Lecture #22
Water Resources & Hydrology II: Wells, Withdrawals and Contaminant Transport

Reading: Mihelcic & Zimmerman, Chapter 7
Darcy’s Law

- Groundwater flow, or flow through porous media
  - Used to determine the rate at which water or other fluids flow in the sub-surface region
  - Also applicable to flow through engineered system having pores
    - Air Filters
    - Sand beds
    - Packed towers
Groundwater flow

- Balance of forces, but frame of reference is reversed
- Water flowing though a “field” of particles
Terminology

- **Head**
  - Height to which water rises within a well
    - At water table for an “unconfined” aquifer
    - Above water table for “confined” aquifers

- **Hydraulic Gradient**
  - The difference in head between two points in a aquifer separated in horizontal space

$$\text{Hydraulic Gradient} = \frac{dh}{dx}$$
**Terminology**

- **Porosity**
  - The fraction of total volume of soil or rock that is empty pore space
  - **Typical values**
    - 5-30% for sandstone rock
    - 25-50% for sand deposits
    - 5-50% for Karst limestone formations
    - 40-70% for clay deposits

\[ \eta = \frac{\text{volumes of pores}}{\text{total volume}} \]
Darcy’s Law

- Obtained theoretically by setting drag forces equal to resistive forces
- Determined experimentally by Henri Darcy (1803-1858)

\[ Q = -KA \frac{dh}{dx} = K \left( \frac{\Delta h}{L} \right) A \]

Flow per unit cross-sectional area is directly proportional to the hydraulic gradient
Hydraulic Conductivity, K

- Proportionality constant between hydraulic gradient and flow/area ratio
- A property of the medium through which flow is occurring (and of the fluid)
  - Very High for gravel: 0.2 to 0.5 cm/s
  - High for sand: $3 \times 10^{-3}$ to $5 \times 10^{-2}$ cm/s
  - Low for clays: $\sim 2 \times 10^{-7}$ cm/s
  - Almost zero for synthetic barriers: $<10^{-11}$ for high density polyethylene membranes
- Measured by pumping tests
## Hydraulic Conductivity - Table

### Typical Values of Aquifer Parameters

<table>
<thead>
<tr>
<th>Aquifer Material</th>
<th>Porosity (%)</th>
<th>Typical Values for Hydraulic Conductivity (m · s⁻¹)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Clay</td>
<td>55</td>
<td>2.3 x 10⁻⁹</td>
</tr>
<tr>
<td>Loam</td>
<td>35</td>
<td>6.0 x 10⁻⁶</td>
</tr>
<tr>
<td>Fine sand</td>
<td>45</td>
<td>2.9 x 10⁻⁵</td>
</tr>
<tr>
<td>Medium sand</td>
<td>37</td>
<td>1.4 x 10⁻⁴</td>
</tr>
<tr>
<td>Coarse sand</td>
<td>30</td>
<td>5.2 x 10⁻⁴</td>
</tr>
<tr>
<td>Sand and gravel</td>
<td>20</td>
<td>6.0 x 10⁻⁴</td>
</tr>
<tr>
<td>Gravel</td>
<td>25</td>
<td>3.1 x 10⁻³</td>
</tr>
<tr>
<td>Slate</td>
<td>&lt;5</td>
<td>9.2 x 10⁻¹⁰</td>
</tr>
<tr>
<td>Granite</td>
<td>&lt;1</td>
<td>1.2 x 10⁻¹⁰</td>
</tr>
<tr>
<td>Sandstone</td>
<td>15</td>
<td>5.8 x 10⁻⁷</td>
</tr>
<tr>
<td>Limestone</td>
<td>15</td>
<td>1.1 x 10⁻⁵</td>
</tr>
<tr>
<td>Fractured rock</td>
<td>5</td>
<td>1 x 10⁻⁸ - 1 x 10⁻⁴</td>
</tr>
</tbody>
</table>
Darcy Velocity

- just a re-arrangement of Darcy’s Law
  \[ v_d = \frac{Q}{A} = -K\left(\frac{dh}{dx}\right) \]

- Not the true (or linear or seepage) velocity of groundwater flow because flow can only occur in pores
  \[ v_{true} = \frac{L}{\tau} = \frac{L}{\eta V} = \frac{QL}{\eta V} = \frac{1}{\eta} \frac{Q}{A} \]

- combining
  \[ v_{true} = \frac{1}{\eta} v_d \quad \text{or} \quad v_{water} = \frac{1}{\eta} v \]
Velocities Illustrated

- Pipe with soil core

![Diagram of a pipe with soil core showing different velocities: Darcy Velocity, True Velocity, and Empty Air Velocity.](image)
Example 7-9

An aquifer material of coarse sand has piezometric surfaces of 10 cm and 8 cm above a datum and these are spaced 10 cm apart. If the cross sectional area is 10 cm\(^2\), what is the linear velocity of the water?

- Hydraulic gradient:
  \[
  \frac{\Delta h}{L} = \frac{10\text{cm} - 8\text{cm}}{10\text{cm}} = 0.2\text{ cm/cm}
  \]

- From Table 7-4, K for coarse sand is \(5.2 \times 10^{-4}\), so the Darcy velocity is:
  \[
  v = K \frac{\Delta h}{L} = \left(5.2 \times 10^{-4} \text{ m/s}\right) \times \frac{0.2\text{ cm}}{\text{cm}} = 1.04 \times 10^{-4} \text{ m/s}
  \]

- Assuming that the porosity is 30\% or 0.3 (Table 7-4):
  \[
  v_{\text{water}} = \frac{v}{\eta} = \frac{1.04 \times 10^{-4} \text{ m/s}}{0.3} = 3.47 \times 10^{-4} \text{ m/s}
  \]

Note a different K was used in text
Definitions

- **Specific Yield** – the fraction of water in an aquifer that will drain by gravity
  - Less than porosity due to capillary forces
  - See Table 7-5 in D&M for typical values

- **Transmissibility (T)** – flow expected from a 1 m wide cross section of aquifer (full depth) when the hydraulic gradient is 1 m/m.
  - $T = \frac{K}{D}$
  - Where $D$ is the aquifer depth and $K$ is hydraulic conductivity
Drawdown I

- Unconfined aquifer
  - D&M: Figure 7-31a
  - Showing cone of depression
**Drawdown II**

- **Confined aquifer**
  - D&M: Figure 7-31b
Cones of Depression

- Conductivity
  - Low K
    - Deep, shallow cone

- Overlapping
Flow Model

- Well in confined aquifer
  \[ Q = \frac{2\pi KD(h_2 - h_1)}{\ln(r_2 / r_1)} \]
  Where: \( h_x \) is the height of the piezometric surface at distance "\( r_x \)" from the well

- In an unconfined aquifer
  - D is replaced by average height of water table \((h_2 + h_1)/2\), so:
  \[ Q = \frac{\pi K(h_2^2 - h_1^2)}{\ln(r_2 / r_1)} \]
  See examples: 7-10 and 7-11 in D&M
Contaminant Flow

- Separate Phase flow – low solubility compounds
  - Low density:
    - LNAPL – light non-aqueous phase liquid
  - High density: HNAPL

- Dissolved contaminant
  - Flows with water, but subject to retardation
    - Caused by adsorption to aquifer materials

See D&M section 9-7, pg.389-393
Adsorption in Groundwater

- Based on relative affinity of contaminant for aquifer to water
  - Defined by partition coefficient, $K_d$:
    
    $K_d = \frac{C_s \left( \text{moles}_{\text{adsorbed}} / \text{kg - soil} \right)}{C_w \left( \text{moles}_{\text{dissolved}} / \text{L - water} \right)}$

- And more fundamentally the Kd can be related to the soil organic fraction ($f_{oc}$) and an organic partition coefficient ($K_{OC}$):
  
  $K_d = K_{OC} f_{oc}$

See also pg 392 in D&M 2nd ed.

Equ 2-89, pg 76 in D&M 2nd ed.
Relative Velocities

- The retardation coefficient, $R$, is defined as the ratio of water velocity to contaminant velocity
  \[ R \equiv \frac{V_{\text{water}}}{V_{\text{cont}}} \]

- And since only the dissolved fraction of the contaminant actually moves
  \[ V'_{\text{cont}} = V'_{\text{water}} \left( \frac{\text{moles}_{\text{dissolved}}}{\text{moles}_{\text{dissolved}} + \text{moles}_{\text{adsorbed}}} \right) \]

Equation 9-42, pg 391 in D&M 2nd ed.
Relating $R$ to $K_d$

- So

$$\frac{v'_{cont}}{v'_w} = \left( \frac{\text{moles}_{dissolved}}{\text{moles}_{dissolved} + \text{moles}_{adsorbed}} \right)$$

- And therefore

$$R \equiv \frac{v'_w}{v'_{cont}} = \frac{\text{moles}_{dissolved} + \text{moles}_{adsorbed}}{\text{moles}_{dissolved}} = 1 + \frac{\text{moles}_{adsorbed}}{\text{moles}_{dissolved}}$$

- And we can parse the last term:

$$\frac{\text{moles}_{adsorbed}}{\text{moles}_{dissolved}} = \frac{C_s \left( \text{moles}_{adsorbed} / \text{kg} - \text{soil} \right)}{C_w \left( \text{moles}_{dissolved} / \text{L} - \text{water} \right)} \left( \frac{Y \left( \text{L} - \text{aquifer} / \text{L} - \text{water} \right)}{X \left( \text{L} - \text{aquifer} / \text{kg} - \text{soil} \right)} \right)$$
Note that the fundamental partition coefficient is:

\[ K_d = \frac{C_s (\text{moles}_{\text{adsorbed}} / \text{kg} - \text{soil})}{C_w (\text{moles}_{\text{dissolved}} / \text{L} - \text{water})} \]

So:

\[ \frac{\text{moles}_{\text{adsorbed}}}{\text{moles}_{\text{dissolved}}} = K_d \left( \frac{Y (L - \text{aquifer} / L - \text{water})}{X (L - \text{aquifer} / \text{kg} - \text{soil})} \right) \]

And then

\[ R = 1 + K_d \frac{Y}{X} \]
where:

\[ Y(L-aquifer/L-water) = \frac{1}{\eta} \]

\[ X(L-aquifer/kg-soil) = \frac{1-\eta}{\rho_s} = \frac{1}{\rho_b} \]

Where:

- \( \rho_s \) is density of soil particles without pores
  - usually \( \sim 2-3 \) g/cm\(^3\)
- \( \rho_b \) is the bulk soil density with pores

So, then

\[ R = 1 + K_d \left( \frac{\rho_s}{\eta(1-\eta)} \right) = 1 + K_d \left( \frac{\rho_b}{\eta} \right) \]

Compare to Equ 9-43, pg 391 in D&M 2nd ed.
Retardation in Groundwater & solute movement

\[ R = 1 + \frac{\rho_b}{\eta} K_d \]

\( \rho \) = Soil bulk mass density
\( \eta \) = void fraction
(a) Percentage of the population served by drinking-water system source. (b) Percentage of drinking-water systems by supply source. (c) Number of drinking-water systems (in thousands) by size. (d) Population served (in millions of people) by drinking-water system size.

To next lecture