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CEE 370 Environmental Engineering Principles

Lecture #22

Water Resources & Hydrology II: Wells, Withdrawals and Contaminant Transport

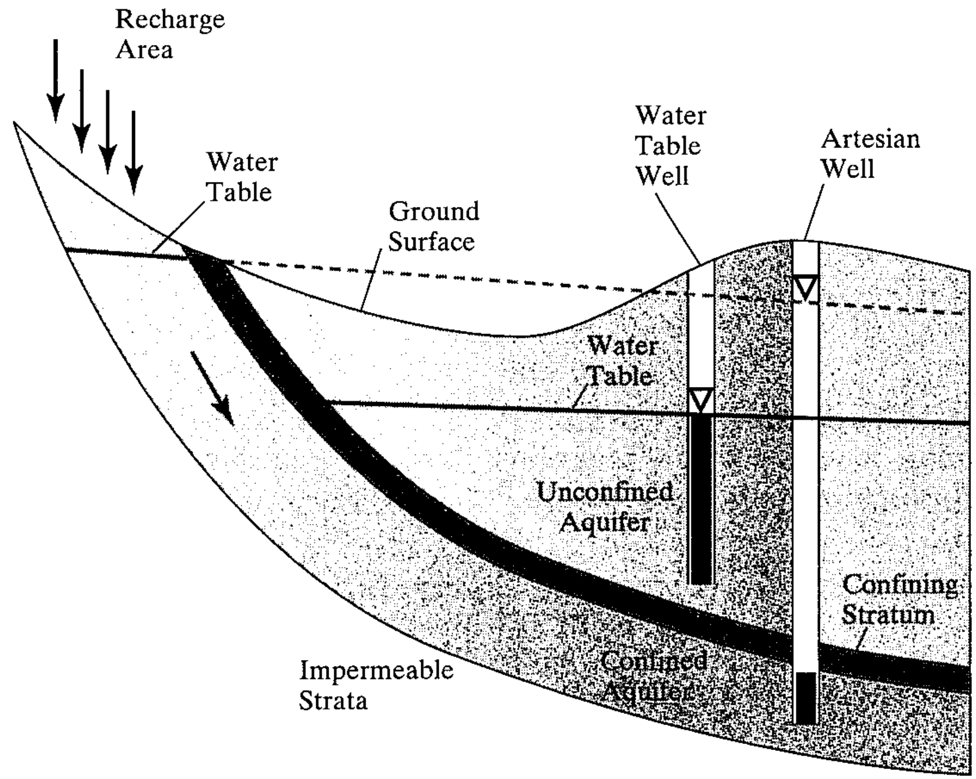
Reading: Mihelcic & Zimmerman, Chapter 7

Darcy's Law

- Groundwater flow, or flow through porous media
  - Used to determine the rate at which water or other fluids flow in the sub-surface region
  - Also applicable to flow through engineered system having pores
    - Air Filters
    - Sand beds
    - Packed towers

Groundwater flow

- Balance of forces, but frame of reference is reversed
  - Water flowing though a “field” of particles



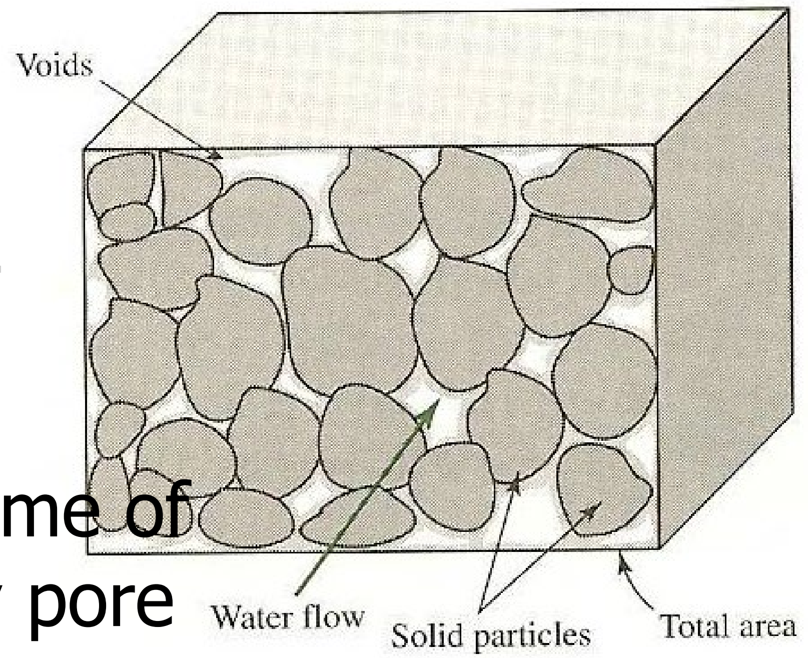
Terminology

- Head
  - Height to which water rises within a well
    - At water table for an “unconfined” aquifer
    - Above water table for “confined” aquifers
- Hydraulic Gradient
  - The difference in head between two points in a aquifer separated in horizontal space

$$\text{Hydraulic Gradient} = \frac{dh}{dx}$$

Terminology

- Porosity
  - The fraction of total volume of soil or rock that is empty pore space
    - Typical values
      - 5-30% for sandstone rock
      - 25-50% for sand deposits
      - 5-50% for Karst limestone formations
      - 40-70% for clay deposits

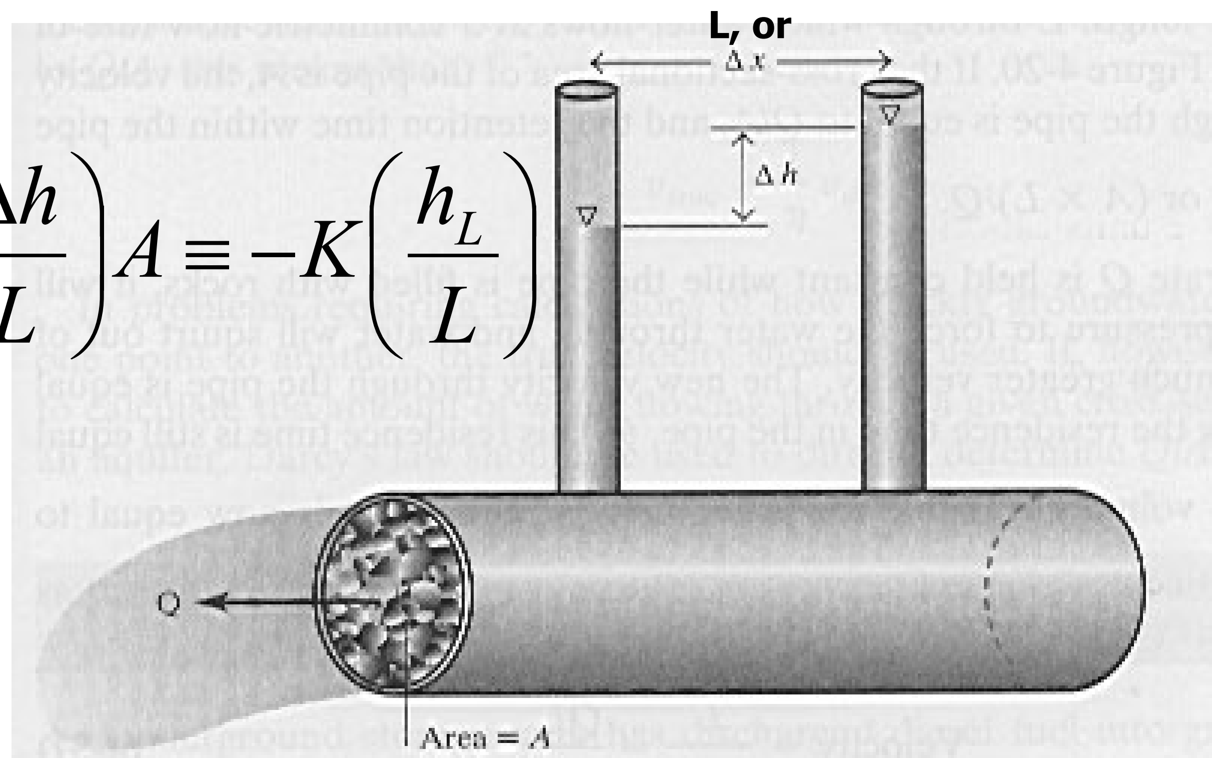


$$\eta = \frac{\text{volumes of pores}}{\text{total volume}}$$

Darcy's Law

- Obtained theoretically by setting drag forces equal to resistive forces
- Determined experimentally by Henri Darcy (1803-1858)

$$Q = -KA\frac{dh}{dx} \equiv -K\left(\frac{\Delta h}{L}\right)A \equiv -K\left(\frac{h_L}{L}\right)$$



Flow per unit cross-sectional area is directly proportional to the hydraulic gradient

Hydraulic Conductivity, K

- Proportionality constant between hydraulic gradient and flow/area ratio
- A property of the medium through which flow is occurring (and of the fluid)
  - Very High for gravel: 0.2 to 0.5 cm/s
  - High for sand: $3x10^{-3}$ to $5x10^{-2}$ cm/s
  - Low for clays: ~$2x10^{-7}$ cm/s
  - Almost zero for synthetic barriers: <10^{-11} for high density polyethylene membranes
- Measured by pumping tests

Hydraulic Conductivity - Table

- Compare with M&Z Table 7.23

Typical Values of Aquifer Parameters

Aquifer Material	Porosity (%)	Typical Values for Hydraulic Conductivity ($m \cdot s^{-1}$)
Clay	55	2.3×10^{-9}
Loam	35	6.0×10^{-6}
Fine sand	45	2.9×10^{-5}
Medium sand	37	1.4×10^{-4}
Coarse sand	30	5.2×10^{-4}
Sand and gravel	20	6.0×10^{-4}
Gravel	25	3.1×10^{-3}
Slate	<5	9.2×10^{-10}
Granite	<1	1.2×10^{-10}
Sandstone	15	5.8×10^{-7}
Limestone	15	1.1×10^{-5}
Fractured rock	5	1×10^{-8}–1×10^{-4}

Darcy Velocity

- re-arrangement of Darcy's Law gives the Darcy Velocity, ʋ

M&Z Equ #7.20

$$v_d = \frac{Q}{A} = -K\left(\frac{dh}{dx}\right)$$

or

$$v = \frac{Q}{A} = -K\left(\frac{\Delta h}{L}\right)$$

M&Z

- Not the true (or linear or seepage) velocity of groundwater flow because flow can only occur in pores

- combining $$v_{true} \equiv v_a = \frac{L}{\tau} = \frac{L}{\frac{\eta V}{Q}} = \frac{QL}{\eta V} = \frac{1}{\eta}\frac{Q}{A}$$

$$v_{true} \equiv v_a = \frac{1}{\eta} v_d$$

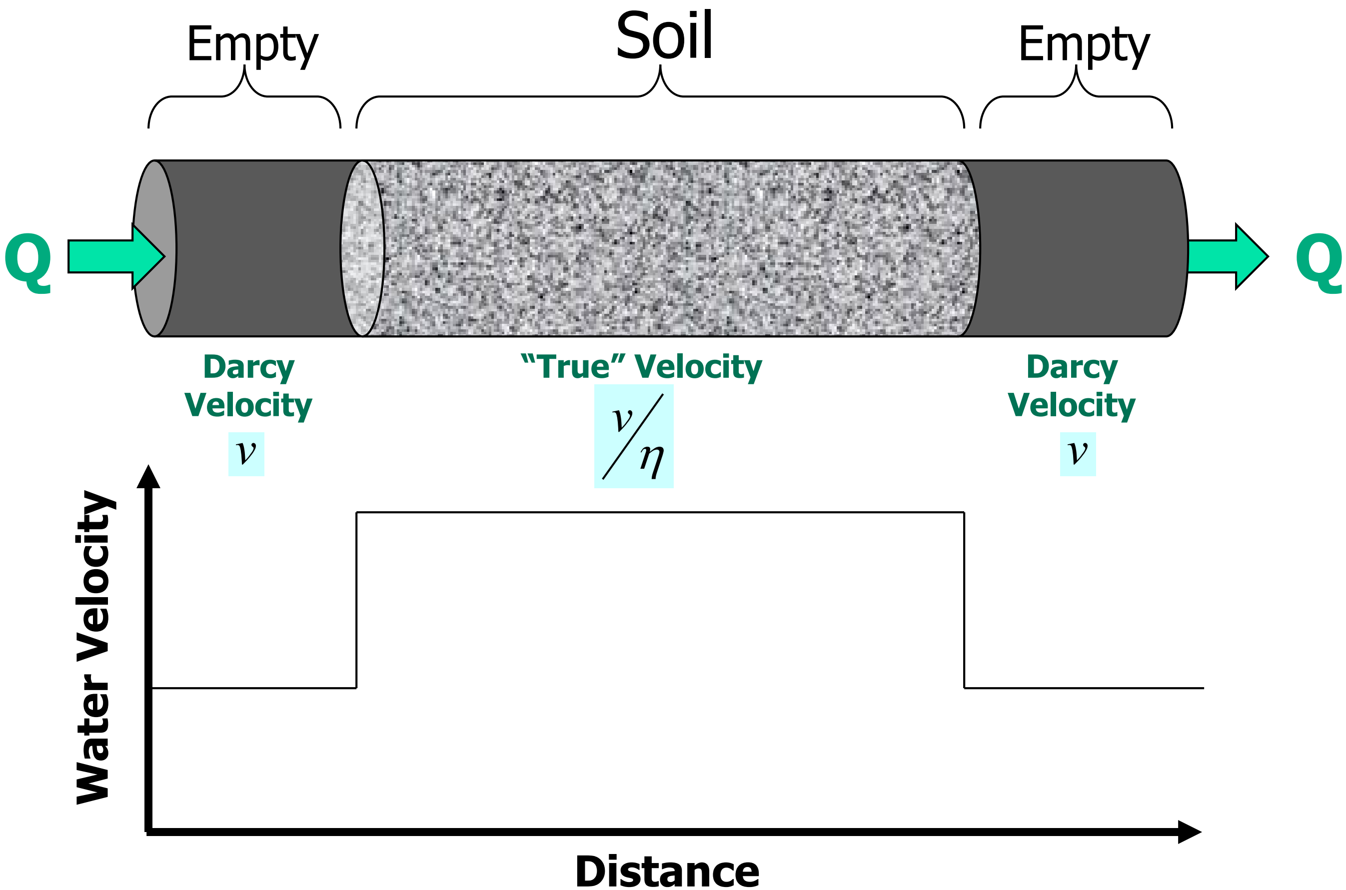
or

$$v_a = \frac{1}{\eta} v$$

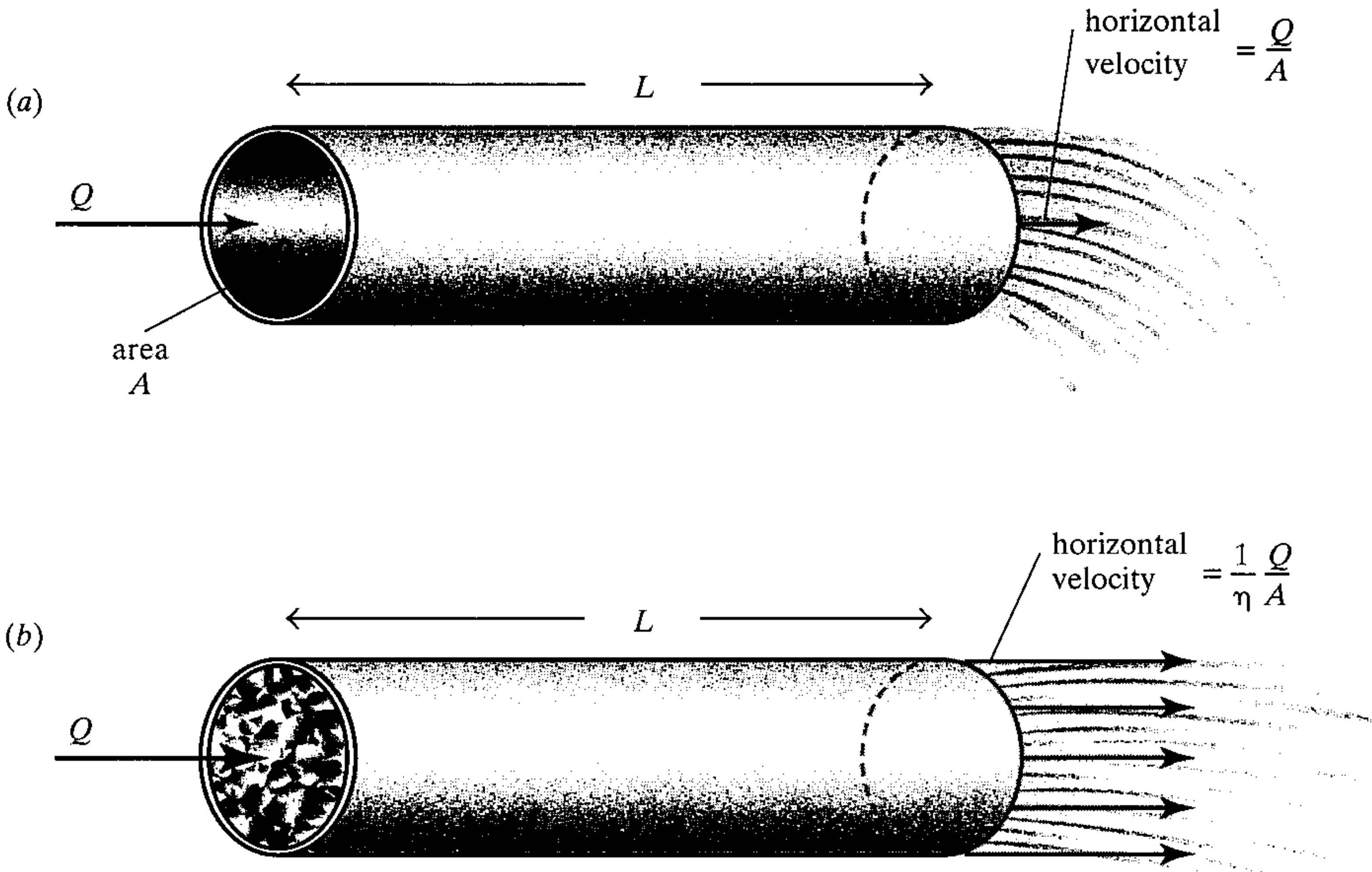
M&Z Equ #7.21

Velocities Illustrated

- Pipe with soil core



Alternative illustration



Example C

- An aquifer material of coarse sand has piezometric surfaces of 10 cm and 8 cm above a datum and these are spaced 10 cm apart. If the cross sectional area is 10 cm^2, what is the linear velocity of the water?
  - Hydraulic gradient: $$\frac{\Delta h}{L} = \frac{10cm - 8cm}{10cm} = 0.2 \, {}^{cm}\!/_{cm}$$
  - From the prior table, K for coarse sand is 5.2 x 10^{-4}, so the Darcy velocity is:

$$v = K\frac{\Delta h}{L} = \left(5.2x10^{-4} \, {}^{m}\!/_{s}\right)0.2 \, {}^{cm}\!/_{cm} = 1.04x10^{-4} \, {}^{m}\!/_{s}$$

  - Assuming that the porosity is 30% or 0.3 (prior Table):

$$v'_{water} = \frac{v}{\eta} = \frac{1.04x10^{-4} \, {}^{m}\!/_{s}}{0.3} = 3.47x10^{-4} \, {}^{m}\!/_{s}$$

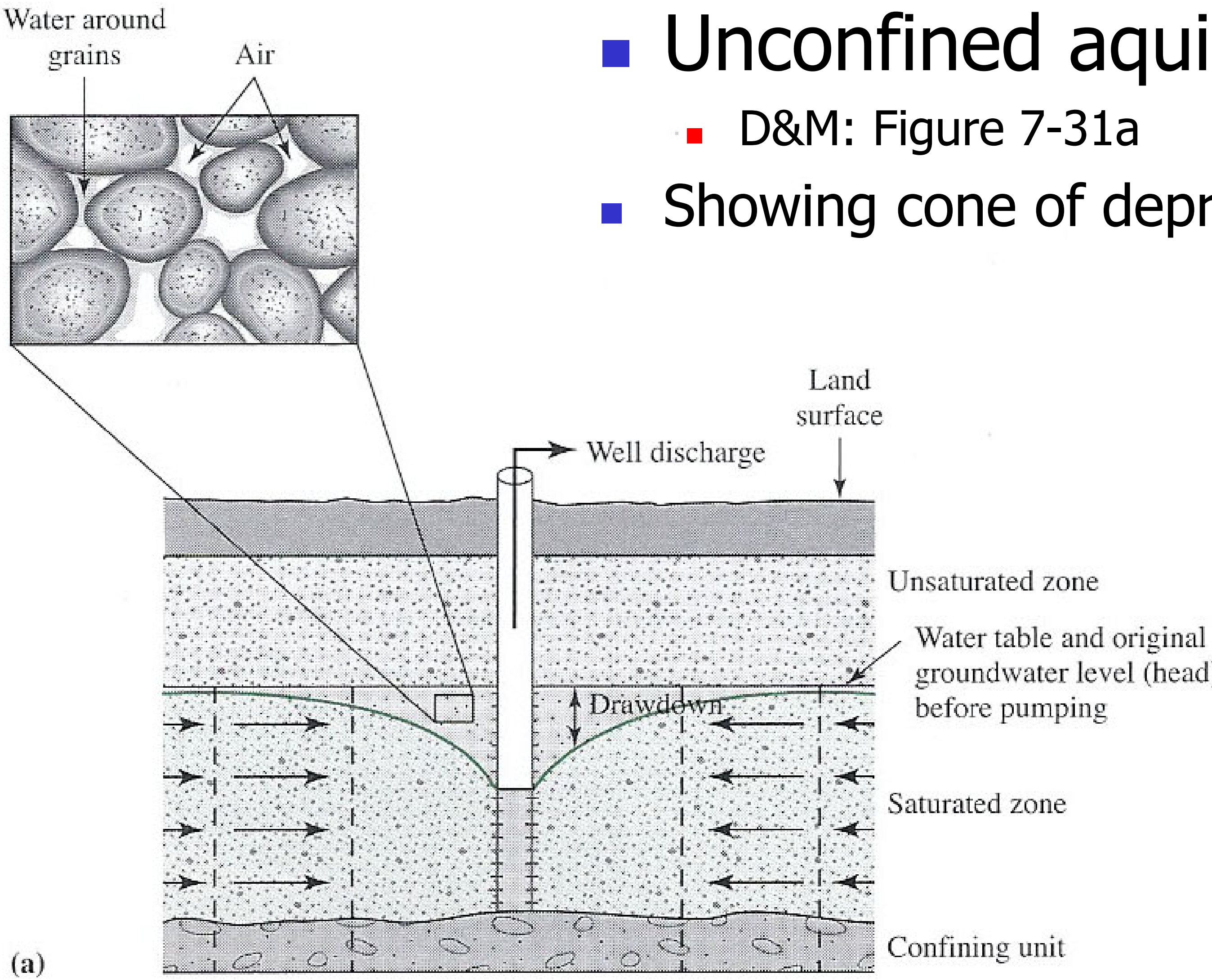
See M&Z, example 7.9, part a

Definitions

- Specific Yield – the fraction of water in an aquifer that will drain by gravity
  - Less than porosity due to capillary forces
  - See Table 7-5 in D&M for typical values
- Transmissivity (T) – flow expected from a 1 m wide cross section of aquifer (full depth) when the hydraulic gradient is 1 m/m.
  - T=K*D
    - Where D is the aquifer depth and K is hydraulic conductivity

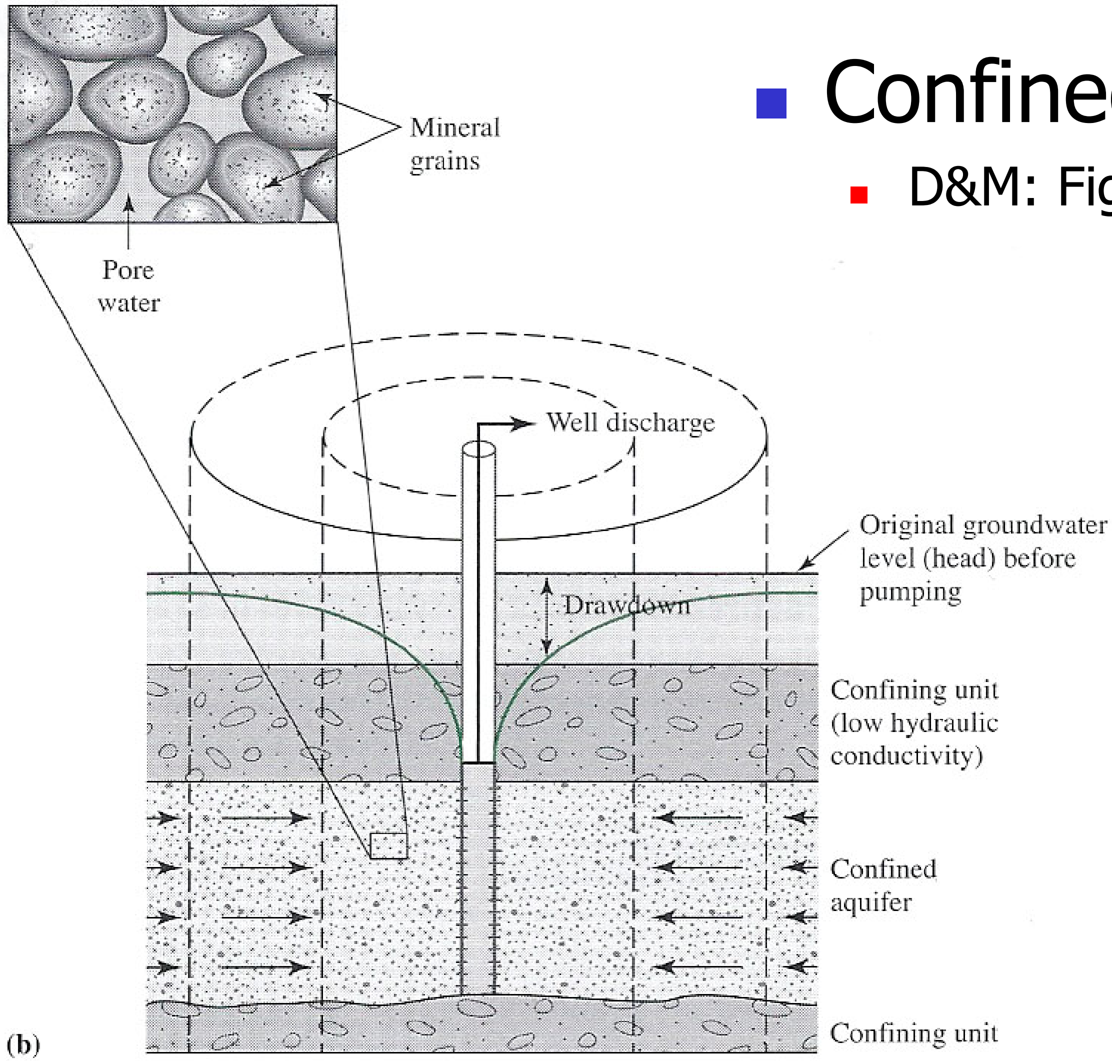
Drawdown I

- Unconfined aquifer
  - D&M: Figure 7-31a
- Showing cone of depression



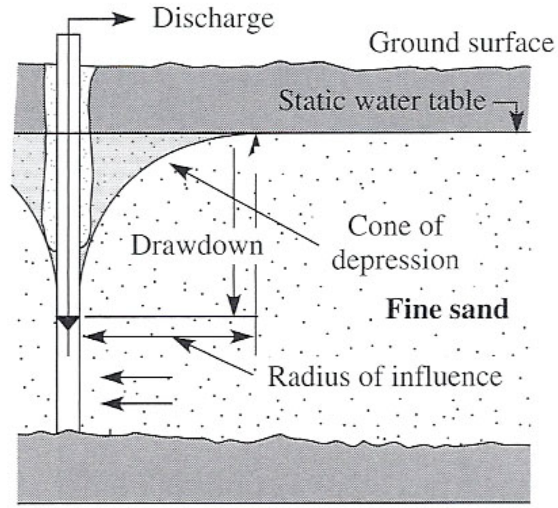
Drawdown II

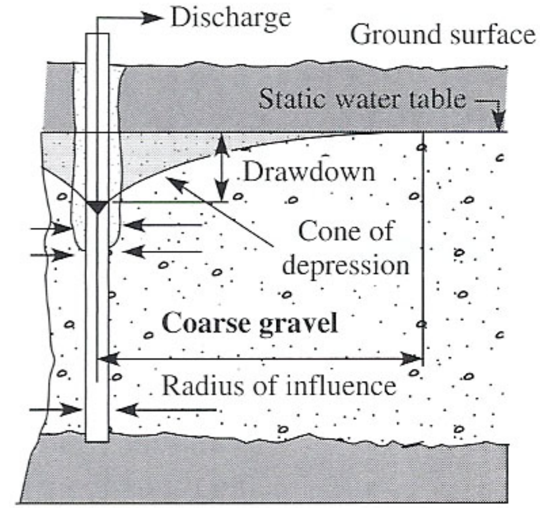
- Confined aquifer
  - D&M: Figure 7-31b



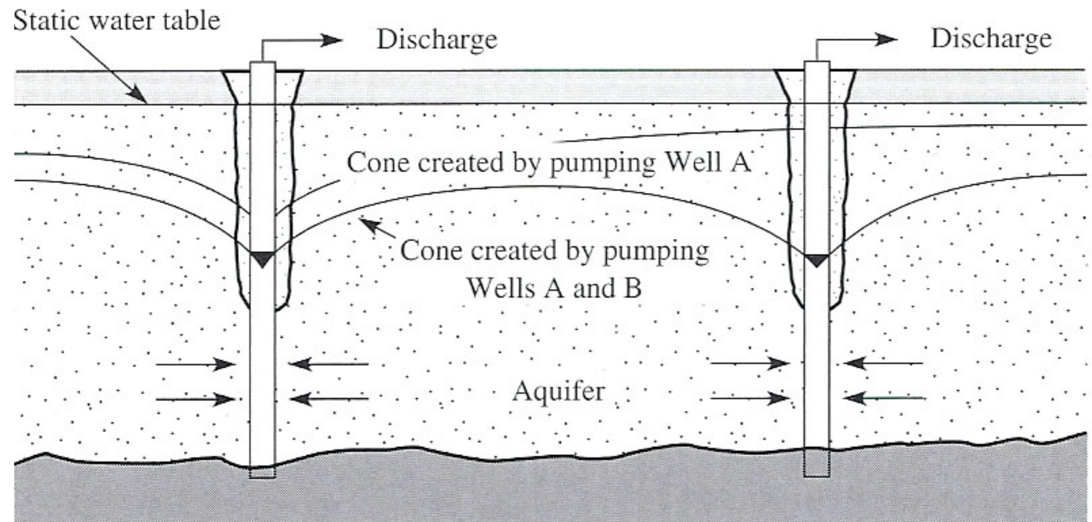
Cones of Depression

- Conductivity
  - Low K
    - Deep, shallow cone





- overlapping



Flow Model

- Well in confined aquifer

$$Q = \frac{2\pi KD(h_2 - h_1)}{\ln(r_2 / r_1)}$$

Where: h_x is the height of the piezometric surface at distance "r_x" from the well

- In an unconfined aquifer
  - D is replaced by average height of water table $(h_2+h_1)/2$, so:

$$Q = \frac{\pi K(h_2^2 - h_1^2)}{\ln(r_2 / r_1)}$$

See examples: 7-10 and 7-11 in D&M

Contaminant Flow

- Separate Phase flow – low solubility compounds
  - Low density:
    - LNAPL – light non-aqueous phase liquid
  - High density: HNAPL
- Dissolved contaminant
  - Flows with water, but subject to retardation
    - Caused by adsorption to aquifer materials

See D&M section 9-7, pg.389-393

Adsorption in Groundwater

- Based on relative affinity of contaminant for aquifer to water
  - Defined by partition coefficient, K_p:

$$K_p = \frac{C_s(moles_{adsorbed}/kg-soil)}{C_w(moles_{dissolved}/L-water)}$$

Equ 2-89, pg 76 in D&M 2nd ed.
Similar to Equ 3.32, pg 95 in M&Z

  - And more fundamentally the Kd can be related to the soil organic fraction (f_{oc}) and an organic partition coefficient (K_{OC}):

$$K_p = K_{oc} f_{oc}$$

See also pg 392 in D&M 2nd ed.
Similar to Equ 3.33, pg 95 in M&Z

Relative Velocities

- The retardation coefficient, R, is defined as the ratio of water velocity to contaminant velocity

$$R_f \equiv \frac{v'_{water}}{v'_{cont}}$$

Equ 9-42, pg 391 in D&M 2nd ed.

- And since only the dissolved fraction of the contaminant actually moves

$$v'_{cont} = v'_{water}\left(\frac{moles_{dissolved}}{moles_{dissolved} + moles_{adsorbed}}\right)$$

Relating R to K_d

- So $$v'_{cont} \big/ v'_{water} = \left(\frac{moles_{dissolved}}{moles_{dissolved} + moles_{adsorbed}} \right)$$

- And therefore

$$R_f \equiv \frac{v'_{water}}{v'_{cont}} = \frac{moles_{dissolved} + moles_{adsorbed}}{moles_{dissolved}} = 1 + \frac{moles_{adsorbed}}{moles_{dissolved}}$$

- And we can parse the last term:

$$\frac{moles_{adsorbed}}{moles_{dissolved}} = \frac{C_s(moles_{adsorbed} / kg - soil)}{C_w(moles_{dissolved} / L - water)} \left(\frac{Y(L - aquifer / L - water)}{X(L - aquifer / kg - soil)} \right)$$

cont

- Note that the fundamental partition coefficient is:

$$K_p \equiv \frac{C_s(moles_{adsorbed} / kg - soil)}{C_w(moles_{dissolved} / L - water)}$$

- So:

$$\frac{moles_{adsorbed}}{moles_{dissolved}} = K_p\left(\frac{Y(L - aquifer / L - water)}{X(L - aquifer / kg - soil)}\right)$$

- And then

$$R_f = 1 + K_p \frac{Y}{X}$$

cont

- where:

$$Y\left({}^{L-aquifer}\!\!\diagup_{L-water}\right) = {}^{1}\!\diagup_{\eta} \qquad X\left({}^{L-aquifer}\!\!\diagup_{kg-soil}\right) = \frac{1-\eta}{\rho_s} = \frac{1}{\rho_b}$$

  - Where:
    - ρ_s is density of soil particles without pores
      - usually ~2-3 g/cm^3
    - ρ_b is the bulk soil density with pores
- So, then

$$R_f = 1 + K_p\left(\frac{\rho_s}{\eta(1-\eta)}\right) = 1 + K_p\left(\frac{\rho_b}{\eta}\right)$$

M&Z Equ #7.23

Compare to Equ 9-43, pg 391 in D&M 2nd ed.

See M&Z, example 7.9, part b

$$f_d = \frac{1}{1+K_p m}$$

Estimation of partition coefficients

- Relationship to organic fraction

$$K_p = f_{oc} K_{oc}$$

$$\left(\frac{mg-tox./g-C}{mg-tox./m^3}\right) or \left(\frac{m^3}{g-C}\right)$$

- and properties of organic fraction

$$K_{oc} = 6.17x10^{-7} K_{ow}$$

Octanol:water partition coefficient

- combining, we get:

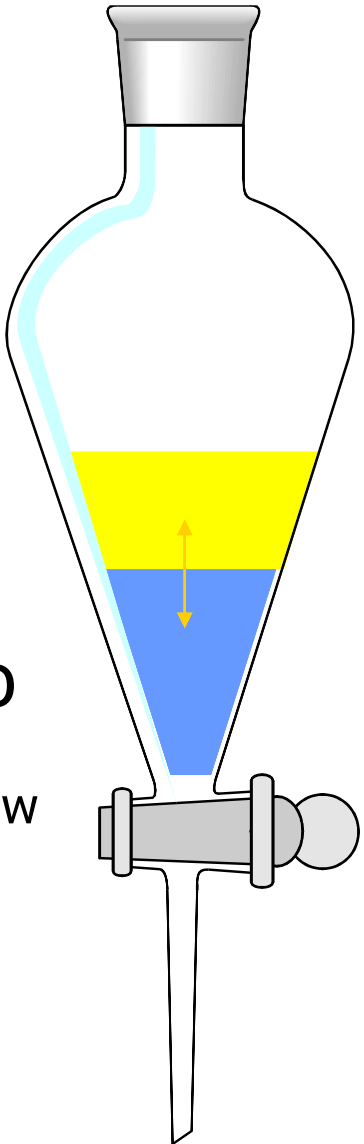
$$K_p = 6.17x10^{-7} f_{oc} K_{ow}$$

$$\left(\frac{mg-tox./m^3-Oct.}{mg-tox./m^3-H_2O}\right)$$

Karickhoff et al., 1979; Wat. Res. 13:241

Octanol:water partitioning

- 2 liquid phases in a separatory funnel that don't mix
  - octanol
  - water
- Add contaminant to flask
- Shake and allow contaminant to reach equilibrium between the two
- Measure concentration in each (K_{ow} is the ratio)



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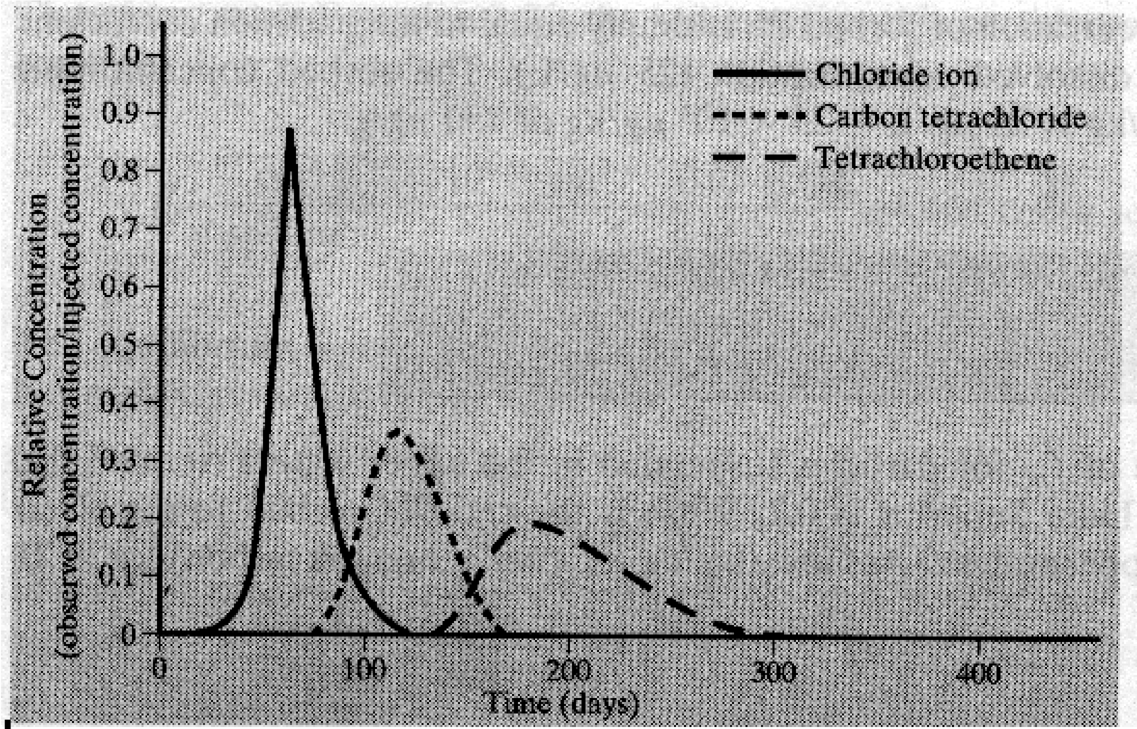
- Retardation in Groundwater & solute movement



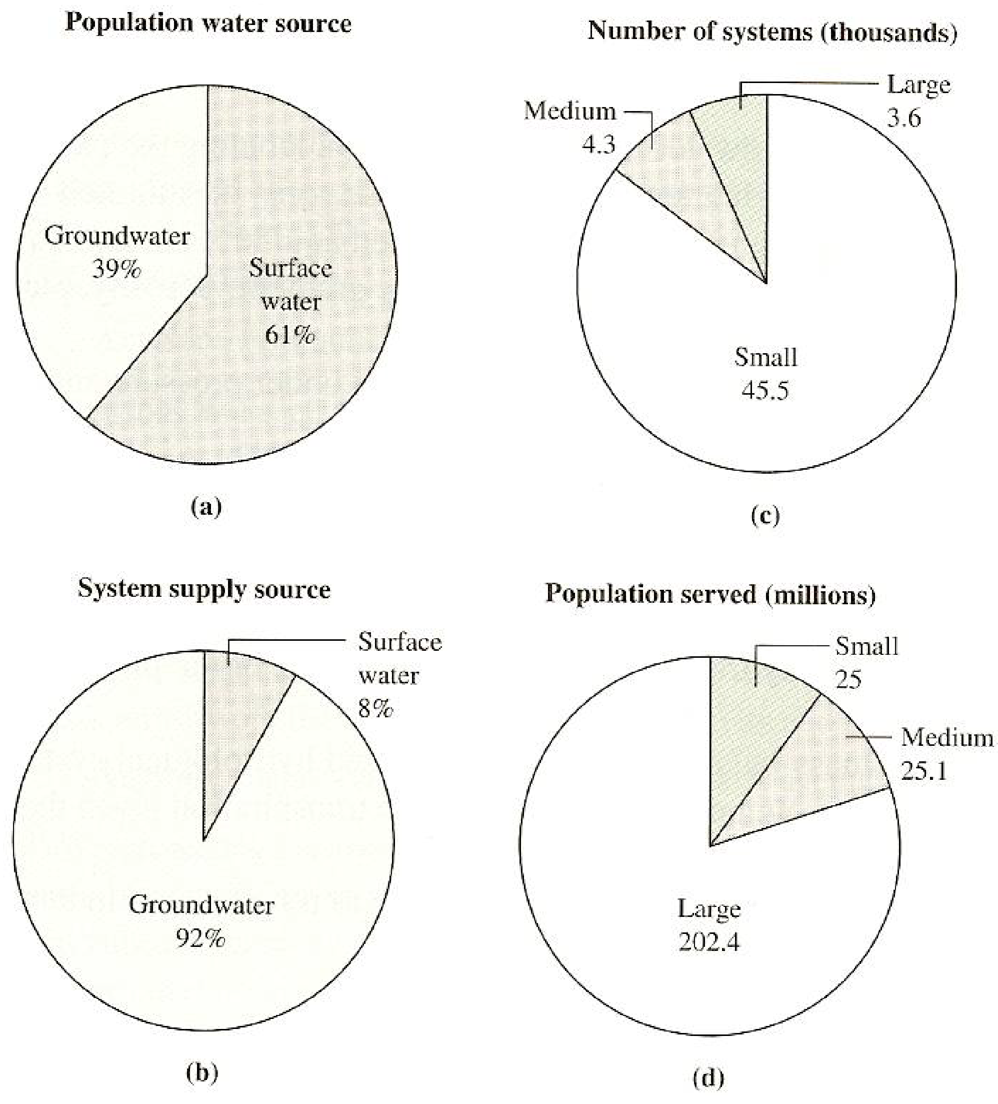
$$R_f = 1 + \frac{\rho_b}{\eta} K_p$$

ρ=Soil bulk mass density

η= void fraction

FIGURE 6–27

(a) Percentage of the population served by drinking-water system source. **(b)** Percentage of drinking-water systems by supply source. **(c)** Number of drinking-water systems (in thousands) by size. **(d)** Population served (in millions of people) by drinking-water system size. (*Source: 1997 National Public Water Systems Compliance Report.* U.S. EPA, Office of Water. Washington, D.C. 20460. (EPA-305-R-99-002). (*Note: Small systems serve 25–3300 people; medium systems serve 3301–10,000 people; large systems serve 10,000+ people.*)



- To next lecture