

CEE 370

Environmental Engineering

Principles

Lecture #22

Water Resources & Hydrology II: Wells, Withdrawals and Contaminant Transport

[Reading: Mihelcic & Zimmerman, Chapter 7](#)

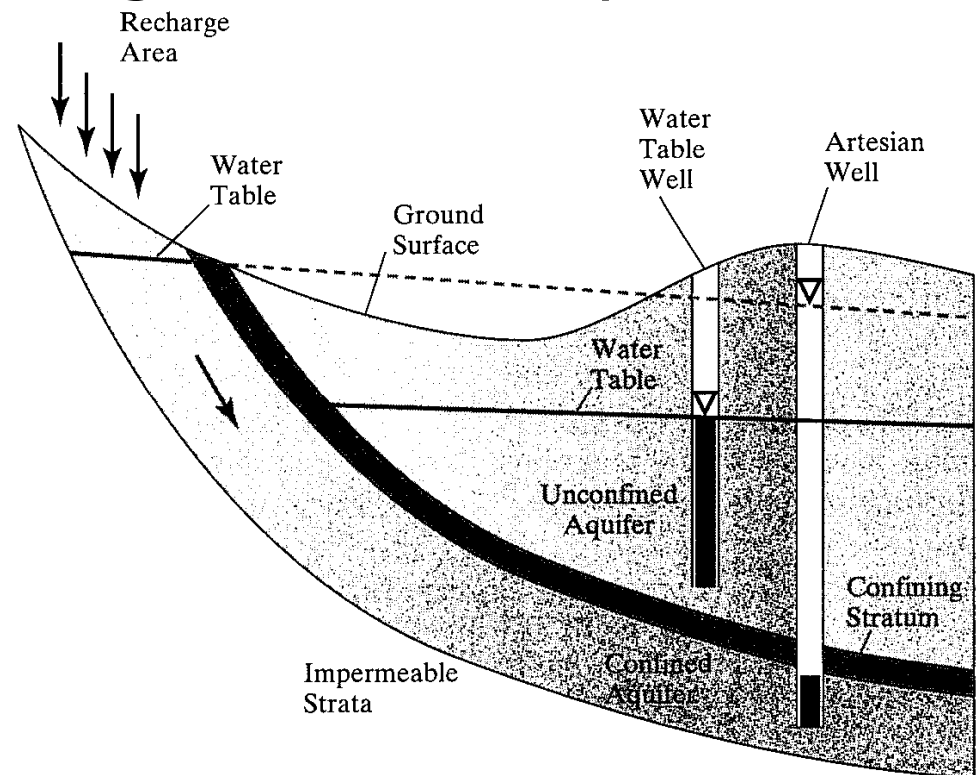


Darcy's Law

- Groundwater flow, or flow through porous media
 - Used to determine the rate at which water or other fluids flow in the sub-surface region
 - Also applicable to flow through engineered system having pores
 - Air Filters
 - Sand beds
 - Packed towers

Groundwater flow

- Balance of forces, but frame of reference is reversed
 - Water flowing through a “field” of particles





Terminology

- Head
 - Height to which water rises within a well
 - At water table for an “unconfined” aquifer
 - Above water table for “confined” aquifers
- Hydraulic Gradient
 - The difference in head between two points in a aquifer separated in horizontal space

$$\text{Hydraulic Gradient} = \frac{dh}{dx}$$

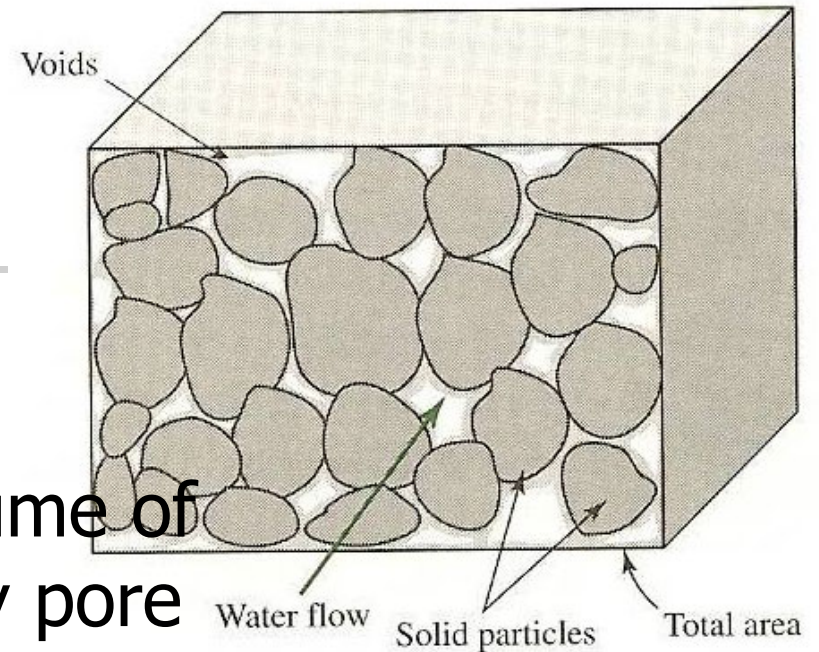
Terminology

■ Porosity

- The fraction of total volume of soil or rock that is empty pore space

- Typical values

- 5-30% for sandstone rock
- 25-50% for sand deposits
- 5-50% for Karst limestone formations
- 40-70% for clay deposits



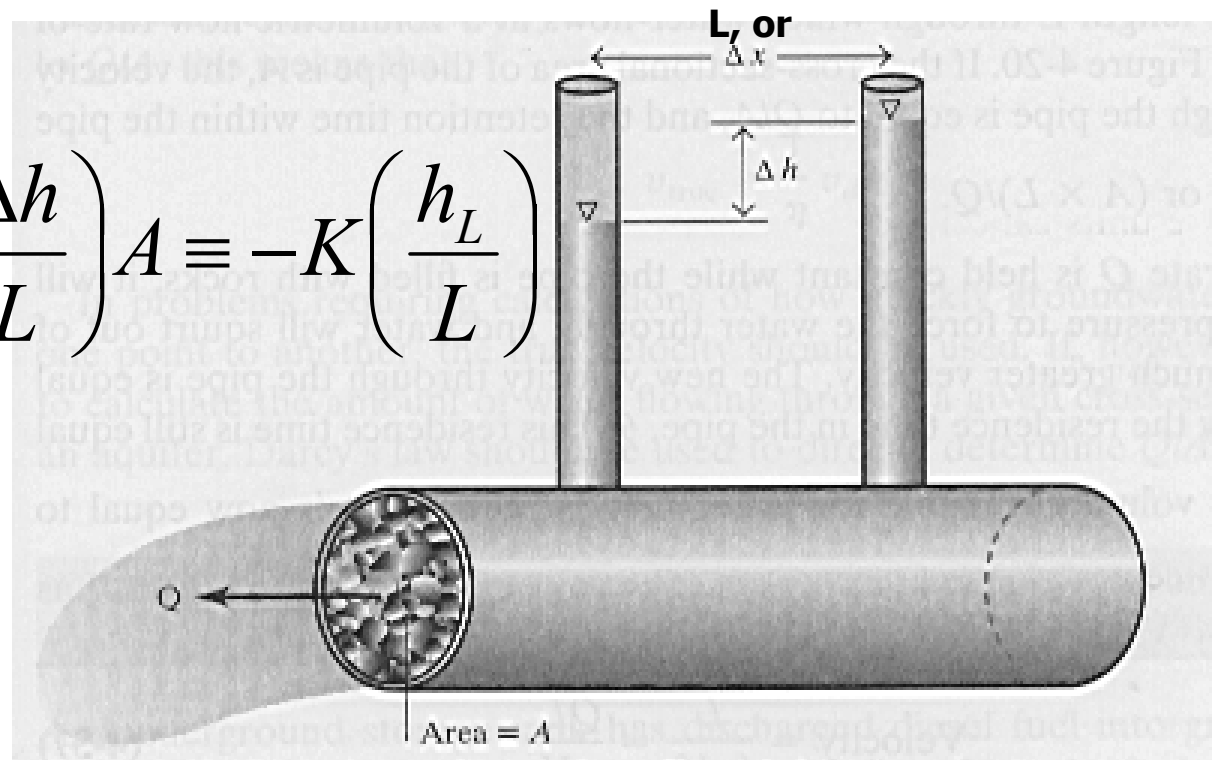
$$\eta = \frac{\text{volumes of pores}}{\text{total volume}}$$

Darcy's Law

- Obtained theoretically by setting drag forces equal to resistive forces
- Determined experimentally by Henri Darcy (1803-1858)

$$Q = -KA \frac{dh}{dx} \equiv -K \left(\frac{\Delta h}{L} \right) A \equiv -K \left(\frac{h_L}{L} \right)$$

Flow per unit cross-sectional area is directly proportional to the hydraulic gradient





Hydraulic Conductivity, K

- Proportionality constant between hydraulic gradient and flow/area ratio
- A property of the medium through which flow is occurring (and of the fluid)
 - Very High for gravel: 0.2 to 0.5 cm/s
 - High for sand: 3×10^{-3} to 5×10^{-2} cm/s
 - Low for clays: $\sim 2 \times 10^{-7}$ cm/s
 - Almost zero for synthetic barriers: $< 10^{-11}$ for high density polyethylene membranes
- Measured by pumping tests



Hydraulic Conductivity - Table

- Compare with M&Z Table 7.23

Typical Values of Aquifer Parameters

Aquifer Material	Porosity (%)	Typical Values for Hydraulic Conductivity ($\text{m} \cdot \text{s}^{-1}$)
Clay	55	2.3×10^{-9}
Loam	35	6.0×10^{-6}
Fine sand	45	2.9×10^{-5}
Medium sand	37	1.4×10^{-4}
Coarse sand	30	5.2×10^{-4}
Sand and gravel	20	6.0×10^{-4}
Gravel	25	3.1×10^{-3}
Slate	<5	9.2×10^{-10}
Granite	<1	1.2×10^{-10}
Sandstone	15	5.8×10^{-7}
Limestone	15	1.1×10^{-5}
Fractured rock	5	1×10^{-8} – 1×10^{-4}

Darcy Velocity

- re-arrangement of Darcy's Law gives the Darcy Velocity, v

M&Z Equ #7.20

$$v_d = \frac{Q}{A} = -K \left(\frac{dh}{dx} \right)$$

or

$$v = \frac{Q}{A} = -K \left(\frac{\Delta h}{L} \right)$$

- Not the true (or linear or seepage) velocity of groundwater flow because flow can only occur in pores

- combining

$$v_{true} \equiv v_a = \frac{L}{\tau} = \frac{L}{\frac{\eta V}{Q}} = \frac{QL}{\eta V} = \frac{1}{\eta} \frac{Q}{A}$$

$$v_{true} \equiv v_a = \frac{1}{\eta} v_d$$

or

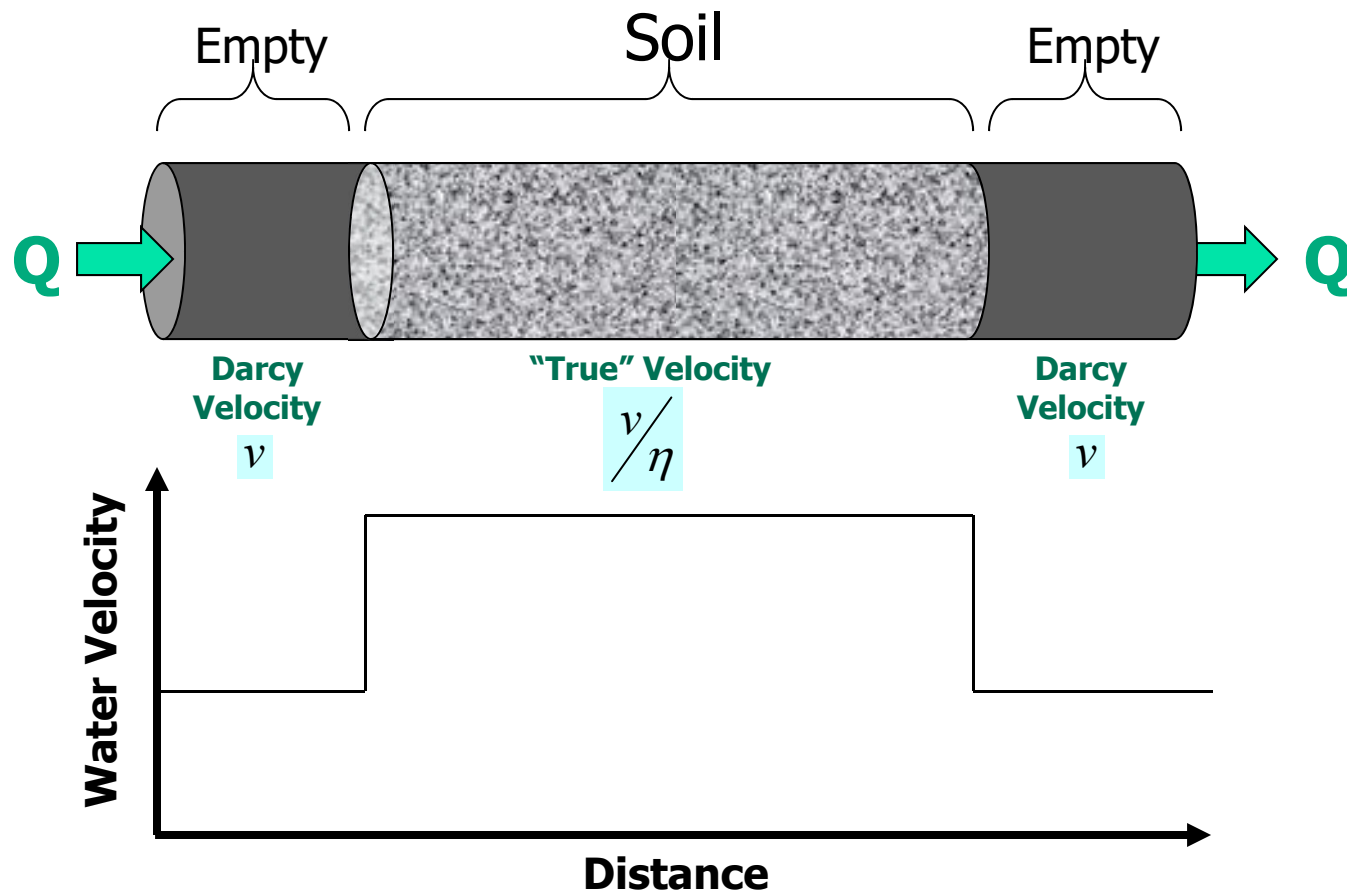
$$v_a = \frac{1}{\eta} v$$

M&Z

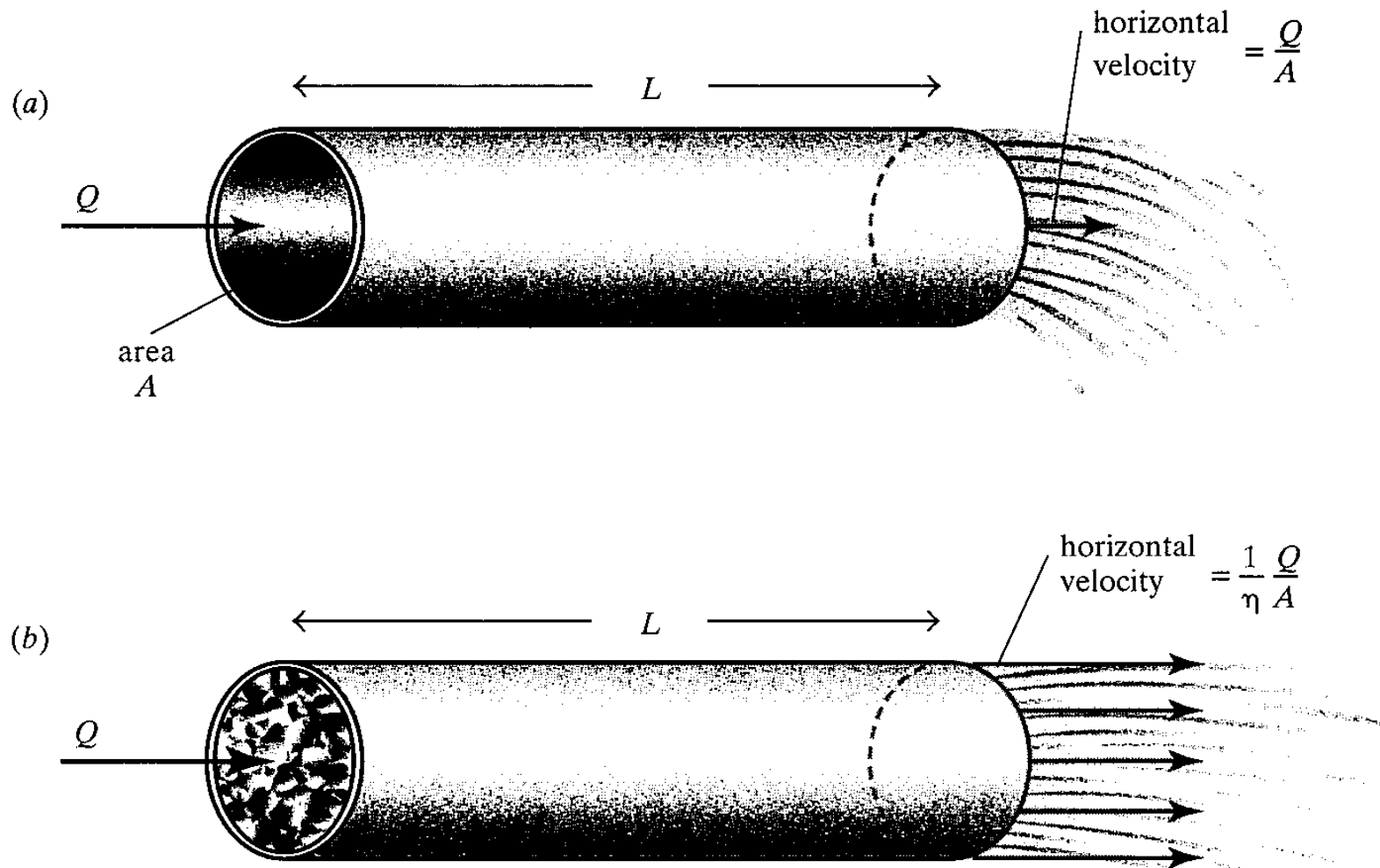
M&Z Equ #7.21

Velocities Illustrated

- Pipe with soil core



Alternative illustration





Example C

- An aquifer material of coarse sand has piezometric surfaces of 10 cm and 8 cm above a datum and these are spaced 10 cm apart. If the cross sectional area is 10 cm², what is the linear velocity of the water?

- Hydraulic gradient:
$$\frac{\Delta h}{L} = \frac{10\text{cm} - 8\text{cm}}{10\text{cm}} = 0.2 \text{ cm/cm}$$

- From the prior table, K for coarse sand is 5.2×10^{-4} , so the Darcy velocity is:

$$v = K \frac{\Delta h}{L} = \left(5.2 \times 10^{-4} \text{ m/s}\right) 0.2 \text{ cm/cm} = 1.04 \times 10^{-4} \text{ m/s}$$

- Assuming that the porosity is 30% or 0.3 (prior Table):

$$v'_{\text{water}} = \frac{v}{\eta} = \frac{1.04 \times 10^{-4} \text{ m/s}}{0.3} = 3.47 \times 10^{-4} \text{ m/s}$$

See M&Z, example 7.9, part a



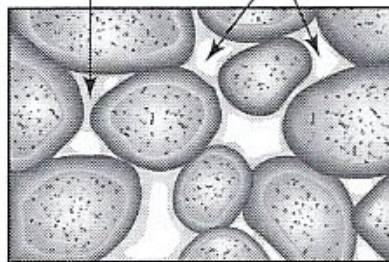
Definitions

- Specific Yield – the fraction of water in an aquifer that will drain by gravity
 - Less than porosity due to capillary forces
 - See Table 7-5 in D&M for typical values
- Transmissivity (T) – flow expected from a 1 m wide cross section of aquifer (full depth) when the hydraulic gradient is 1 m/m.
 - $T=K*D$
 - Where D is the aquifer depth and K is hydraulic conductivity

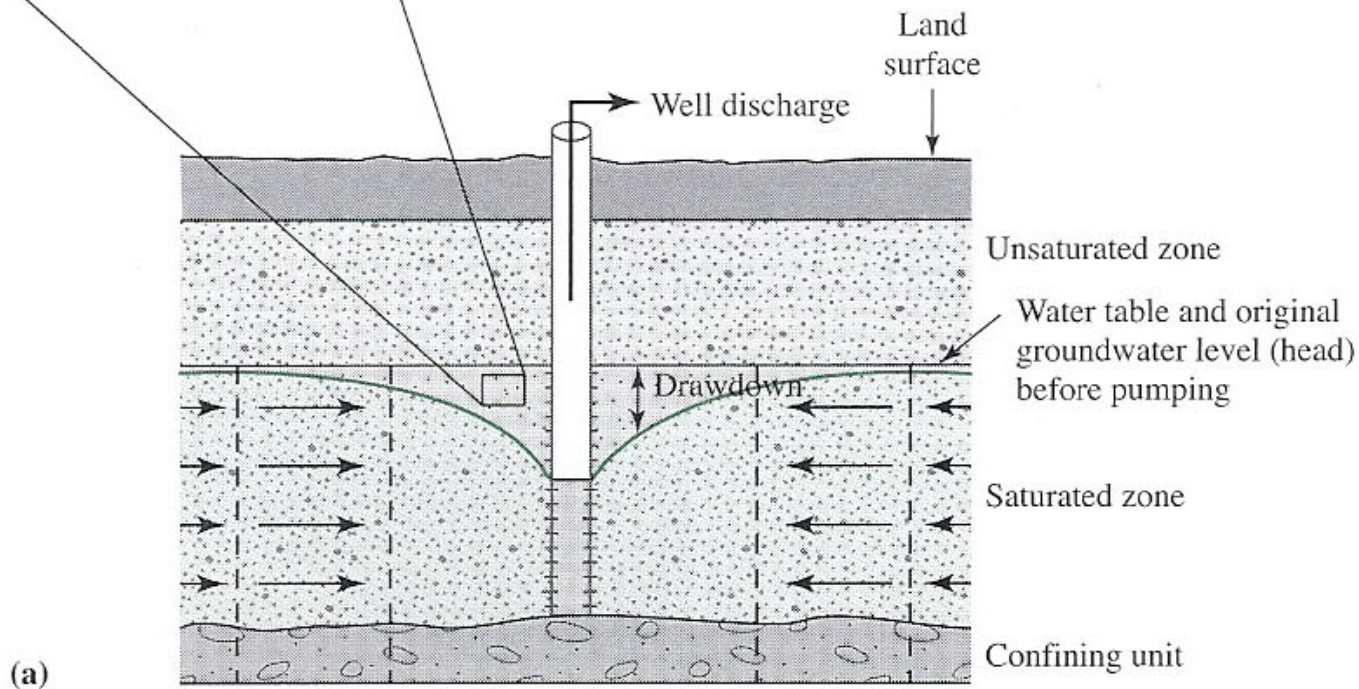
Drawdown I

Water around grains

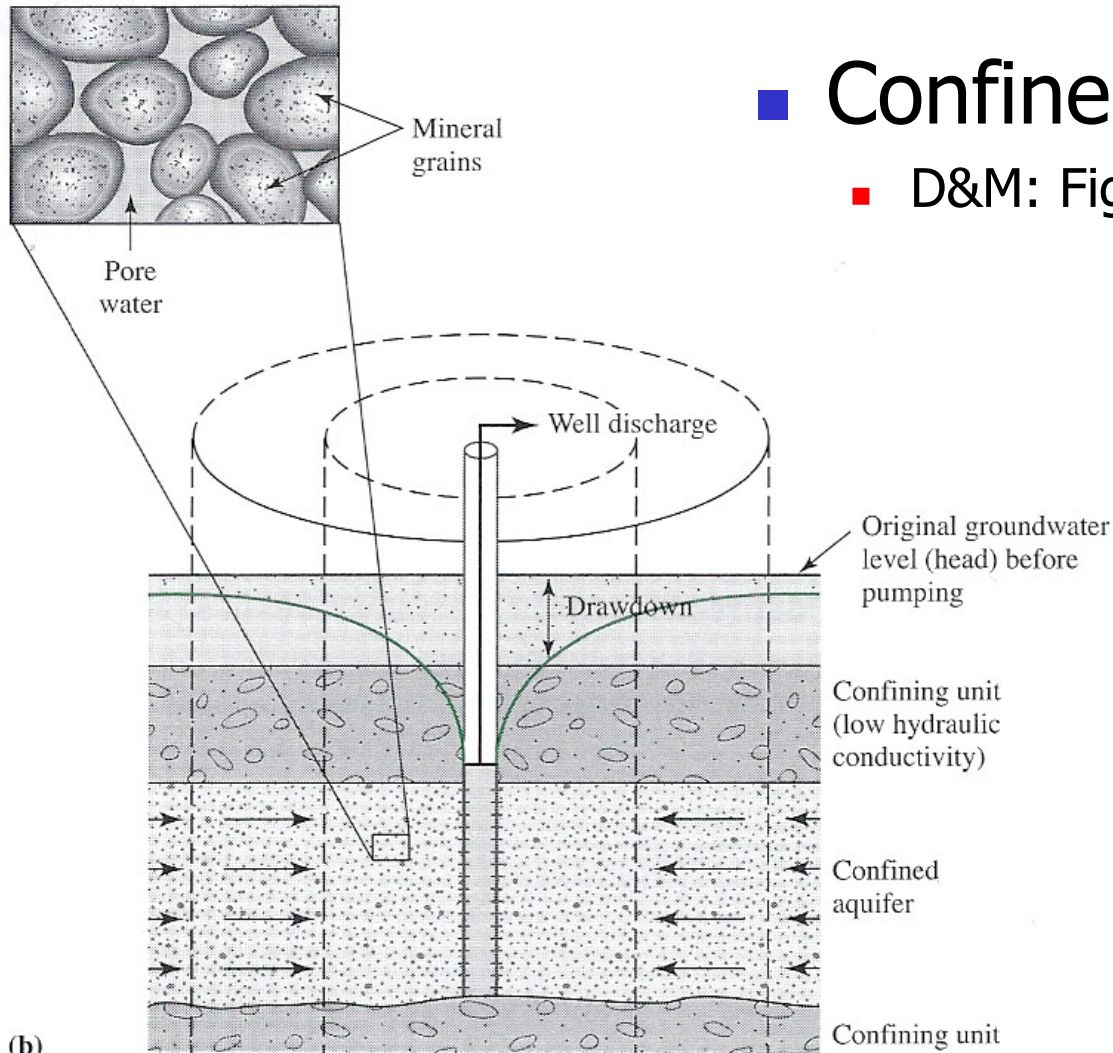
Air



- Unconfined aquifer
 - D&M: Figure 7-31a
- Showing cone of depression



Drawdown II



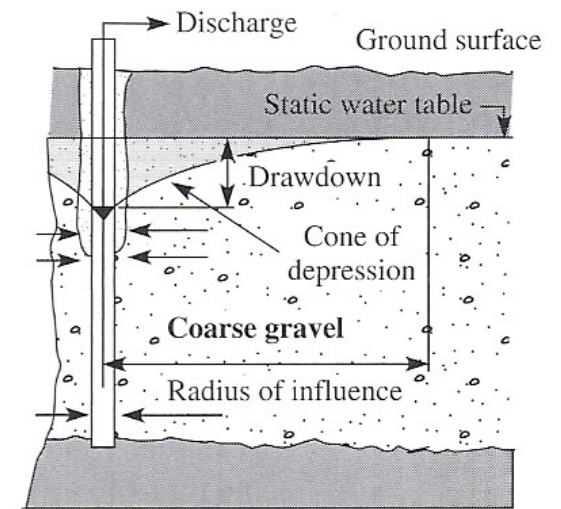
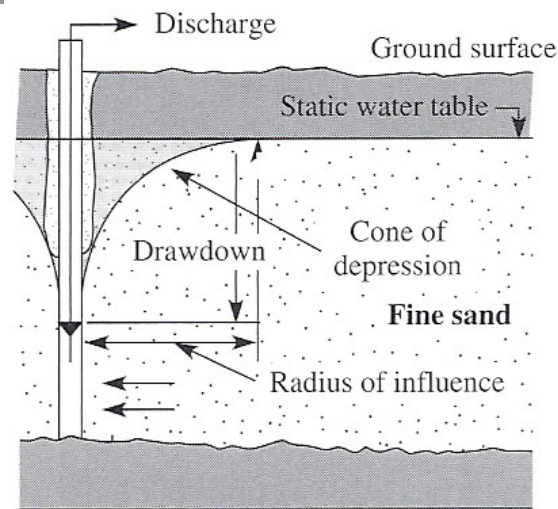
■ Confined aquifer

■ D&M: Figure 7-31b

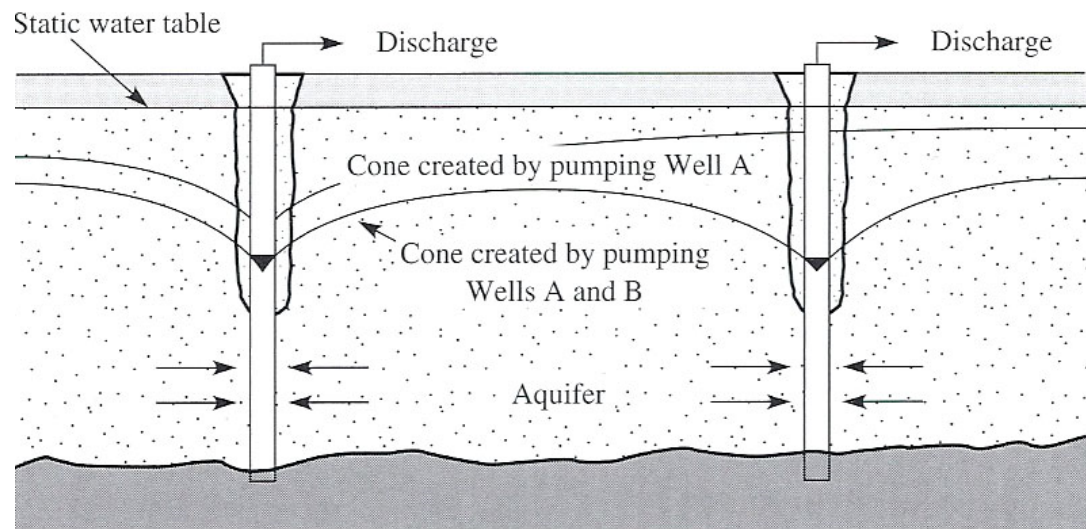
(b)

Cones of Depression

- Conductivity
 - Low K
 - Deep, shallow cone



- overlapping





Flow Model

- Well in confined aquifer

$$Q = \frac{2\pi KD(h_2 - h_1)}{\ln(r_2 / r_1)}$$

Where: h_x is the height of the piezometric surface at distance “ r_x ” from the well

- In an unconfined aquifer

- D is replaced by average height of water table $(h_2 + h_1)/2$, so:

$$Q = \frac{\pi K(h_2^2 - h_1^2)}{\ln(r_2 / r_1)}$$

See examples: 7-10 and 7-11 in D&M



Contaminant Flow

- Separate Phase flow – low solubility compounds
 - Low density:
 - LNAPL – light non-aqueous phase liquid
 - High density: HNAPL
- Dissolved contaminant
 - Flows with water, but subject to retardation
 - Caused by adsorption to aquifer materials

**See D&M section 9-7,
pg.389-393**



Adsorption in Groundwater

- Based on relative affinity of contaminant for aquifer to water

- Defined by partition coefficient, K_p :

$$K_p = \frac{C_s (\text{moles}_{\text{adsorbed}} / \text{kg} - \text{soil})}{C_w (\text{moles}_{\text{dissolved}} / \text{L} - \text{water})}$$

Equ 2-89, pg 76 in
D&M 2nd ed.
Similar to Equ 3.32,
pg 95 in M&Z

- And more fundamentally the K_d can be related to the soil organic fraction (f_{oc}) and an organic partition coefficient (K_{oc}):

$$K_p = K_{oc} f_{oc}$$

See also pg 392 in
D&M 2nd ed.
Similar to Equ 3.33,
pg 95 in M&Z



Relative Velocities

- The retardation coefficient, R_f , is defined as the ratio of water velocity to contaminant velocity

$$R_f \equiv \frac{v'_{water}}{v'_{cont}}$$

Equ 9-42, pg 391 in
D&M 2nd ed.

- And since only the dissolved fraction of the contaminant actually moves

$$v'_{cont} = v'_{water} \left(\frac{\text{moles}_{dissolved}}{\text{moles}_{dissolved} + \text{moles}_{adsorbed}} \right)$$



Relating R to K_d

- So
$$\frac{v'_{cont}}{v'_{water}} = \left(\frac{\text{moles}_{dissolved}}{\text{moles}_{dissolved} + \text{moles}_{adsorbed}} \right)$$

- And therefore

$$R_f \equiv \frac{v'_{water}}{v'_{cont}} = \frac{\text{moles}_{dissolved} + \text{moles}_{adsorbed}}{\text{moles}_{dissolved}} = 1 + \frac{\text{moles}_{adsorbed}}{\text{moles}_{dissolved}}$$

- And we can parse the last term:

$$\frac{\text{moles}_{adsorbed}}{\text{moles}_{dissolved}} = \frac{C_s (\text{moles}_{adsorbed} / \text{kg} - \text{soil})}{C_w (\text{moles}_{dissolved} / \text{L} - \text{water})} \left(\frac{Y(\text{L} - \text{aquifer} / \text{L} - \text{water})}{X(\text{L} - \text{aquifer} / \text{kg} - \text{soil})} \right)$$



cont

- Note that the fundamental partition coefficient is:

$$K_p \equiv \frac{C_s (\text{moles}_{\text{adsorbed}} / \text{kg} - \text{soil})}{C_w (\text{moles}_{\text{dissolved}} / \text{L} - \text{water})}$$

- So:
$$\frac{\text{moles}_{\text{adsorbed}}}{\text{moles}_{\text{dissolved}}} = K_p \left(\frac{Y (\text{L} - \text{aquifer} / \text{L} - \text{water})}{X (\text{L} - \text{aquifer} / \text{kg} - \text{soil})} \right)$$

- And then

$$R_f = 1 + K_p \frac{Y}{X}$$



cont

- where:

$$Y^{(L-aquifer/L-water)} = \frac{1}{\eta}$$

$$X^{(L-aquifer/kg-soil)} = \frac{1-\eta}{\rho_s} = \frac{1}{\rho_b}$$

- Where:

- ρ_s is density of soil particles without pores
 - usually $\sim 2-3 \text{ g/cm}^3$
- ρ_b is the bulk soil density with pores

- So, then

$$R_f = 1 + K_p \left(\frac{\rho_s}{\eta(1-\eta)} \right) = 1 + K_p \left(\frac{\rho_b}{\eta} \right)$$

M&Z Equ #7.23

Compare to Equ 9-43, pg 391 in D&M 2nd ed.

See M&Z, example 7.9, part b

$$f_d = \frac{1}{1 + K_p m}$$

Estimation of partition coefficients

- Relationship to organic fraction

$$K_p = f_{oc} K_{oc} \longrightarrow \left(\frac{mg - tox. / g - C}{mg - tox. / m^3} \right) \text{ or } \left(\frac{m^3}{g - C} \right)$$

- and properties of organic fraction

$$K_{oc} = 6.17 \times 10^{-7} K_{ow}$$

Octanol:water
partition
coefficient

- combining, we get:

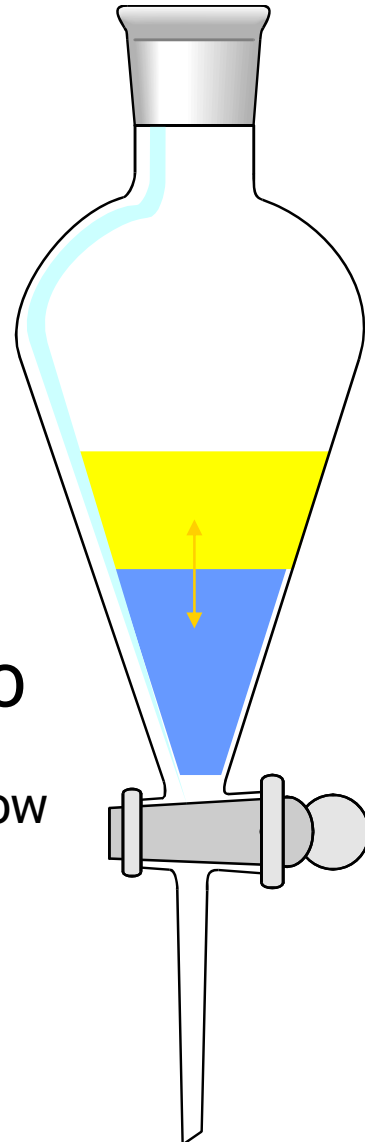
$$K_p = 6.17 \times 10^{-7} f_{oc} K_{ow}$$

$$\left(\frac{mg - tox. / m^3 - Oct.}{mg - tox. / m^3 - H_2O} \right)$$

Karickhoff et al., 1979; [Wat. Res. 13:241](#)

Octanol:water partitioning

- 2 liquid phases in a separatory funnel that don't mix
 - octanol
 - water
- Add contaminant to flask
- Shake and allow contaminant to reach equilibrium between the two
- Measure concentration in each (K_{ow} is the ratio)



cont

- Retardation in Groundwater & solute movement

$$R_f = 1 + \frac{\rho_b}{\eta} K_p$$

ρ = Soil bulk mass density
 η = void fraction

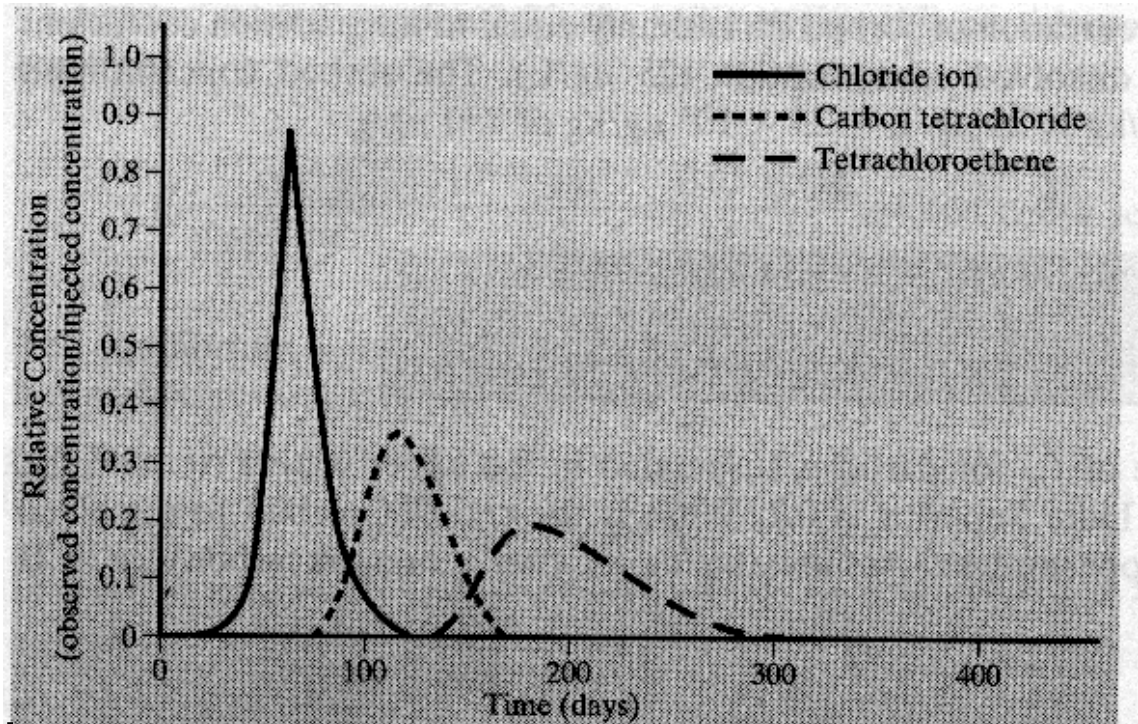
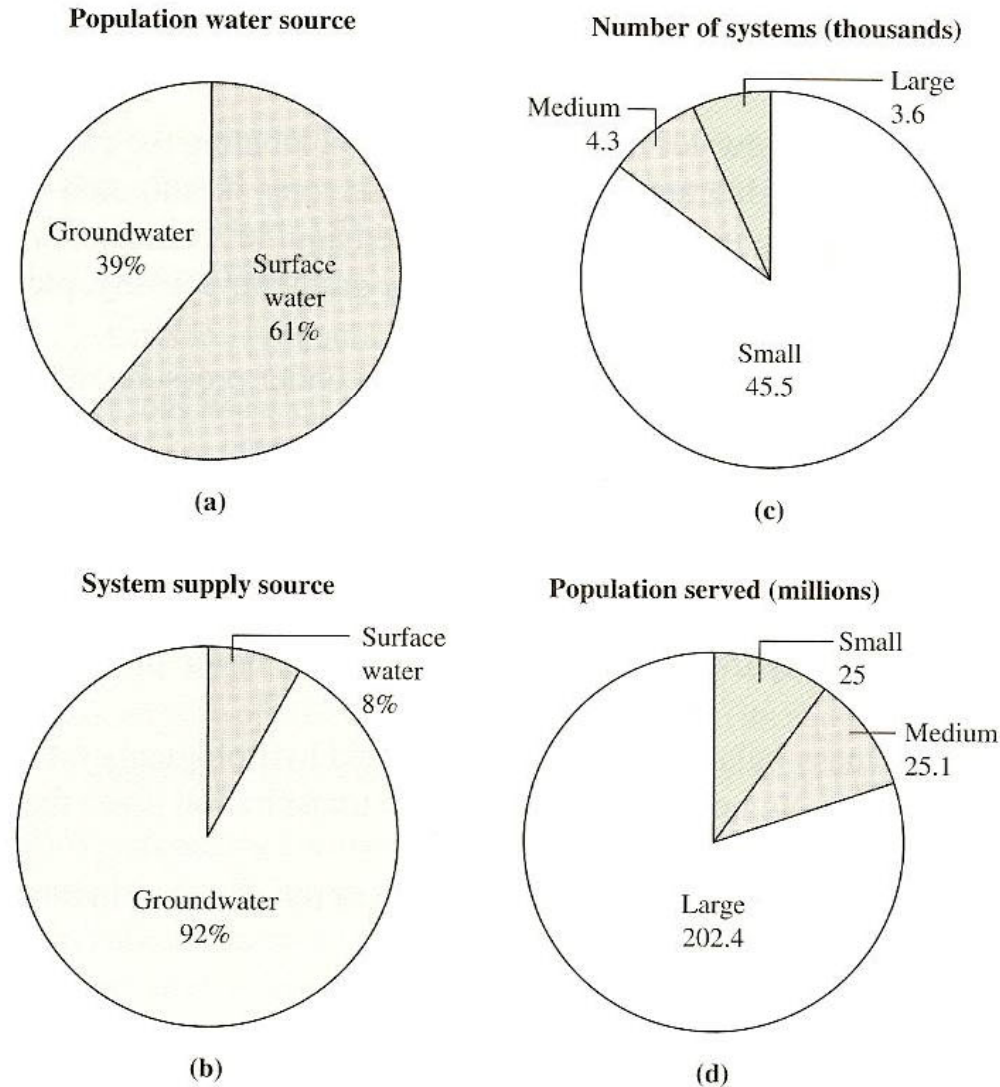
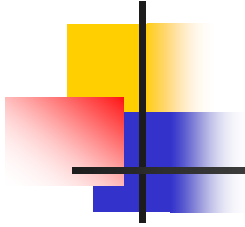


FIGURE 6-27

(a) Percentage of the population served by drinking-water system source. **(b)** Percentage of drinking-water systems by supply source. **(c)** Number of drinking-water systems (in thousands) by size. **(d)** Population served (in millions of people) by drinking-water system size. (Source: 1997 National Public Water Systems Compliance Report. U.S. EPA, Office of Water. Washington, D.C. 20460. (EPA-305-R-99-002). (Note: Small systems serve 25–3300 people; medium systems serve 3301–10,000 people; large systems serve 10,000+ people.)





- To next lecture