Updated: 26 November 2019

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CEE 370
Environmental Engineering Principles

Lecture #32

Wastewater Treatment III:

Process Modeling & Residuals

Reading M&Z: Chapter 9

Reading: Davis & Cornwall, Chapt 6-1 to 6-8

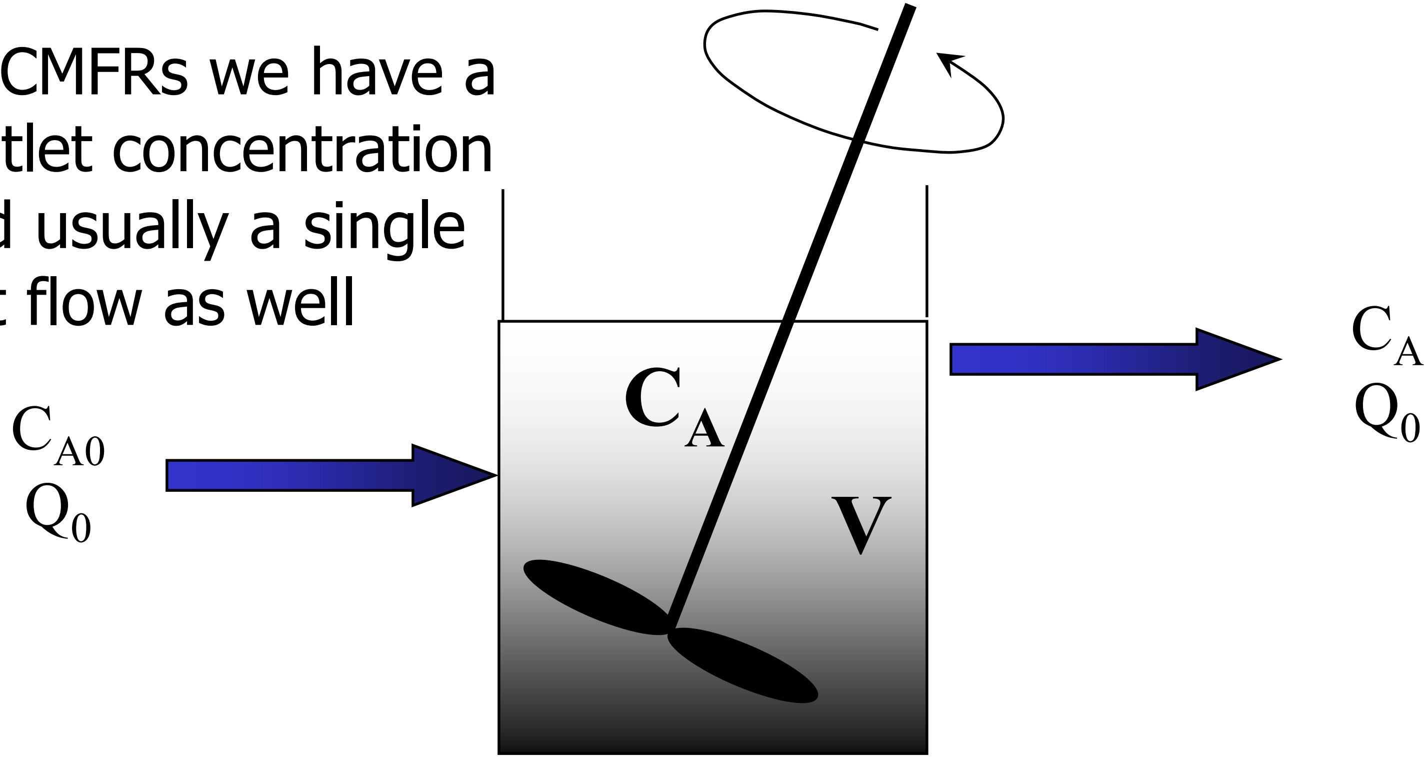
Reading: Davis & Masten, Chapter 11-11 to 11-12

Microbial Biomass in a CMFR

General Reactor mass balance

$$\frac{dm_A}{dt} = \sum_{i=1}^{n}(C_{Ai}Q_i)_{in} - \sum_{j=1}^{n}(C_{Aj}Q_j)_{out} - r_A V$$

But with CMFRs we have a single outlet concentration (C_A) and usually a single inlet flow as well

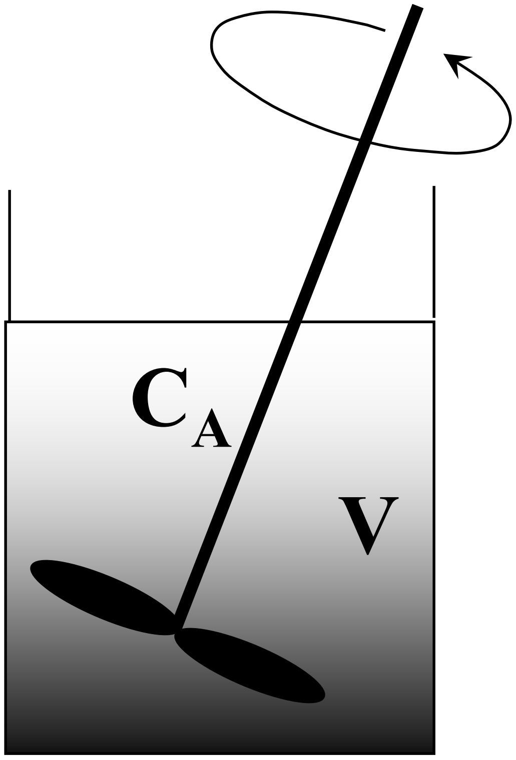


Batch Microbial Growth

General Reactor mass balance

$$\frac{dM_A}{dt} = \overset{0}{\cancel{\sum_{i=1}^{n}(C_{Ai}Q_i)_{in}}} - \overset{0}{\cancel{\sum_{j=1}^{n}(C_{Aj}Q_j)_{out}}} - r_A V$$

Batch reactors are usually filled, allowed to react, then emptied for the next batch



Because there isn't any flow in a batch reactor:

$$\frac{1}{V}\frac{dM_A}{dt} = -r_A$$

And:

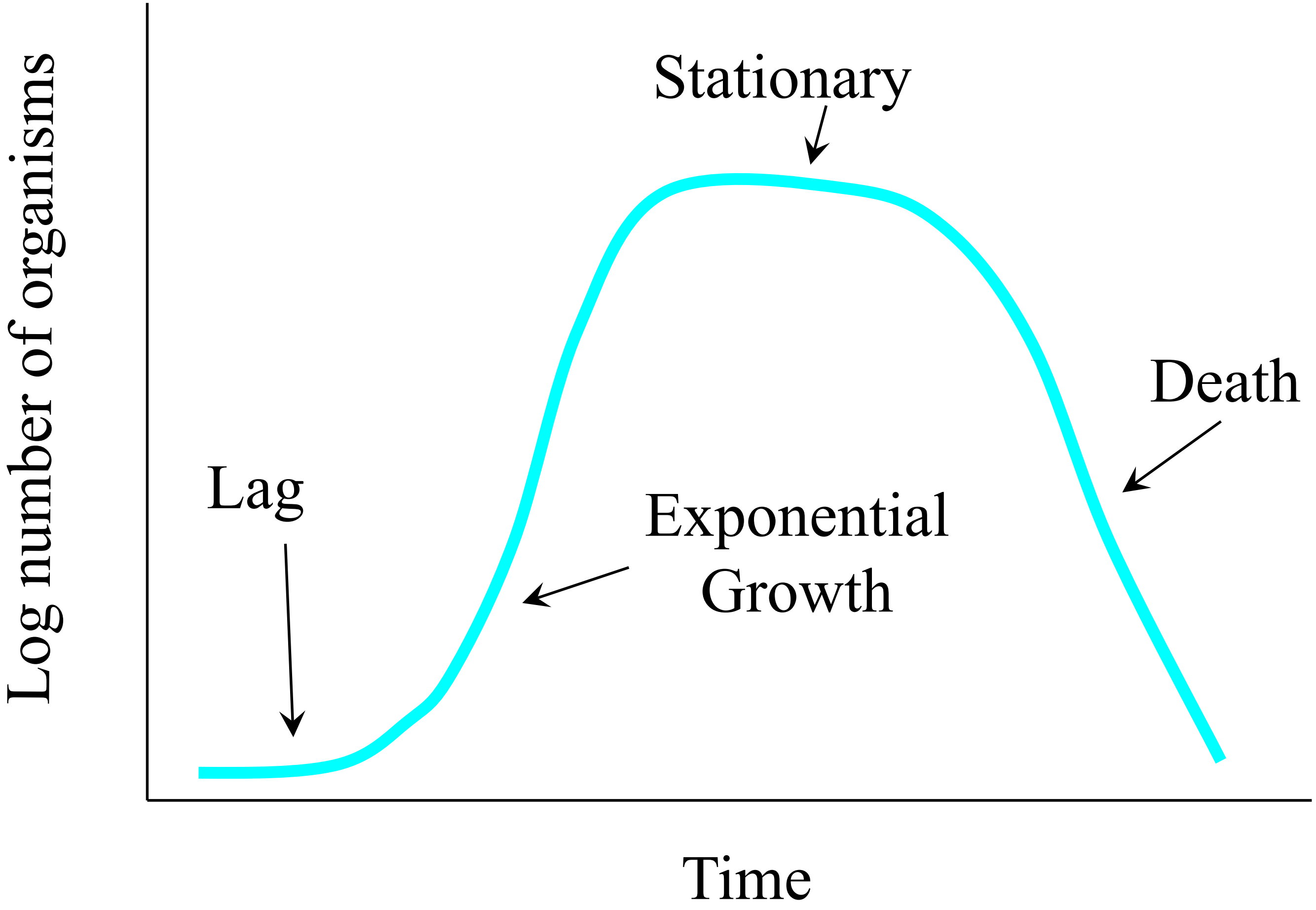
$$\frac{d\overset{X}{C_A}}{dt} = -\overset{kX}{r_A}$$

For 1st order biomass growth

Batch Microbial Growth

- Observed behavior

Covered in lecture #17



Exponential Growth model

Covered in lecture #17

$$\left(\frac{dX}{dt}\right)_{gr} \equiv \mu X$$

where,

D&M Text ⇩	
N	X = concentration of microorganisms at time t
t	t = time
r	μ = proportionality constant or specific growth rate, [time^{-1}]
dN/dt	dX/dt = microbial growth rate, [mass per volume-time]

Exp. Growth (cont.)

Covered in lecture #17

$$\left(\frac{dX}{dt}\right)_{gr} \equiv \mu X \qquad \text{or} \qquad \left(\frac{dX}{X}\right)_{gr} \equiv \mu dt$$

$$\ln\left(\frac{X}{X_o}\right) = \mu t$$

$$\boxed{X = X_o e^{\mu t}}$$

Substrate-limited Growth

- Also known as resource-limited growth
  - THE MONOD MODEL

$$\mu = \mu_{\max} \frac{S}{K_S + S}$$

and

$$\left(\frac{dX}{dt}\right)_{gr} \equiv \mu X = \mu_{\max} \frac{SX}{K_S + S}$$

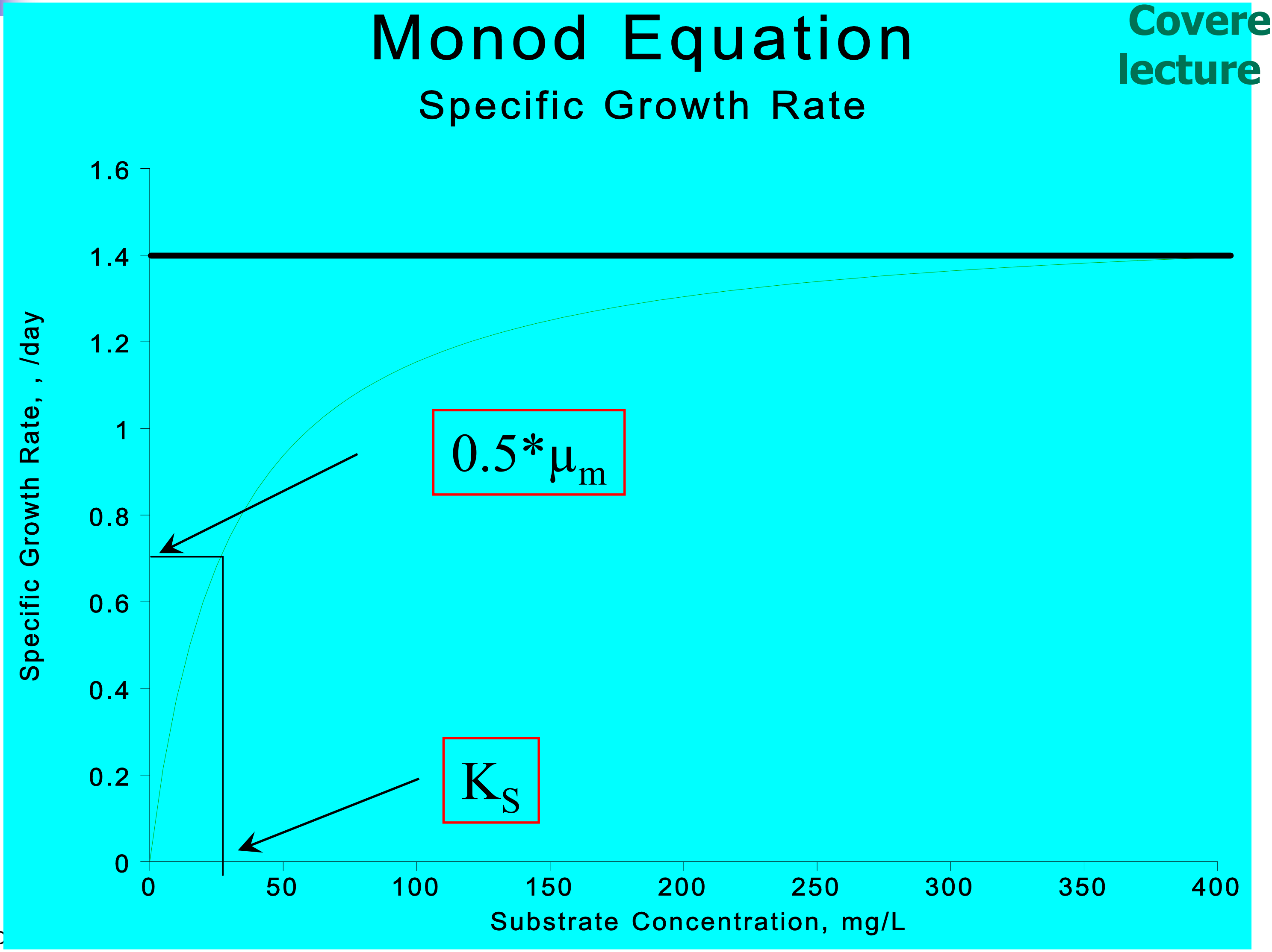
where,

μ_{max} = maximum specific growth rate, [day^{-1}]

S = concentration of limiting substrate, [mg/L]

K_s = Monod or half-velocity constant, or half saturation coefficient, [mg/L]

Monod Kinetics



Covered in lecture #17

Substrate Utilization & Yield

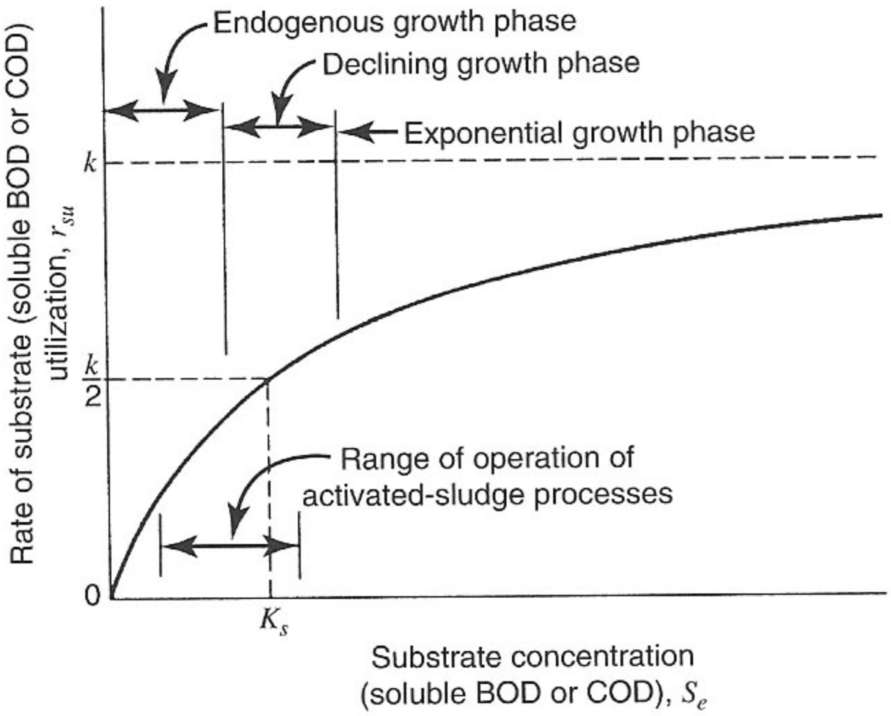
- Related to growth by Y, the yield coefficient
  - Mass of cells produced per mass of substrate utilized

$$Y \equiv \frac{\Delta X}{\Delta S} = \frac{dX/dt}{dS/dt}$$

  - Just pertains to cell growth

$$\left(\frac{dX}{dt}\right)_{gr} = Y\frac{dS}{dt}$$

H&H, Fig 11-38, pp.406



Microbial Growth

$$\left(\frac{dX}{dt}\right)_{gr} = Y\frac{dS}{dt}$$

- Monod kinetics in a chemostat (batch reactor)

$$\left(\frac{dX}{dt}\right)_{gr} \equiv \mu X = \mu_{\max}\frac{SX}{K_S + S} \xrightarrow[\text{\& Divide by Y}]{\text{Substitute for dS}} \frac{dS}{dt} = \frac{\mu_{\max}}{Y}\frac{XS}{K_S + S}$$

$$r_{su} = k\frac{XS_e}{K_S + S_e}$$

- Where
  - dS/dt = r_{su} = actual substrate utilization rate
  - k = maximum substrate utilization rate = μ_{max}/Y
  - S = concentration of substrate (S_e in H&H)
  - K_S = half-saturation constant
  - Y = cell yield = dX/dS

Death

- Bacterial cells also die at a characteristic first order rate with a rate constant, k

$$\left(\frac{dX}{dt}\right)_d = -k_d X$$

- This occurs at all times, and is independent of the substrate concentration

Overall model: chemostat

- Combining growth and death, we have:

$$\left(\frac{dX}{dt}\right)_{net} = \left(\frac{dX}{dt}\right)_{gr} + \left(\frac{dX}{dt}\right)_{d}$$

$$= \mu_{\max}\frac{SX}{K_S + S} - k_d X$$

See: M&Z equ 9.3

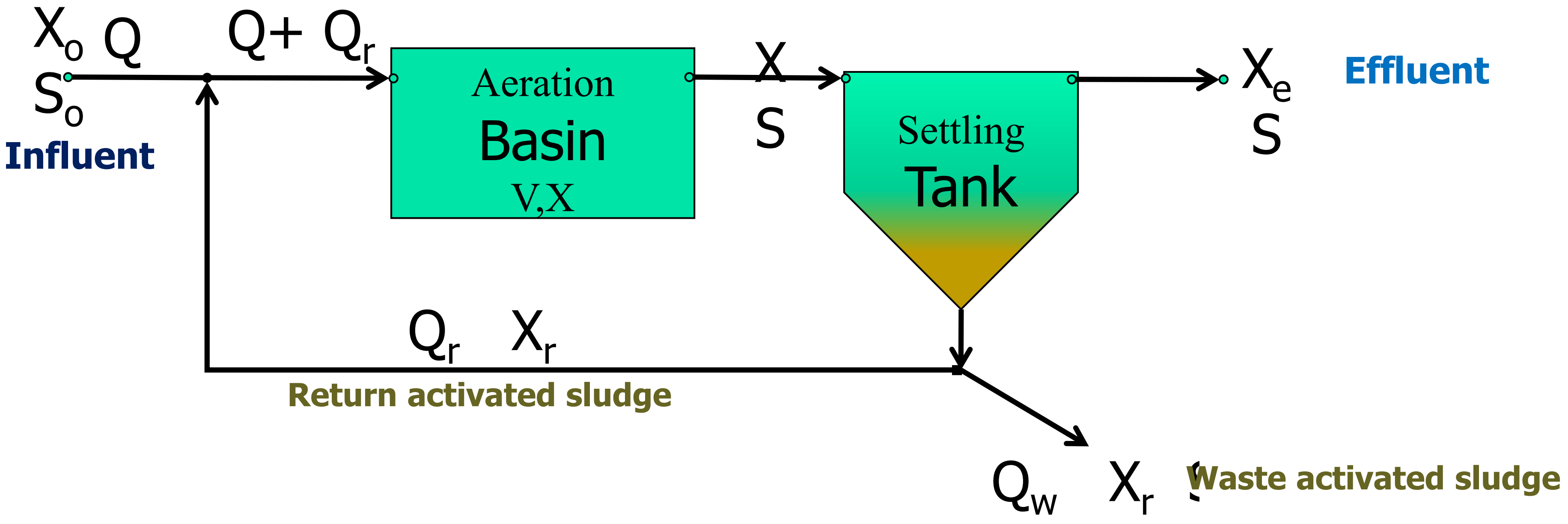
$$Y \equiv \left(\frac{dX}{dt}\right)_{gr} \div \frac{dS}{dt}$$

- And in terms of substrate utilization

$$\left(\frac{dX}{dt}\right)_{net} = Y\left(\frac{dS}{dt}\right) - k_d X$$

Activated Sludge Flow Schematic

- Conventional



X_o Q

S_o

Influent

Q+ Q_r

Aeration

Basin

V,X

X

S

Settling

Tank

X_e

S

Effluent

Q_r X_r

Return activated sludge

Q_w X_r

Waste activated sludge

Efficiency & HRT

- Efficiency of BOD removal

$$E = \frac{(S_o - S)100\%}{S_o}$$

- Hydraulic Retention Time, HRT (Aeration Time)
  - Same as retention time in DWT (t_R)

$$\theta = \frac{V}{Q}$$

  - Actual HRT is a bit different
    - Isn't used as much in design

$$\theta_{act} = \frac{V}{Q + Q_R}$$

SRT – solids retention time & R

- SRT: Primary operation and design parameter
  - How long does biomass stay in system

$$\theta_c = \frac{biomass\ in\ tank}{rate\ of\ biomass\ leaving\ system}$$

$$\theta_c = \frac{XV}{(Q-Q_w)X_e + Q_wX_r} \approx \frac{XV}{Q_wX_r}$$

See: M&Z equ 9.10

  - Typically equals 5-15 days
- Recycle Ratio
  - Values of 0.25-1.0 are typical

$$R = \frac{Q_r}{Q}$$

F:M Ratio and volumetric loading

- Food-to-Microorganism Ratio (F/M)

$$\frac{F}{M} = \frac{Q * BOD}{V * X}$$

$$\frac{F}{M} = \frac{\text{food supplied per day}}{\text{biomass in tank}}$$

$$\frac{F}{M} = \frac{QS_o}{XV}$$

M&Z equ 9.16

  - Typical values are 0.2-0.6 in complete mixed AS
- BOD volumetric Loading

$$Loading = \frac{QS_o}{V}$$

  - Typically 50-120 lb BOD/day/1000ft^3 tank volume

Act. Sludge: Biomass Model

- Steady State mass balance on biomass

$$V\frac{dX}{dt} = 0 = QX_o - Q_eX_e - Q_wX_r + V\left(\frac{dX}{dt}\right)_{batch}$$

$$\left(\frac{dX}{dt}\right)_{net} = \left(\frac{dX}{dt}\right)_{gr} + \left(\frac{dX}{dt}\right)_{d}$$

$$= \mu_{\max}\frac{SX}{K_S + S} - k_dX$$

From chemostat model

  - Incorporating the chemostat model gets:

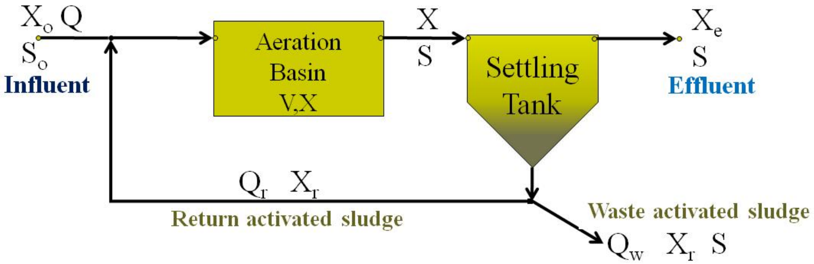
$$V\frac{dX}{dt} = 0 = QX_o - Q_eX_e - Q_wX_r + V\left(\mu_{\max}\frac{SX}{K_S + S} - k_dX\right)$$

  - And simplifying

$$-QX_o + Q_eX_e + Q_wX_r = V\left(\mu_{\max}\frac{SX}{K_S + S} - k_dX\right)$$

    - Finally, we recognize that the amount of solids entering with the WW (i.e., X_o) and leaving in the treated effluent (i.e., X_e) is quite small and can be neglected

Biomass Model II



- So it becomes

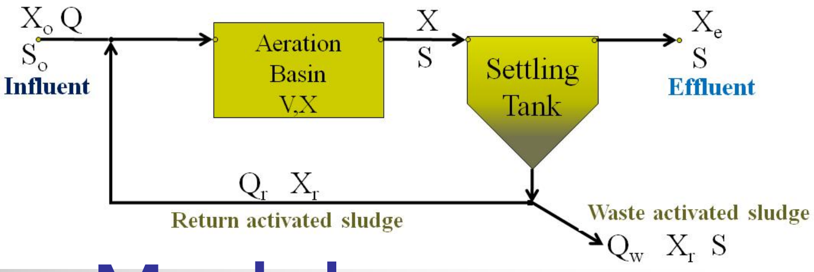
$$Q_w X_r = V\left(\mu_{\max}\frac{SX}{K_S + S} - k_d X\right)$$

- And rearranging

$$\frac{1}{\theta_c} \approx \qquad \frac{Q_w X_r}{VX} = \mu_{\max}\frac{S}{K_S + S} - k_d$$

Earlier equation for SRT

$$\theta_c = \frac{XV}{(Q - Q_w)X_e + Q_w X_r} \approx \boxed{\frac{XV}{Q_w X_r}}$$



Act. Sludge: Substrate Model

- Steady state mass balance on subst

$$V\frac{dS}{dt} = 0 = QS_o - Q_eS - Q_wS + V\left(\frac{dS}{dt}\right)_{batch}$$

$$\frac{dS}{dt} = \frac{\mu_{\max}}{Y}\frac{XS}{K_S + S}$$

From chemostat model

  - Substituting and noting that $Q_e = Q - Q_w$

$$QS_o = QS - Q_wS + Q_wS - V\left(\frac{\mu_{\max}}{Y}\frac{XS}{K_S + S}\right)$$

  - And further simplifying

$$Q(S_o - S) = V\left(\frac{\mu_{\max}}{Y}\frac{XS}{K_S + S}\right)$$

Merging the biomass & substrate models

- If we divide the previous equation by V and X

$$\frac{Q(S_o - S)}{VX} = \frac{\mu_{\max}}{Y}\frac{S}{K_S + S}$$

$$Q(S_o - S) = V\left(\frac{\mu_{\max}}{Y}\frac{XS}{K_S + S}\right)$$

- Multiply both sides by Y

$$\frac{YQ(S_o - S)}{VX} = \mu_{\max}\frac{S}{K_S + S}$$

M&Z equ 9.8

- Now insert the LH term into the earlier equation based on biomass

⇨

$$\frac{1}{\theta_c} = \frac{Q_w X_r}{VX} = \mu_{\max}\frac{S}{K_S + S} - k_d$$

$$\frac{1}{\theta_c} = \frac{Q_w X_r}{VX} = \frac{YQ(S_o - S)}{VX} - k_d$$

M&Z equ 9.9

Combined model II

- Now recognize that Q/V is the reciprocal of the HRT

$$\frac{1}{\theta_c} = \frac{1}{\theta}\frac{Y(S_o - S)}{X} - k_d$$

Question

- All else being equal, as SRT goes up:
  1. Settleability goes down
  2. F/M goes down
  3. Waste sludge return ratio must go down
  4. Endogenous respiration becomes less important
  5. Sludge yield increases

Aeration: Loadings

- Food-to-Microorganism Ratio (F/M)

$$\frac{F}{M} = \frac{Q * BOD}{V * X}$$

- Sludge Age or mean cell residence time (θ_c)

$$\theta_c = \frac{VX}{(X_e Q_e) + (X_W Q_W)}$$

$$\approx \frac{VX}{X_W Q_W}$$

- Where
  - Q=WW flow
  - V=volume of aeration tank
  - X=MLVSS=mixed liquor volatile suspended solids (biomass concentration)
  - X_e=VSS_e = suspended solids in wastewater effluent
  - X_W=VSS_w = suspended solids in waste sludge
  - Q_w = flow of waste sludge
- SS is sometimes used instead of VSS

Operating Criteria

- Loading, biomass, retention time, etc

H&H, Table11-4, pp.395

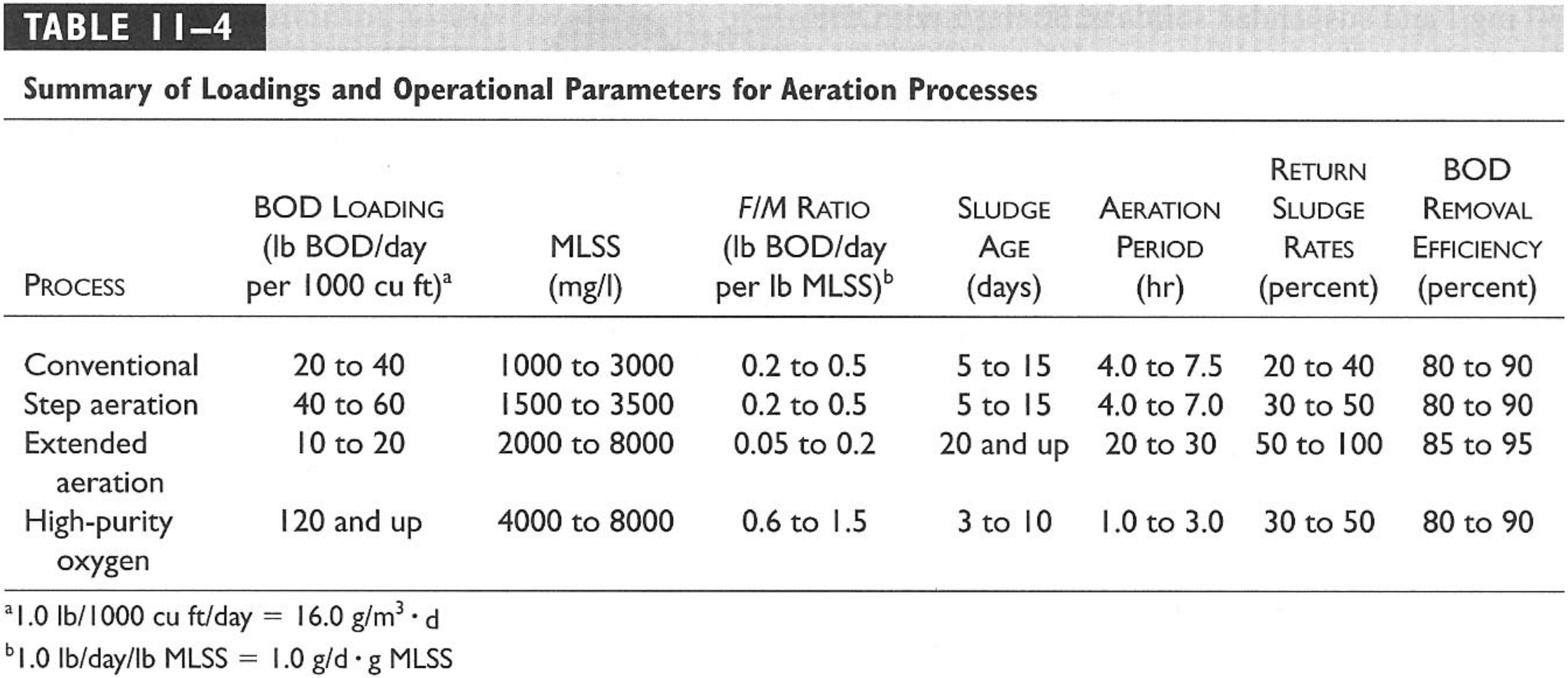


TABLE 11–4

Summary of Loadings and Operational Parameters for Aeration Processes

PROCESS	BOD LOADING (lb BOD/day per 1000 cu ft)[a]	MLSS (mg/l)	*F/M* RATIO (lb BOD/day per lb MLSS)[b]	SLUDGE AGE (days)	AERATION PERIOD (hr)	RETURN SLUDGE RATES (percent)	BOD REMOVAL EFFICIENCY (percent)
Conventional	20 to 40	1000 to 3000	0.2 to 0.5	5 to 15	4.0 to 7.5	20 to 40	80 to 90
Step aeration	40 to 60	1500 to 3500	0.2 to 0.5	5 to 15	4.0 to 7.0	30 to 50	80 to 90
Extended aeration	10 to 20	2000 to 8000	0.05 to 0.2	20 and up	20 to 30	50 to 100	85 to 95
High-purity oxygen	120 and up	4000 to 8000	0.6 to 1.5	3 to 10	1.0 to 3.0	30 to 50	80 to 90

[a] 1.0 lb/1000 cu ft/day = 16.0 $g/m^3 \cdot d$

[b] 1.0 lb/day/lb MLSS = 1.0 $g/d \cdot g$ MLSS

Activated Sludge

- Mixed liquor
- Return Activated sludge

Aeration!

1. Surface aerators
2. Bubble diffusers

Updated: 26 November 2019

CEE 370 Environmental Engineering Principles

Lecture #32b

Wastewater Treatment IIIb:

Process Modeling & Residuals

Reading: M&Z Chapter 9.11

Other Reading: Davis & Cornwall, Chapt 6-1 to 6-8 and Davis & Masten, Chapter 11-11 to 11-12

Anaerobic Digester Problem

Anaerobic digesters are commonly used in wastewater treatment. The biological process produces both carbon dioxide and methane gases. A laboratory worker plans to make a "synthetic" digester gas. There is currently 2 L of methane gas at 1.5 atm and 1 L of carbon dioxide gas at 1 atm in the lab. If these two samples are mixed in a 4 L tank, what will be the partial pressures of the individual gases? The total pressure?

Example 4.4 from Ray

Solution to Anaerobic Digester Problem

First, we must find the partial pressures of the individual gases using the ideal gas law:

$$P_1 V_1 = nRT = P_2 V_2$$

or

$$P_2 = P_1\left(\frac{V_1}{V_2}\right)$$

For methane gas

$$P_2 = 1.5 \text{ atm}\left(\frac{2 \text{ L}}{4 \text{ L}}\right) = 0.75 \text{ atm}$$

For carbon dioxide gas:

$$P_2 = 1 \text{ atm}\left(\frac{1 \text{ L}}{4 \text{ L}}\right) = 0.25 \text{ atm}$$

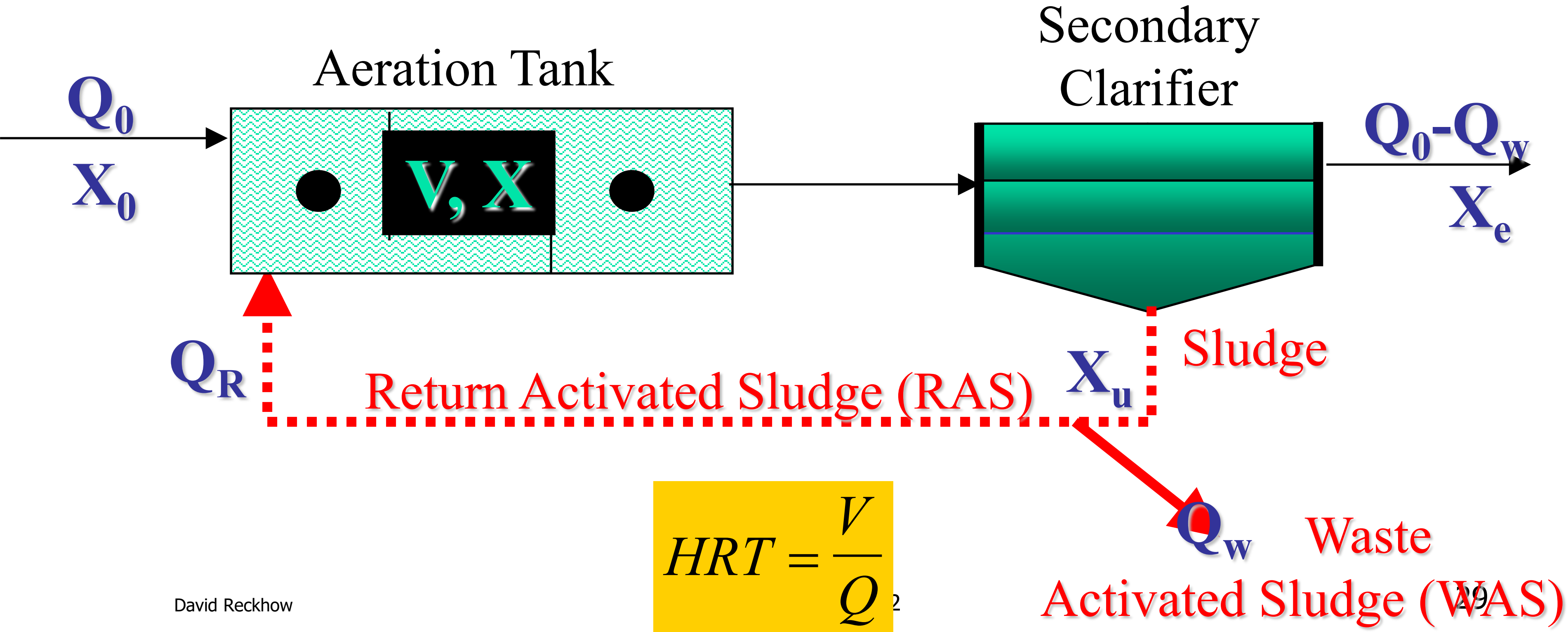
And the total is:

$$P_t = P_{CH_4} + P_{CO_2} = 1 \text{ atm}$$

SRT=solids retention time

Solids Balance

$$SRT = \frac{XV}{Q_w X_u} = \frac{\text{mass of organisms in tank}}{\text{mass of organisms removed per day}}$$



$$HRT = \frac{V}{Q}$$

We will cover this in CEE 471

Solids Mass Balance

- Consider aeration tank and clarifier together
  - Biomass in + biomass produced due to growth = biomass out

$$Q_0X_0 + V\frac{dX}{dt} = (Q_0 - Q_w)X_e + Q_wX_w$$

  - Now using the combined growth equation without limitation to carrying capacity:

$$\frac{dX}{dt} = \left[\mu_{\max}\left(\frac{S}{K_s + S}\right) - k_d\right]X$$

  - Combining and assuming X_0 and X_e to be negligible:

$$\frac{\mu_{\max}S}{K_s + S} = \frac{Q_wX_w}{VX} + k_d$$

Substrate Mass Balance

We cover this in detail in CEE 471

- Consider aeration tank and clarifier together
  - Substrate in + substrate consumed by biomass = substrate out

$$Q_0 S_0 + V\frac{dS}{dt} = (Q_0 - Q_w)S + Q_w S$$

Note that effluent and waste sludge substrate concentrations are considered the same

  - Now using the combined substrate utilization equation without limitation to carrying capacity:

$$\frac{dS}{dt} = -\frac{1}{Y}\left[\mu_{\max}\left(\frac{S}{K_s + S}\right) - k_d\right]X$$

  - Combining and rearranging:

$$\frac{\mu_{\max} S}{K_s + S} = \frac{Q_0 Y}{VX}(S_0 - S)$$

We cover this in CEE 471

Combined Mass Balances

- In summary the solids and substrate mass balance equations are:

$$\frac{\mu_{\max} S}{K_s + S} = \frac{Q_w X_w}{VX} + k_d \qquad\qquad \frac{\mu_{\max} S}{K_s + S} = \frac{Q_0 Y}{VX}\left(S_0 - S\right)$$

- These can be easily combined (left hand terms are the same):

$$\boxed{\frac{Q_w X_w}{VX}} = \frac{Q_0 Y}{VX}\left(S_0 - S\right) - k_d$$

$$\boxed{\frac{1}{\Theta_c}}$$

The mean cell residence time, or sludge age

Sludge Treatment

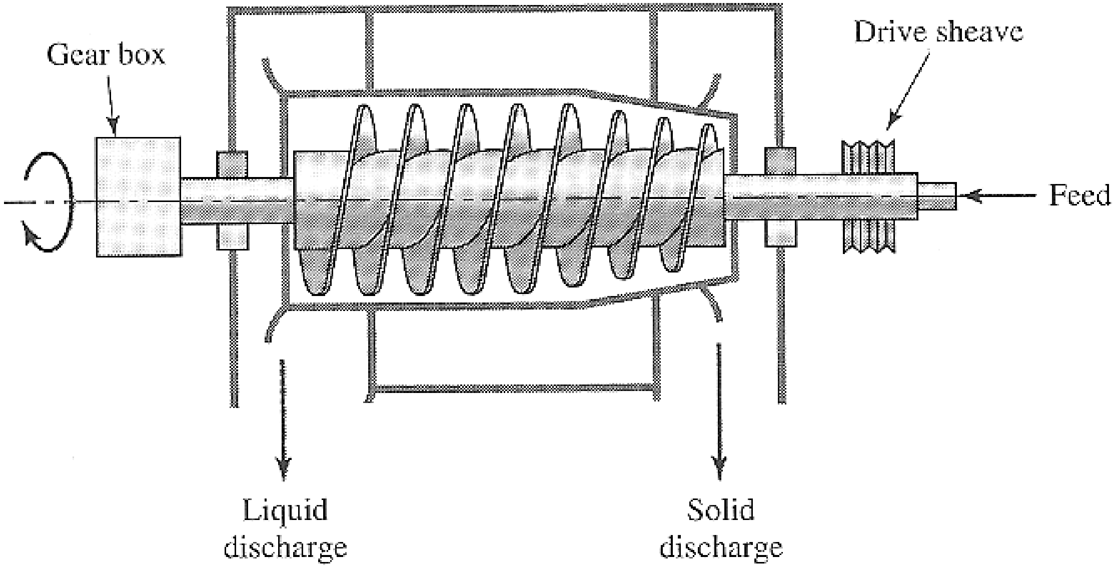
- Depends on type of sludge
- Typical process train
  - Thickening or dewatering
  - Conditioning
  - Stabilization (usually for wastewater)
  - Disposal
- Nonmechanical methods
  - Lagoons
  - Sand-drying beds
  - Freeze treatment
- Mechanical methods
  - Centrifugation
  - Vacuum filtration
  - Belt filter press
  - Plate filters

See also Lecture #30

- Centrifuge

FIGURE 9–21

Solid bowl centrifuge.



From Lecture #30

Vacuum Filter

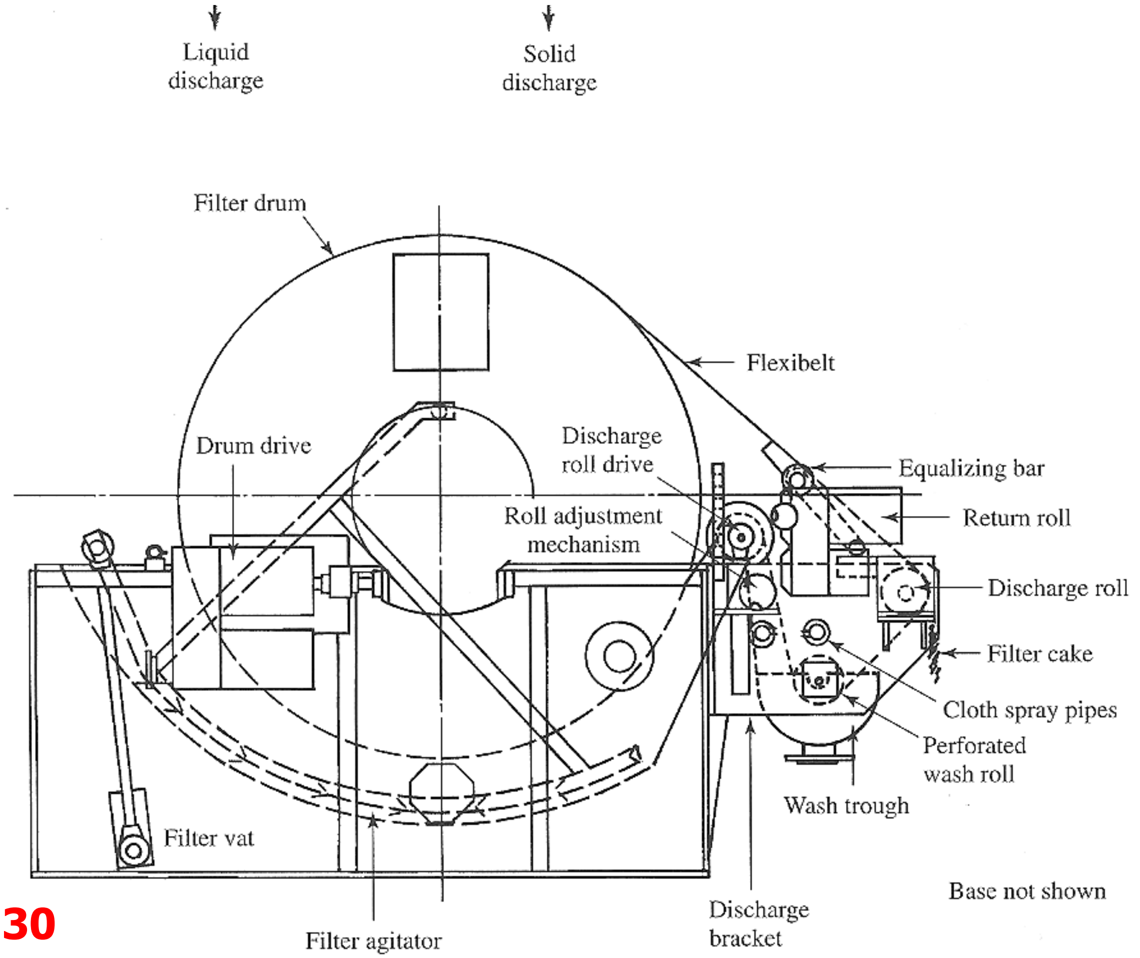


FIGURE 9–22

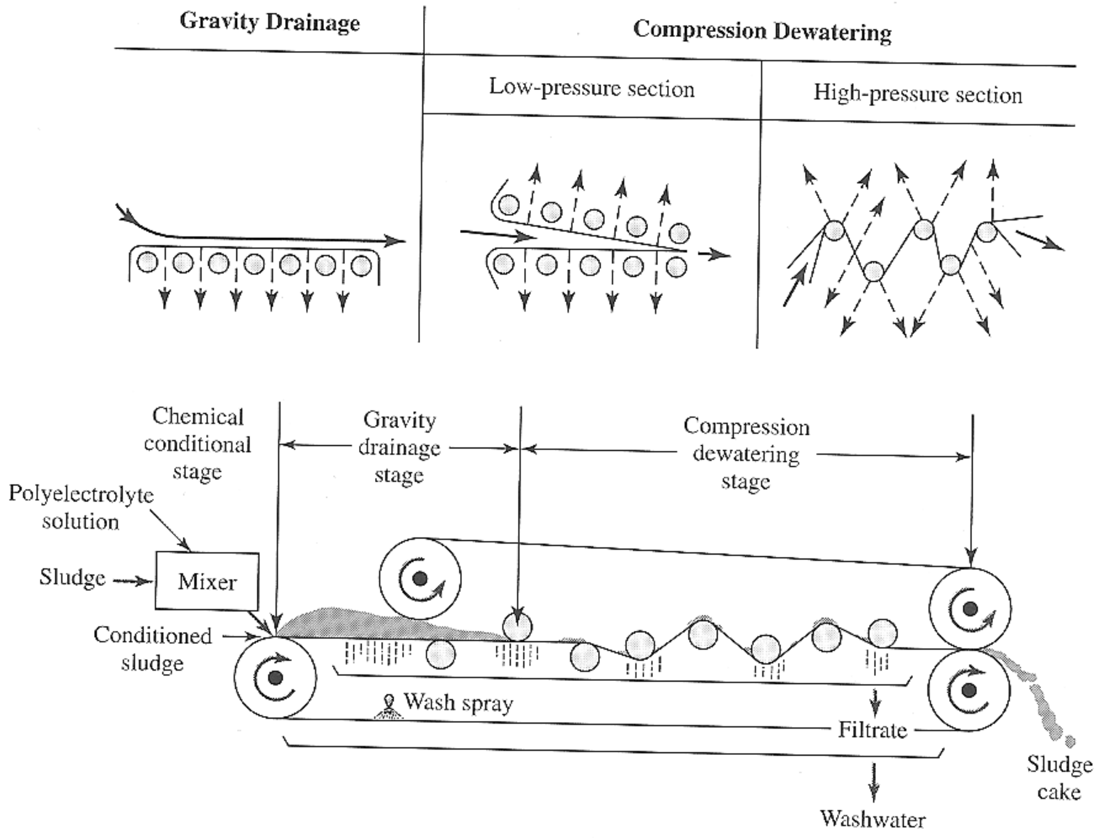
Vacuum filter. (*Courtesy of Komline-Sanderson Engineering Corporation.*)

From Lecture #30

Belt Filter Press

FIGURE 9–23

Continuous belt filter press. (*Source:* U.S. Environmental Protection Agency, *Process Design Manual, Sludge Treatment and Disposal,* 1979.)



From Lecture #30

- To next lecture