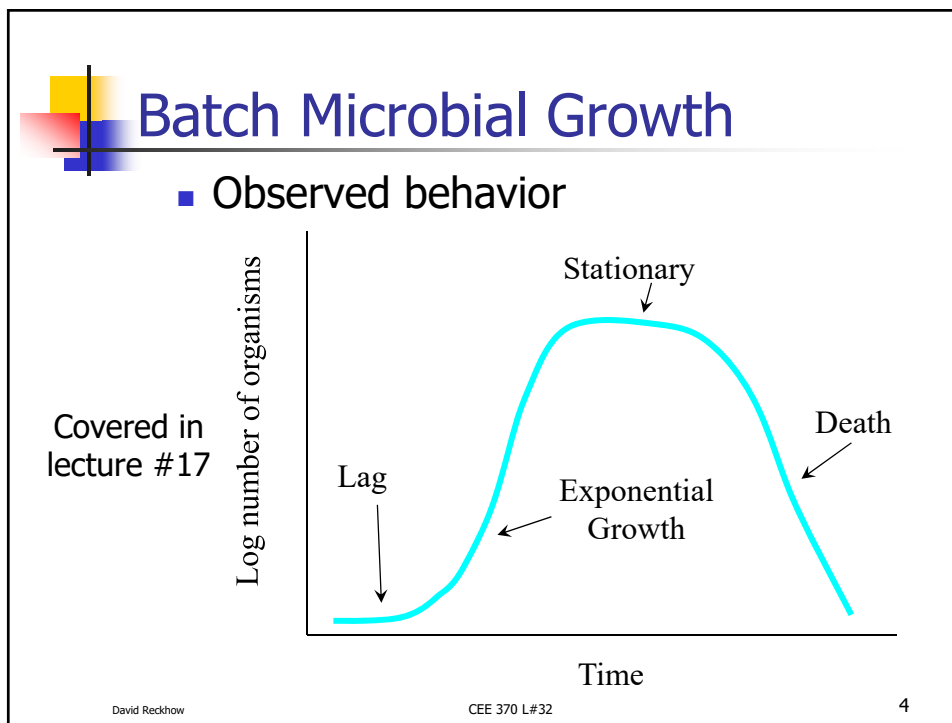
And:


$$\frac{dC_A}{dt} = -r_A$$

For 1st order biomass growth



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Exponential Growth model

Covered in lecture #17

$$\left(\frac{dX}{dt} \right)_{gr} \equiv \mu X$$

D&M Text

↓


N **X** = concentration of microorganisms at time **t**

t **t** = time

r **μ** = proportionality constant or specific growth rate, [time⁻¹]

dN/dt **dX/dt** = microbial growth rate, [mass per volume-time]

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Exp. Growth (cont.)

Covered in lecture #17

$$\left(\frac{dX}{dt} \right)_{gr} \equiv \mu X \quad \text{or} \quad \left(\frac{dX}{X} \right)_{gr} \equiv \mu dt$$

$$\ln \left(\frac{X}{X_0} \right) = \mu t$$

$$X = X_0 e^{\mu t}$$

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Substrate-limited Growth

- Also known as resource-limited growth
- THE MONOD MODEL

$$\mu = \mu_{\max} \frac{S}{K_s + S} \quad \text{and} \quad \left(\frac{dX}{dt} \right)_{gr} \equiv \mu X = \mu_{\max} \frac{SX}{K_s + S}$$

where,

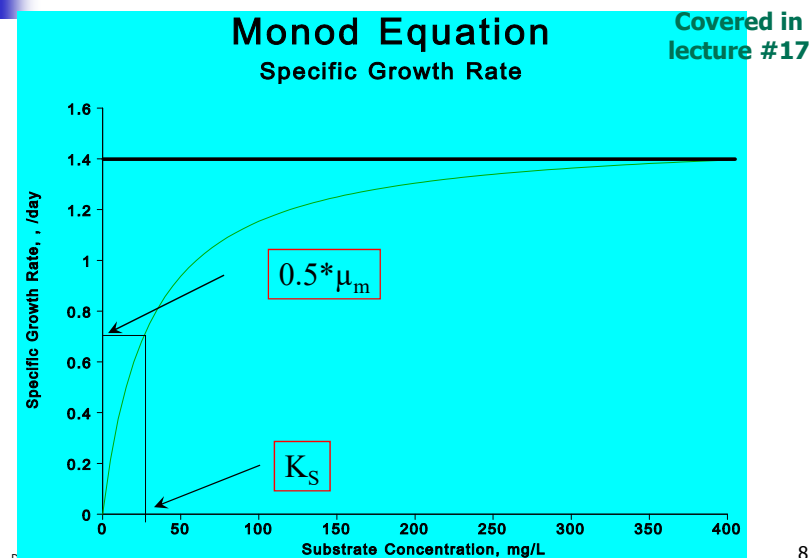
μ_{\max} = maximum specific growth rate, [day⁻¹]
 S = concentration of limiting substrate, [mg/L]
 K_s = Monod or half-velocity constant, or half saturation coefficient, [mg/L]

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Monod Kinetics



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Substrate Utilization & Yield

- Related to growth by Y , the yield coefficient

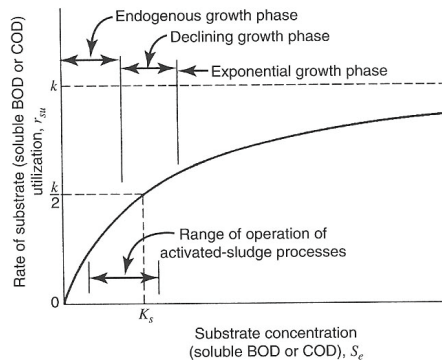
- Mass of cells produced per mass of substrate utilized

H&H, Fig 11-38, pp.406

$$Y \equiv \frac{\Delta X}{\Delta S} = \frac{dX/dt}{dS/dt}$$

- Just pertains to cell growth

$$\left(\frac{dX}{dt}\right)_{gr} = Y \frac{dS}{dt}$$



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Microbial Growth

$$\left(\frac{dX}{dt}\right)_{gr} = Y \frac{dS}{dt}$$

- Monod kinetics in a chemostat (batch reactor)

$$\left(\frac{dX}{dt}\right)_{gr} \equiv \mu X = \mu_{max} \frac{SX}{K_s + S} \xrightarrow[\text{\& Divide by Y}]{\text{\text{Substitute for dS}}} \frac{dS}{dt} = \frac{\mu_{max}}{Y} \frac{XS}{K_s + S}$$

- Where

- $dS/dt = r_{su}$ = actual substrate utilization rate
- k = maximum substrate utilization rate = μ_{max}/Y
- S = concentration of substrate (S_e in H&H)
- K_s = half-saturation constant
- Y = cell yield = dX/dS

$$r_{su} = k \frac{XS_e}{K_s + S_e}$$

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Death

- Bacterial cells also die at a characteristic first order rate with a rate constant, k

$$\left(\frac{dX}{dt}\right)_d = -k_d X$$

- This occurs at all times, and is independent of the substrate concentration

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Overall model: chemostat

- Combining growth and death, we have:

$$\begin{aligned}\left(\frac{dX}{dt}\right)_{net} &= \left(\frac{dX}{dt}\right)_{gr} + \left(\frac{dX}{dt}\right)_d \\ &= \mu_{max} \frac{SX}{K_S + S} - k_d X\end{aligned}$$

See: M&Z equ 9.3

- And in terms of substrate utilization

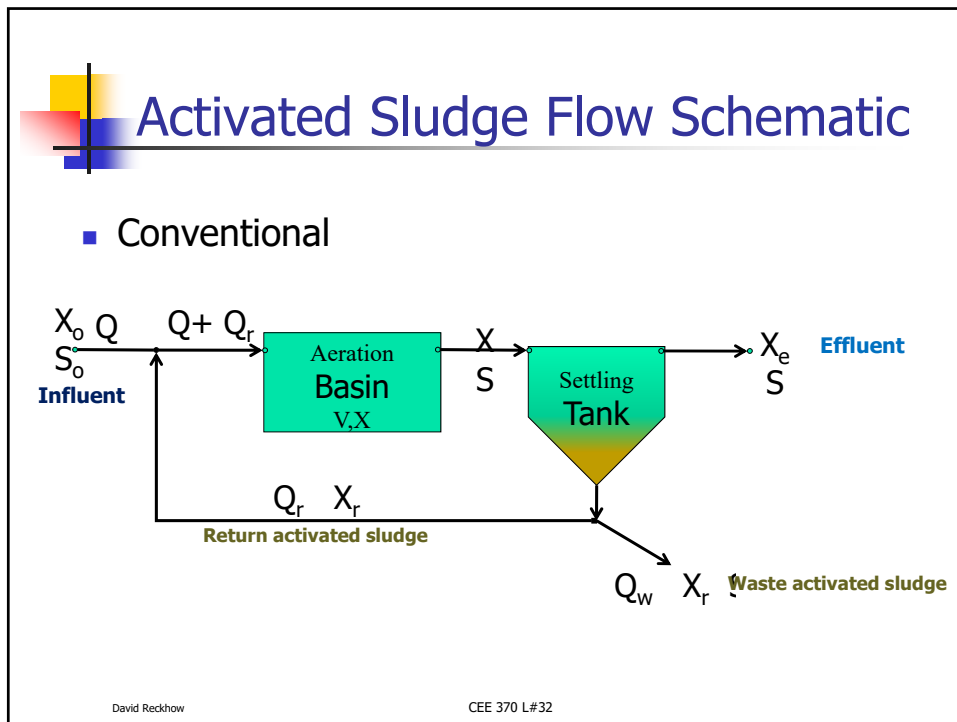
$$\left(\frac{dX}{dt}\right)_{net} = Y \left(\frac{dS}{dt}\right) - k_d X$$

$$Y \equiv \left(\frac{dX}{dt}\right)_{gr} \div \frac{dS}{dt}$$

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Efficiency & HRT

- Efficiency of BOD removal

$$E = \frac{(S_o - S)100\%}{S_o}$$
- Hydraulic Retention Time, HRT (Aeration Time)
 - Same as retention time in DWT (t_R)
 - Actual HRT is a bit different
 - Isn't used as much in design
$$\theta = \frac{V}{Q}$$

$$\theta_{act} = \frac{V}{Q + Q_R}$$

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SRT – solids retention time & R

- SRT: Primary operation and design parameter
 - How long does biomass stay in system


$$\theta_c = \frac{\text{biomass in tank}}{\text{rate of biomass leaving system}}$$

$$\theta_c = \frac{XV}{(Q - Q_w)X_e + Q_w X_r} \approx \frac{XV}{Q_w X_r}$$

See: M&Z equ 9.10
 - Typically equals 5-15 days
- Recycle Ratio

$$R = \frac{Q_r}{Q}$$
 - Values of 0.25-1.0 are typical

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F:M Ratio and volumetric loading

- Food-to-Microorganism Ratio (F/M)

$$\frac{F}{M} = \frac{\text{food supplied per day}}{\text{biomass in tank}}$$

$$\frac{F}{M} = \frac{QS_o}{XV}$$

M&Z equ 9.16
- Typical values are 0.2-0.6 in complete mixed AS
- BOD volumetric Loading

$$\text{Loading} = \frac{QS_o}{V}$$
 - Typically 50-120 lb BOD/day/1000ft³ tank volume

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Act. Sludge: Biomass Model

- Steady State mass balance on biomass

$$V \frac{dX}{dt} = 0 = QX_o - Q_e X_e - Q_w X_r + V \left(\frac{dX}{dt} \right)_{batch}$$

$$\left(\frac{dX}{dt} \right)_{net} = \left(\frac{dX}{dt} \right)_{gr} + \left(\frac{dX}{dt} \right)_d$$

$$= \mu_{max} \frac{SX}{K_S + S} - k_d X$$

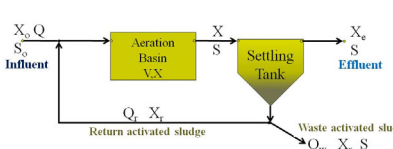
From chemostat model
- Incorporating the chemostat model gets:

$$V \frac{dX}{dt} = 0 = QX_o - Q_e X_e - Q_w X_r + V \left(\mu_{max} \frac{SX}{K_S + S} - k_d X \right)$$
- And simplifying

$$-QX_o + Q_e X_e + Q_w X_r = V \left(\mu_{max} \frac{SX}{K_S + S} - k_d X \right)$$
 - Finally, we recognize that the amount of solids entering with the WW (i.e., X_o) and leaving in the treated effluent (i.e., X_e) is quite small and can be neglected

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Biomass Model II



- So it becomes

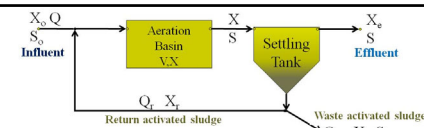
$$Q_w X_r = V \left(\mu_{max} \frac{SX}{K_S + S} - k_d X \right)$$
- And rearranging

$$\frac{1}{\theta_c} \approx \frac{Q_w X_r}{VX} = \mu_{max} \frac{S}{K_S + S} - k_d$$

Earlier equation for SRT

$$\theta_c = \frac{XV}{(Q - Q_w)X_e + Q_w X_r} \approx \frac{XV}{Q_w X_r}$$

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Act. Sludge: Substrate Model

- Steady state mass balance on substrate

$$V \frac{dS}{dt} = 0 = QS_o - Q_e S - Q_w S + V \left(\frac{dS}{dt} \right)_{\text{batch}}$$

$\frac{dS}{dt} = \frac{\mu_{\max}}{Y} \frac{XS}{K_s + S}$
 From chemostat model
- Substituting and noting that $Q_e = Q - Q_w$

$$QS_o = QS - Q_w S + Q_w S - V \left(\frac{\mu_{\max}}{Y} \frac{XS}{K_s + S} \right)$$
- And further simplifying

$$Q(S_o - S) = V \left(\frac{\mu_{\max}}{Y} \frac{XS}{K_s + S} \right)$$

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Merging the biomass & substrate models

- If we divide the previous equation by V and X

$$\frac{Q(S_o - S)}{VX} = \frac{\mu_{\max}}{Y} \frac{S}{K_s + S}$$

$Q(S_o - S) = V \left(\frac{\mu_{\max}}{Y} \frac{XS}{K_s + S} \right)$
- Multiply both sides by Y

$$\frac{YQ(S_o - S)}{VX} = \mu_{\max} \frac{S}{K_s + S}$$

M&Z equ 9.8
- Now insert the LH term into the earlier equation based on biomass

$$\frac{1}{\theta_c} = \frac{Q_w X_r}{VX} = \mu_{\max} \frac{S}{K_s + S} - k_d$$

$\frac{1}{\theta_c} = \frac{Q_w X_r}{VX} = \mu_{\max} \frac{S}{K_s + S} - k_d$

M&Z equ 9.9

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Combined model II

- Now recognize that Q/V is the reciprocal of the HRT

$$\frac{1}{\theta_c} = \frac{1}{\theta} \frac{Y(S_o - S)}{X} - k_d$$

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Question

- All else being equal, as SRT goes up:
 1. Settleability goes down
 2. F/M goes down
 3. Waste sludge return ratio must go down
 4. Endogenous respiration becomes less important
 5. Sludge yield increases



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Aeration: Loadings

Food-to-Microorganism Ratio (F/M)

$$\frac{F}{M} = \frac{Q * BOD}{V * X}$$

Sludge Age or mean cell residence time (θ_c)

$$\theta_c = \frac{VX}{(X_e Q_e) + (X_w Q_w)}$$

$$\approx \frac{VX}{X_w Q_w}$$

Where

- Q=WW flow
- V=volume of aeration tank
- X=MLVSS=mixed liquor volatile suspended solids (biomass concentration)
- X_e =VSS_e = suspended solids in wastewater effluent
- X_w =VSS_w = suspended solids in waste sludge
- Q_w = flow of waste sludge
- SS is sometimes used instead of VSS

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Operating Criteria

Loading, biomass, retention time, etc


H&H, Table11-4, pp.395

TABLE 11-4

Summary of Loadings and Operational Parameters for Aeration Processes

PROCESS	BOD LOADING (lb BOD/day per 1000 cu ft) ^a	MLSS (mg/l)	F/M RATIO (lb BOD/day per lb MLSS) ^b	SLUDGE AGE (days)	AERATION PERIOD (hr)	RETURN SLUDGE RATES (percent)	BOD REMOVAL EFFICIENCY (percent)
Conventional	20 to 40	1000 to 3000	0.2 to 0.5	5 to 15	4.0 to 7.5	20 to 40	80 to 90
Step aeration	40 to 60	1500 to 3500	0.2 to 0.5	5 to 15	4.0 to 7.0	30 to 50	80 to 90
Extended aeration	10 to 20	2000 to 8000	0.05 to 0.2	20 and up	20 to 30	50 to 100	85 to 95
High-purity oxygen	120 and up	4000 to 8000	0.6 to 1.5	3 to 10	1.0 to 3.0	30 to 50	80 to 90

^a1.0 lb/1000 cu ft/day = 16.0 g/m³ · d^b1.0 lb/day/lb MLSS = 1.0 g/d · g MLSS





Activated Sludge

- Mixed liquor
- Return Activated sludge

Aeration!


1. Surface aerators
2. Bubble diffusers





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Environmental Engineering Principles

Lecture #32b

Wastewater Treatment IIIb:

Process Modeling & Residuals

Reading: M&Z Chapter 9.11

Other Reading: Davis & Cornwall, Chapt 6-1 to 6-8 and Davis & Masten, Chapter 11-11 to 11-12

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Anaerobic Digester Problem

Anaerobic digesters are commonly used in wastewater treatment. The biological process produces both carbon dioxide and methane gases. A laboratory worker plans to make a "synthetic" digester gas. There is currently 2 L of methane gas at 1.5 atm and 1 L of carbon dioxide gas at 1 atm in the lab. If these two samples are mixed in a 4 L tank, what will be the partial pressures of the individual gases? The total pressure?

Example 4.4 from Ray

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Solution to Anaerobic Digester Problem

First, we must find the partial pressures of the individual gases using the ideal gas law:

$$P_1 V_1 = nRT = P_2 V_2 \quad \text{or} \quad P_2 = P_1 \left(\frac{V_1}{V_2} \right)$$

For methane gas

$$P_2 = 1.5 \text{ atm} \left(\frac{2 \text{ L}}{4 \text{ L}} \right) = 0.75 \text{ atm}$$

For carbon dioxide gas:

$$P_2 = 1 \text{ atm} \left(\frac{1 \text{ L}}{4 \text{ L}} \right) = 0.25 \text{ atm}$$

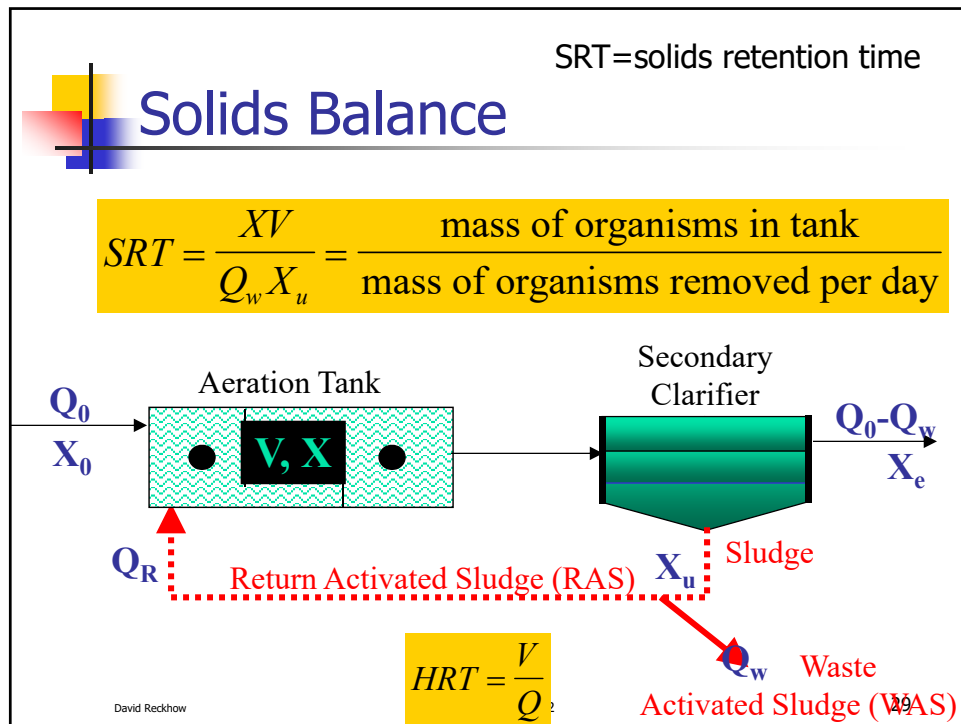
And the total is:

$$P_t = P_{\text{CH}_4} + P_{\text{CO}_2} = 1 \text{ atm}$$

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We will cover this in CEE 471

Solids Mass Balance

- Consider aeration tank and clarifier together
 - Biomass in + biomass produced due to growth = biomass out

$$Q_0 X_0 + V \frac{dX}{dt} = (Q_0 - Q_w) X_e + Q_w X_w$$

- Now using the combined growth equation without limitation to carrying capacity:

$$\frac{dX}{dt} = \left[\mu_{\max} \left(\frac{S}{K_s + S} \right) - k_d \right] X$$
- Combining and assuming X_0 and X_e to be negligible:

$$\frac{\mu_{\max} S}{K_s + S} = \frac{Q_w X_w}{VX} + k_d$$

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We cover this in detail in CEE 471

Substrate Mass Balance

- Consider aeration tank and clarifier together
 - Substrate in + substrate consumed by biomass = substrate out

$$Q_0 S_0 + V \frac{dS}{dt} = (Q_0 - Q_w) S + Q_w S$$

Note that effluent and waste sludge substrate concentrations are considered the same

- Now using the combined substrate utilization equation without limitation to carrying capacity:

$$\frac{dS}{dt} = -\frac{1}{Y} \left[\mu_{\max} \left(\frac{S}{K_s + S} \right) - k_d \right] X$$
- Combining and rearranging:

$$\frac{\mu_{\max} S}{K_s + S} = \frac{Q_0 Y}{VX} (S_0 - S)$$

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We cover this in CEE 471

Combined Mass Balances

- In summary the solids and substrate mass balance equations are:

$$\frac{\mu_{\max} S}{K_s + S} = \frac{Q_w X_w}{VX} + k_d \qquad \frac{\mu_{\max} S}{K_s + S} = \frac{Q_0 Y}{VX} (S_0 - S)$$
- These can be easily combined (left hand terms are the same):

$$\frac{Q_w X_w}{VX} = \frac{Q_0 Y}{VX} (S_0 - S) - k_d$$

$\frac{1}{\Theta_c}$

← The mean cell residence time, or sludge age

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Sludge Treatment

- Depends on type of sludge
- Typical process train
 - Thickening or dewatering
 - Conditioning
 - Stabilization (usually for wastewater)
 - Disposal
- Nonmechanical methods
 - Lagoons
 - Sand-drying beds
 - Freeze treatment
- Mechanical methods
 - Centrifugation
 - Vacuum filtration
 - Belt filter press
 - Plate filters

See also Lecture #30

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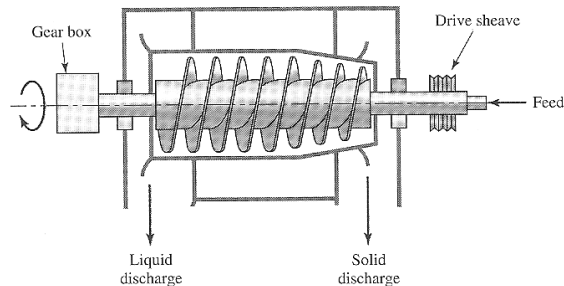
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Centrifuge

FIGURE 9-21

Solid bowl centrifuge.



From Lecture #30

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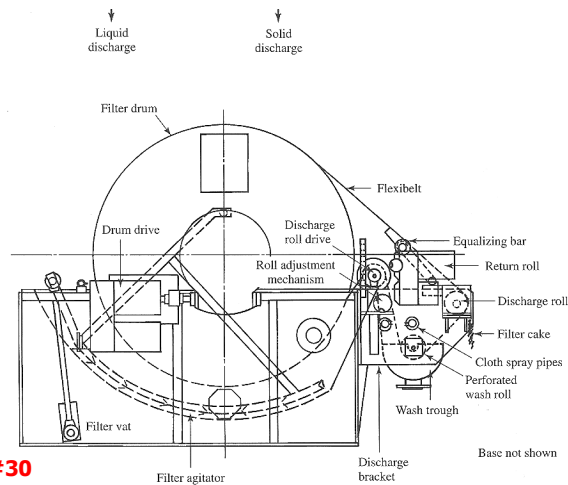
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Vacuum Filter

FIGURE 9-22

Vacuum filter. (Courtesy of Komline-Sanderson Engineering Corporation.)



From Lecture #30

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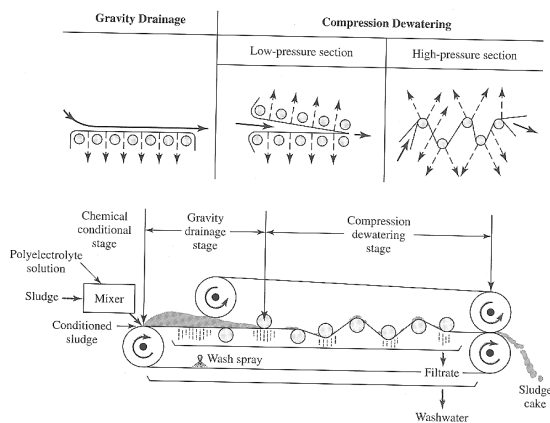
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Belt Filter Press

FIGURE 9-23

Continuous belt filter press. (Source: U.S. Environmental Protection Agency, *Process Design Manual, Sludge Treatment and Disposal*, 1979.)




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- To next lecture

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