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CEE 370 Environmental Engineering Principles

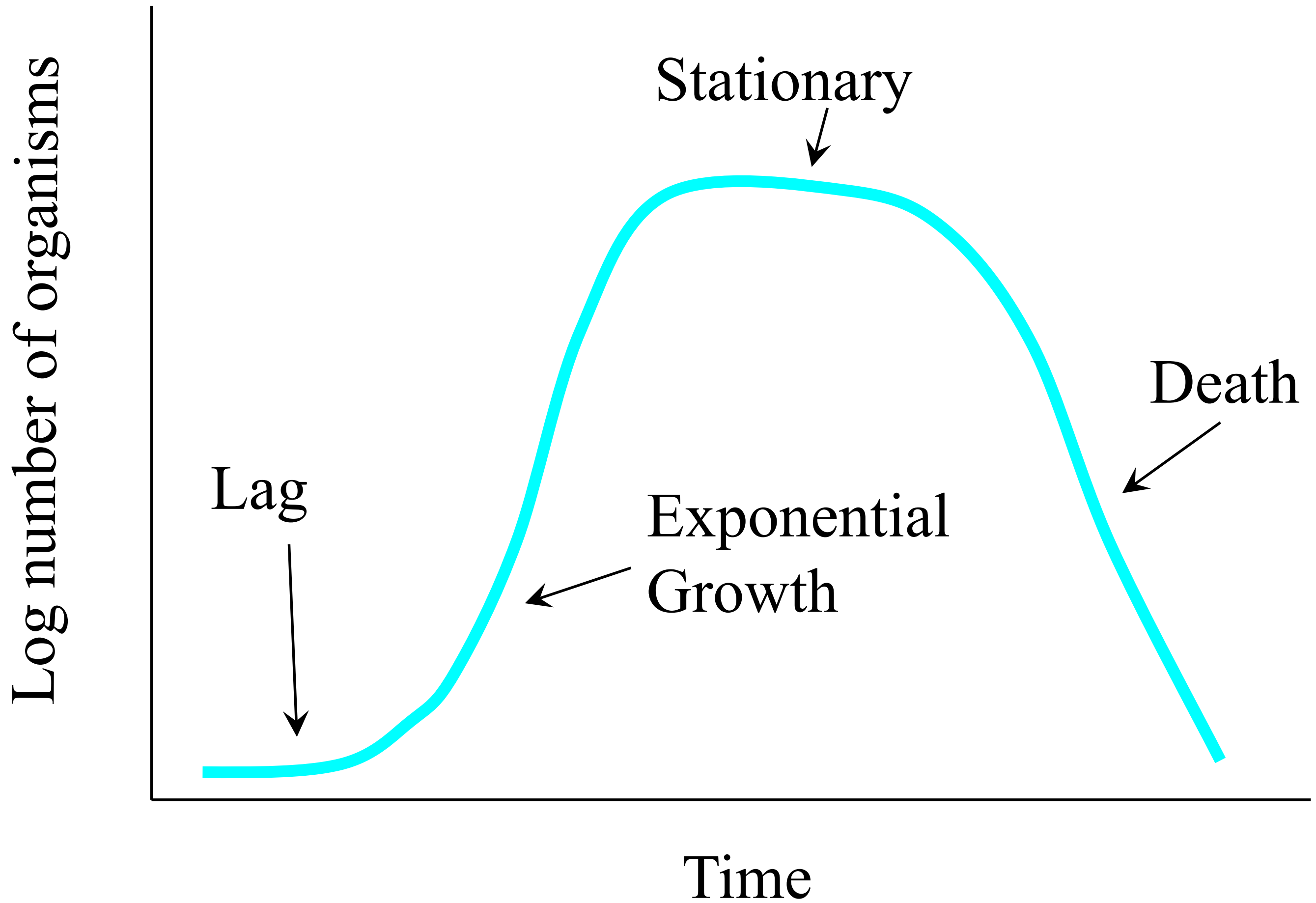
Lecture #17

Ecosystems IV: Microbiology & Biochemical Pathways

Reading: Mihelcic & Zimmerman, Chapter 5
Davis & Masten, Chapter 4

Batch Microbial Growth

- Observed behavior



Exponential Growth model

$$\frac{dN}{dt} = rN$$ or $$\frac{dX}{dt} = \mu X$$

Animals	Bacteria		
			where,
N	X	=	concentration of organisms at time t
t	t	=	time
r	μ	=	proportionality constant or specific growth rate, [time^{-1}]
dN/dt	dX/dt	=	organism growth rate, [mass per volume-time]

Exp. Growth (cont.)

Higher Organisms

$$\frac{dN}{dt} = rN \quad \text{or} \quad \frac{dN}{N} = rdt$$

$$\ln\left(\frac{N}{N_o}\right) = rt$$

$$N = N_o e^{rt}$$

Microorganisms

$$\frac{dX}{dt} = \mu X \quad \text{or} \quad \frac{dX}{X} = \mu dt$$

$$\ln\left(\frac{X}{X_o}\right) = \mu t$$

$$X = X_o e^{\mu t}$$

Exp. Growth Example

A microbial system with an ample substrate and nutrient supply has an initial cell concentration, X_o, of 500 mg/L. The specific <u>growth</u> rate is 0.5 /hour.

a) Estimate the cell concentration after 6 hours, assuming log growth is maintained during the period.

b) Determine the time required for the microbial population to double during this log growth phase.

Solution I

a) To determine the microbial concentration after 6 hours, we substitute into the exp growth model, obtaining,

$$X = X_o e^{\mu t} = 500 \frac{mg}{L} \times e^{(0.5/hr \times 6\ hr)}$$

$$X = 10{,}000 \frac{mg}{L}$$

Thus, in a period of six hours, the microorganisms increase 20 fold.

Solution II

b) To determine the time for the concentration to double, we use the log form. Also, if the concentration doubles, then

$$\frac{X}{X_o} = 2 \qquad \text{or} \qquad \ln\frac{X}{X_o} = \mu t$$

Or, solving for t we obtain,

$$t = \frac{\ln\left(\frac{X}{X_o}\right)}{\mu} = \frac{\ln 2}{0.5\ /\mathrm{hr}}$$

$$t = 1.4\ \mathrm{hr.}$$

Thus, the microbial population can double in only 1.4 hours. By comparison, the human population is currently doubling about every 40 years.

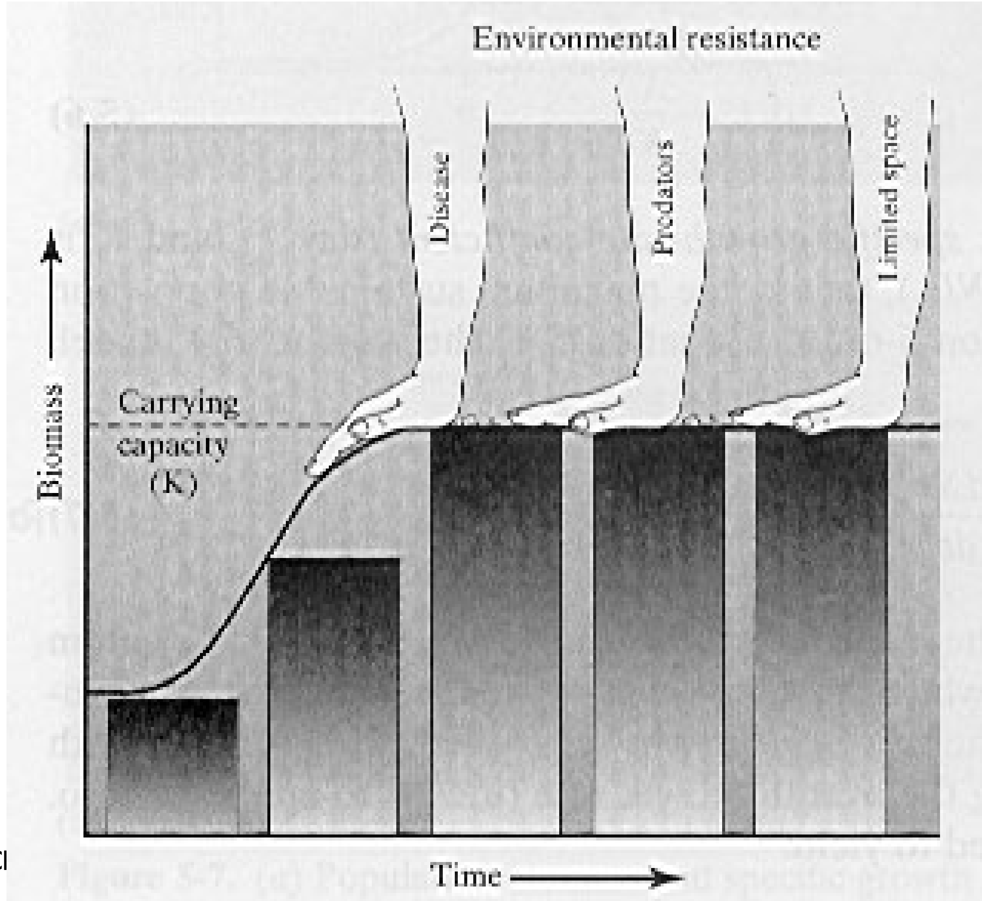
Limitations in population density

- Carrying capacity, K, and the logistic growth model:

$$\mu = \mu_{\max}\left(1 - \frac{X}{K}\right)$$

$$\frac{dX}{dt} = \mu_{\max}\left(1 - \frac{X}{K}\right)X$$

$$X_t = \frac{K}{1 + \left[\left(\frac{K - X_0}{X_0}\right)e^{-\mu_{\max} t}\right]}$$



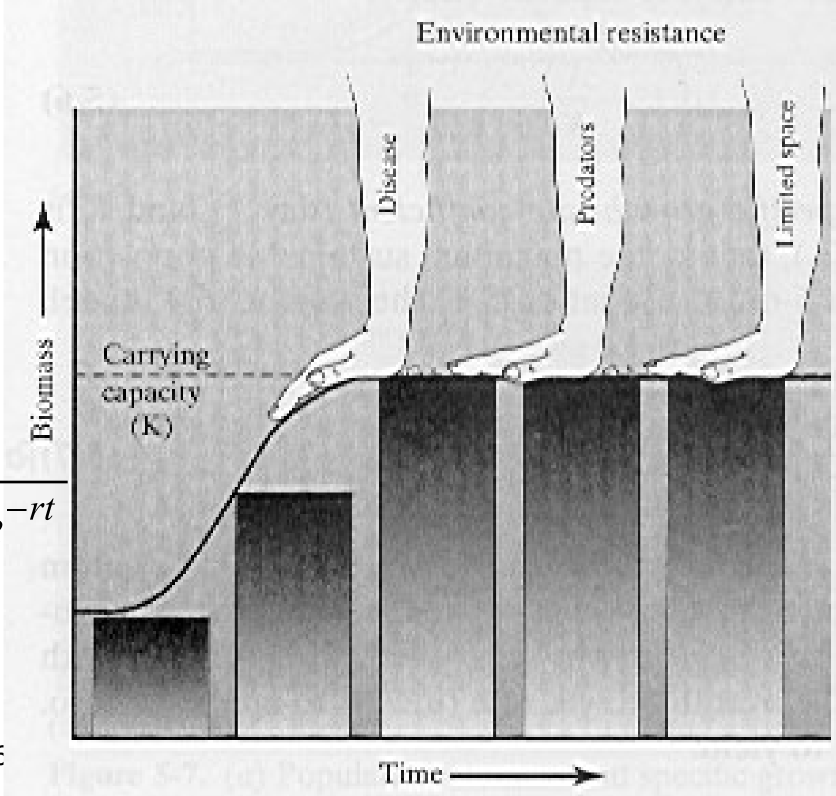
Logistic Model (cont.)

- In many books they use different terms when applying it to animal dynamics:

$$\frac{dX}{dt} = \mu_{\max}\left(1 - \frac{X}{K}\right)X$$

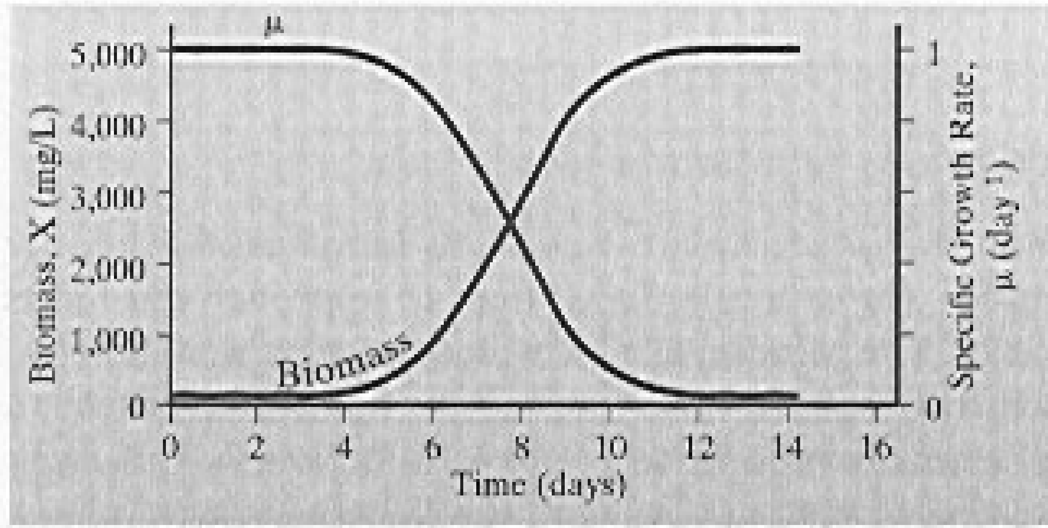
$$\frac{dN}{dt} = r\left(1 - \frac{N}{K}\right)N$$

$$N_t = \frac{K}{1 + \left[\left(\frac{K - N_0}{N_0}\right)e^{-rt}\right]} = \frac{KN_o}{N_o + (K - N_o)e^{-rt}}$$

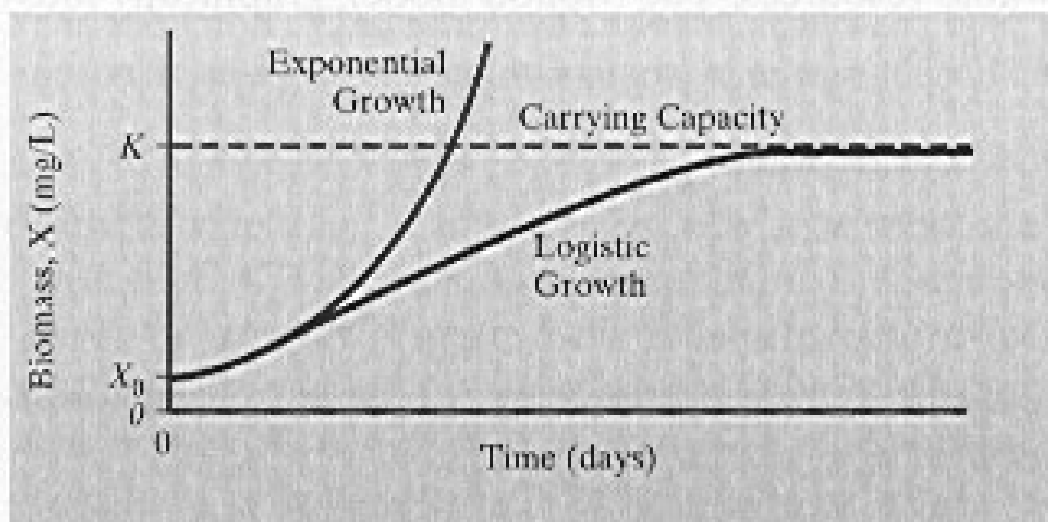


Logistic Growth Example

- Determine 10 day population for:
  - Initial population: X_0 = 2 mg/L
  - Max growth rate: 1 day^{-1}
  - Carrying capacity: 5000 mg/L



(a)



Substrate-limited Growth

- Also known as resource-limited growth
  - THE MONOD MODEL

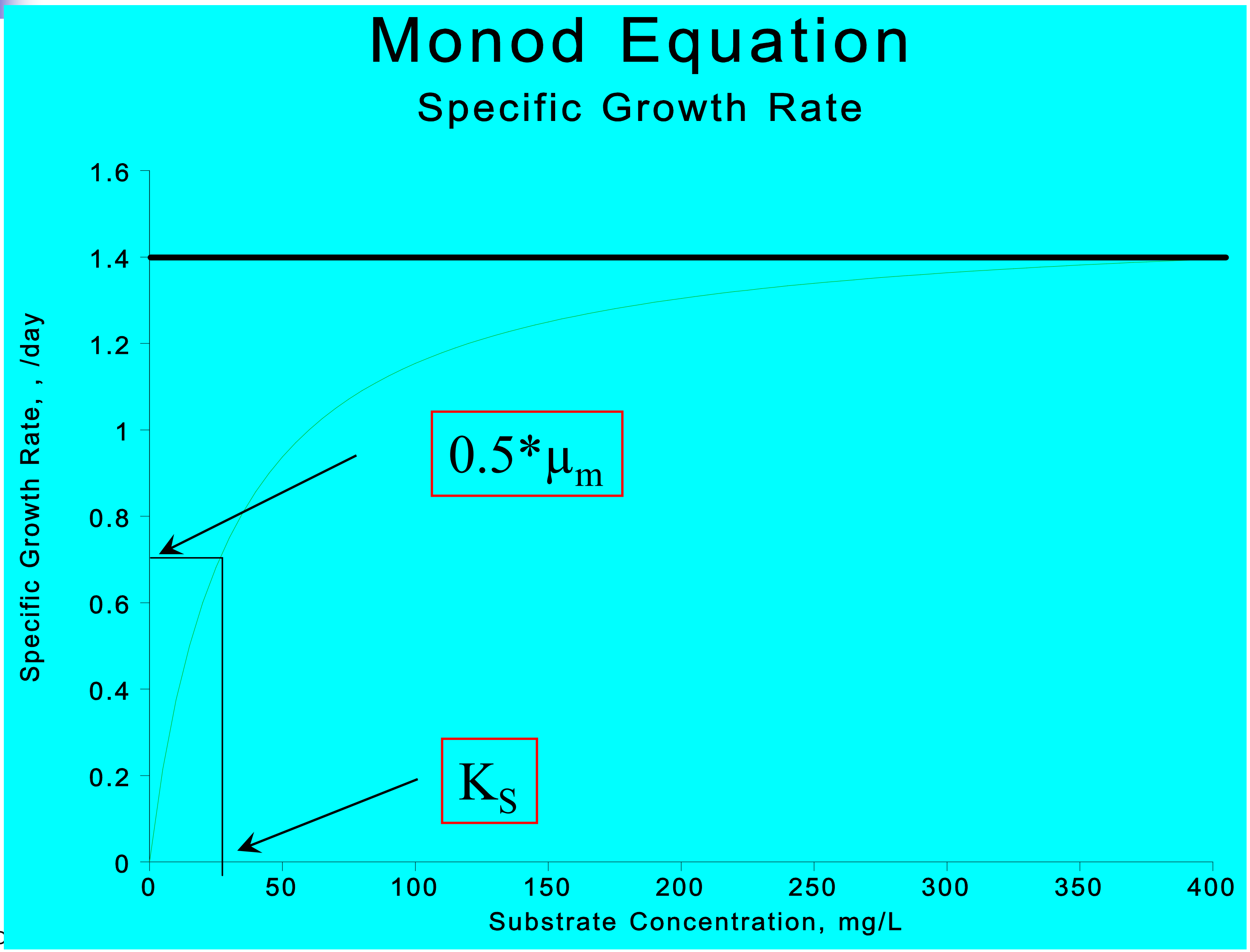
Similar to enzyme kinetic model introduced in lecture #13

$$\left(\frac{dX}{dt}\right)_{growth} \equiv \mu X = \frac{\mu_{max} S}{K_s + S} X$$

where,

μ_m	=	maximum specific growth rate, [day^{-1}]
S	=	concentration of limiting substrate, [mg/L]
K_s	=	Monod or half-velocity constant, or half saturation coefficient, [mg/L]

Monod Kinetics



Decay term

- With microorganisms, when using a substrate-limited growth model, it is also appropriate to consider "decay"
- Decay covers the "cost of doing business", including the energy lost in respiration, cell maintenance & reproduction
- The mode is simple first order $\left(\frac{dX}{dt}\right)_{decay} = -k_d X$
- And combining it with the Monod model

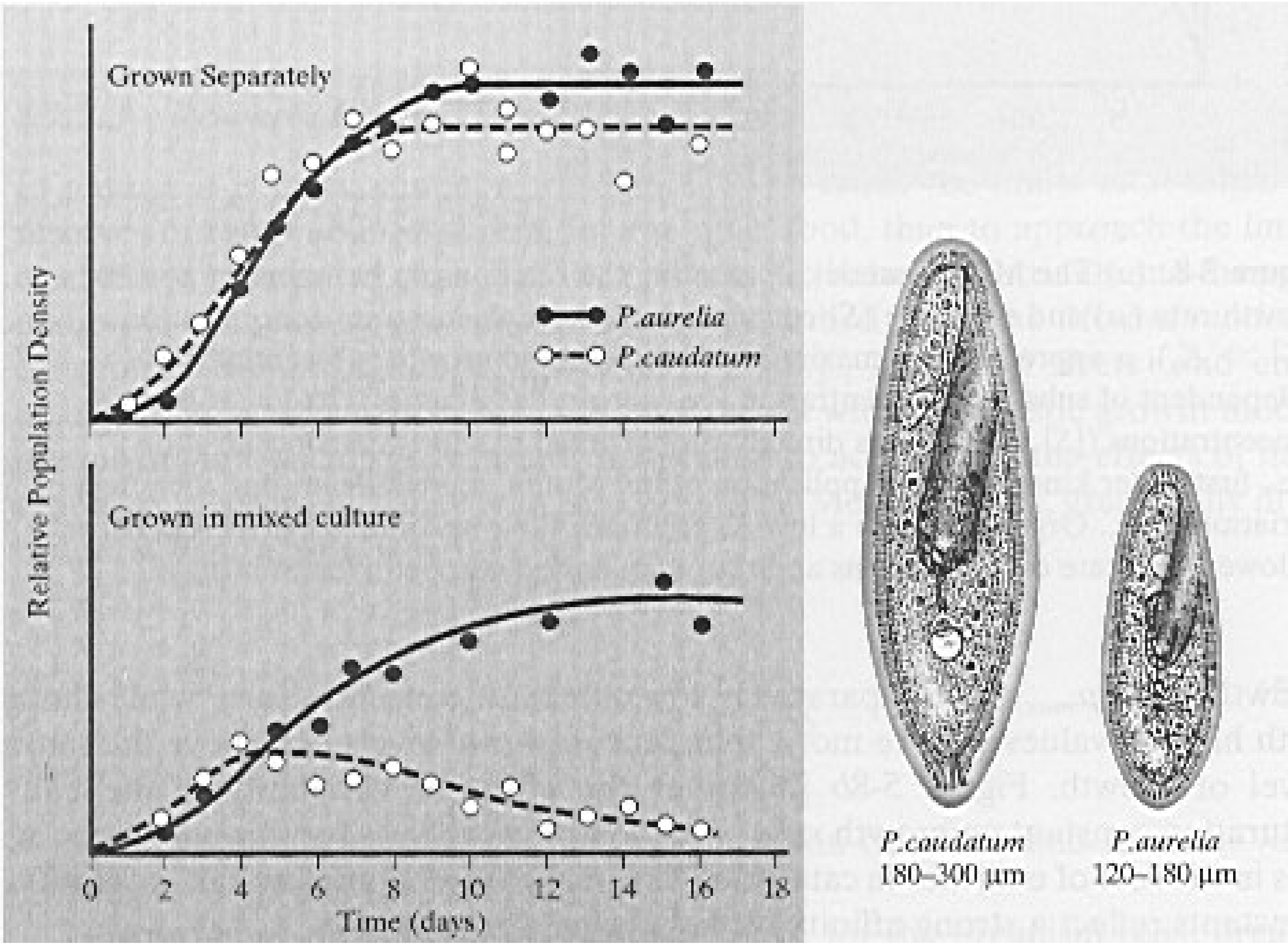
$$\left(\frac{dX}{dt}\right)_{overall} = \frac{\mu_{max}S}{K_s + S}X - k_d X$$

Substrate model

- Yield coefficient $$Y \equiv \frac{\Delta X}{\Delta S}$$
- And $$\frac{dS}{dt} = -\frac{1}{Y}\left(\frac{dX}{dt}\right)$$
- So, combining with the overall growth model

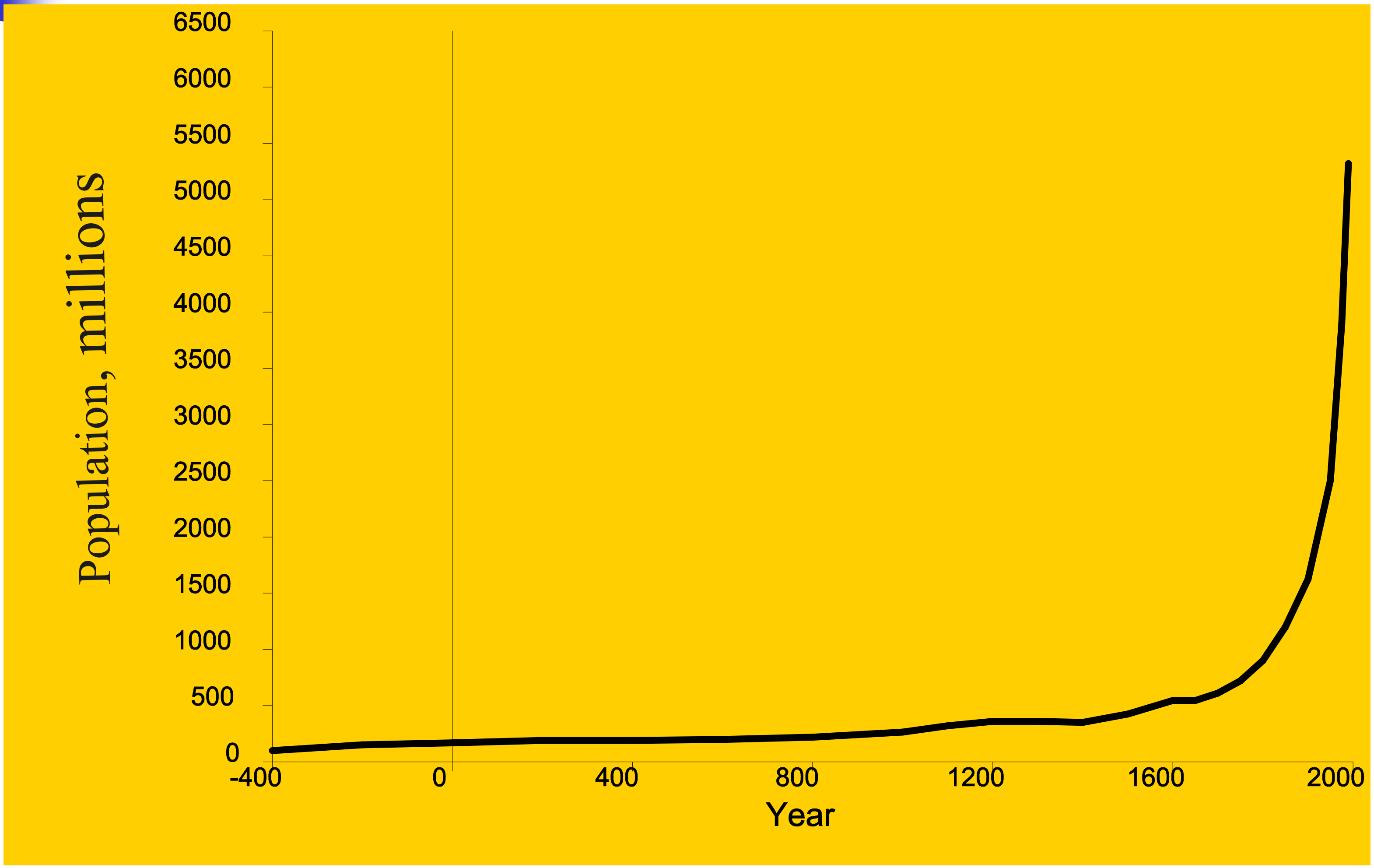
$$\frac{dS}{dt} = \frac{1}{Y}\left(\frac{dX}{dt}\right) \quad = \frac{1}{Y}\left(\frac{\mu_{max}S}{K_s + S}X - k_d X\right)$$

Competition for Substrate



David Reckhow

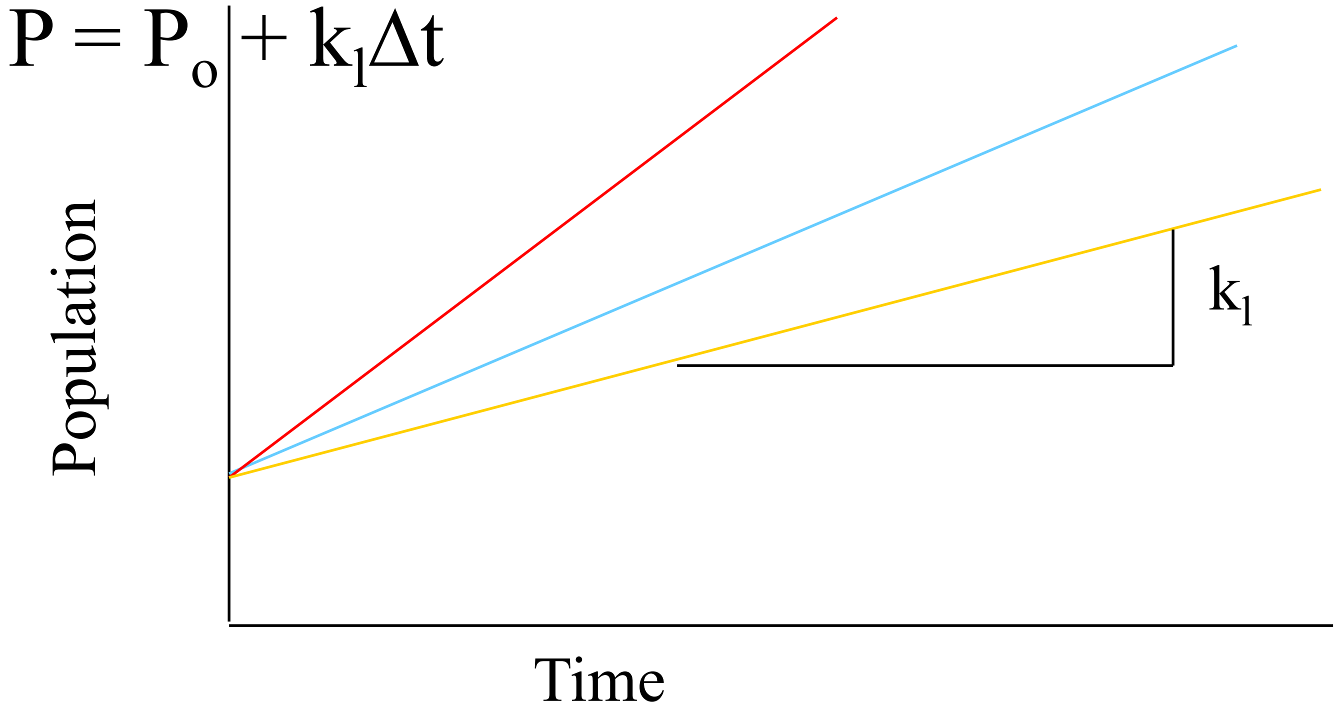
World Population Growth



Human Population Projection

- Arithmetic Model
- Exponential Model
- Decreasing Rate of Increase Model
- Graphical extension
- Graphical comparison
- Ratio method
- Other

Linear Model



Good for some types of growth, but not all

- Babies growth ~ 2 lb/month
- Linear model predicts 1,320 lb by age 55

Exponential Model

Exponential Functions

$$P = P_o \; e^{r\Delta t}$$

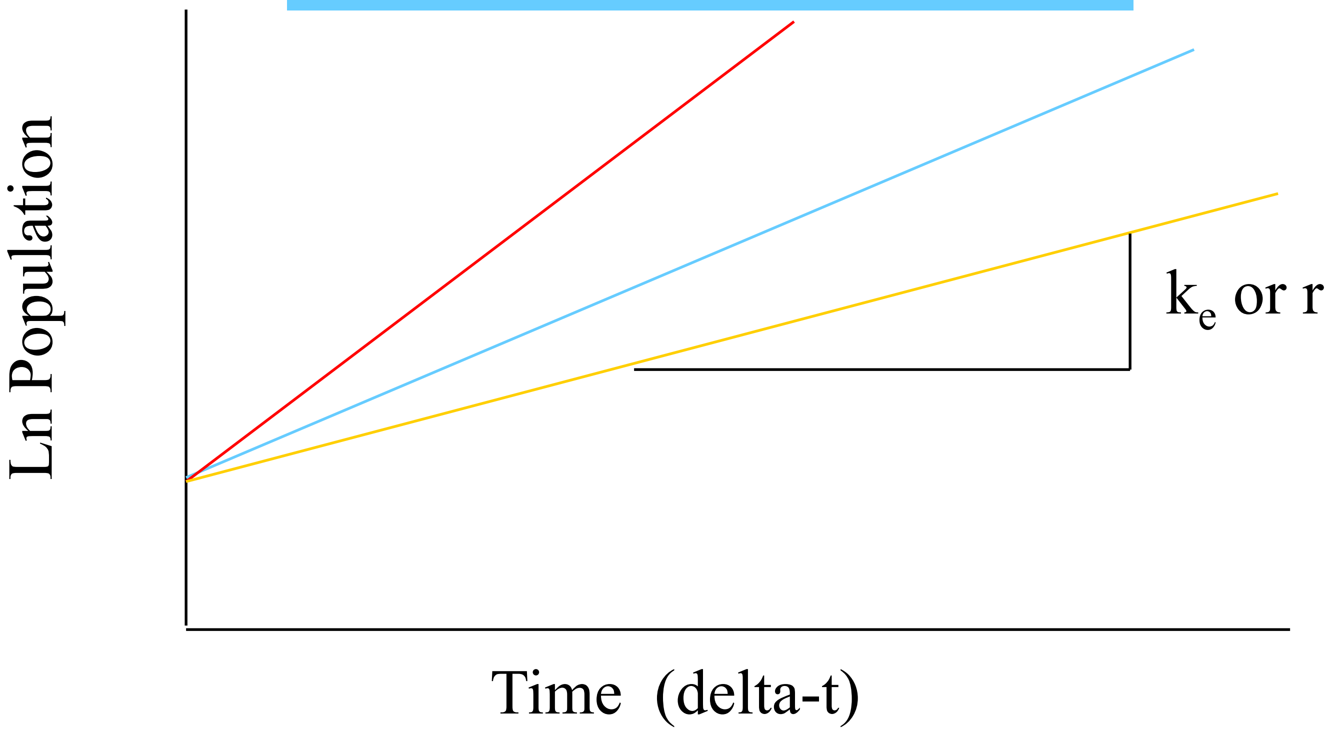
where,

P	=	population at time t,
P_o	=	population at time zero,
r (k_e)	=	population growth rate, years^{-1},
Δt	=	time, years.

Parameter Estimation: Exponential Model

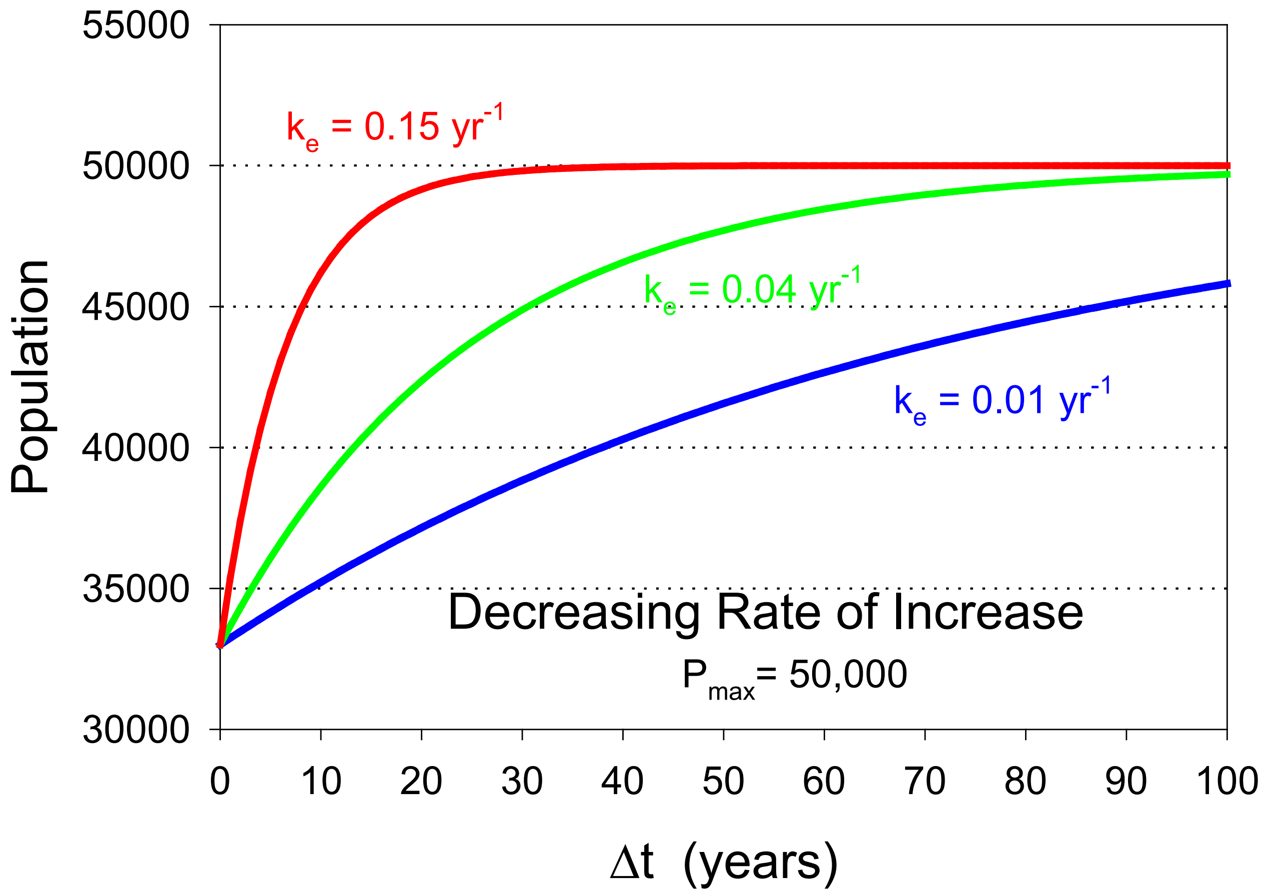
$$Ln(P) = Ln(P_o \; e^{r\Delta t})$$

$$Ln(P) = Ln(P_0) + r\Delta t$$



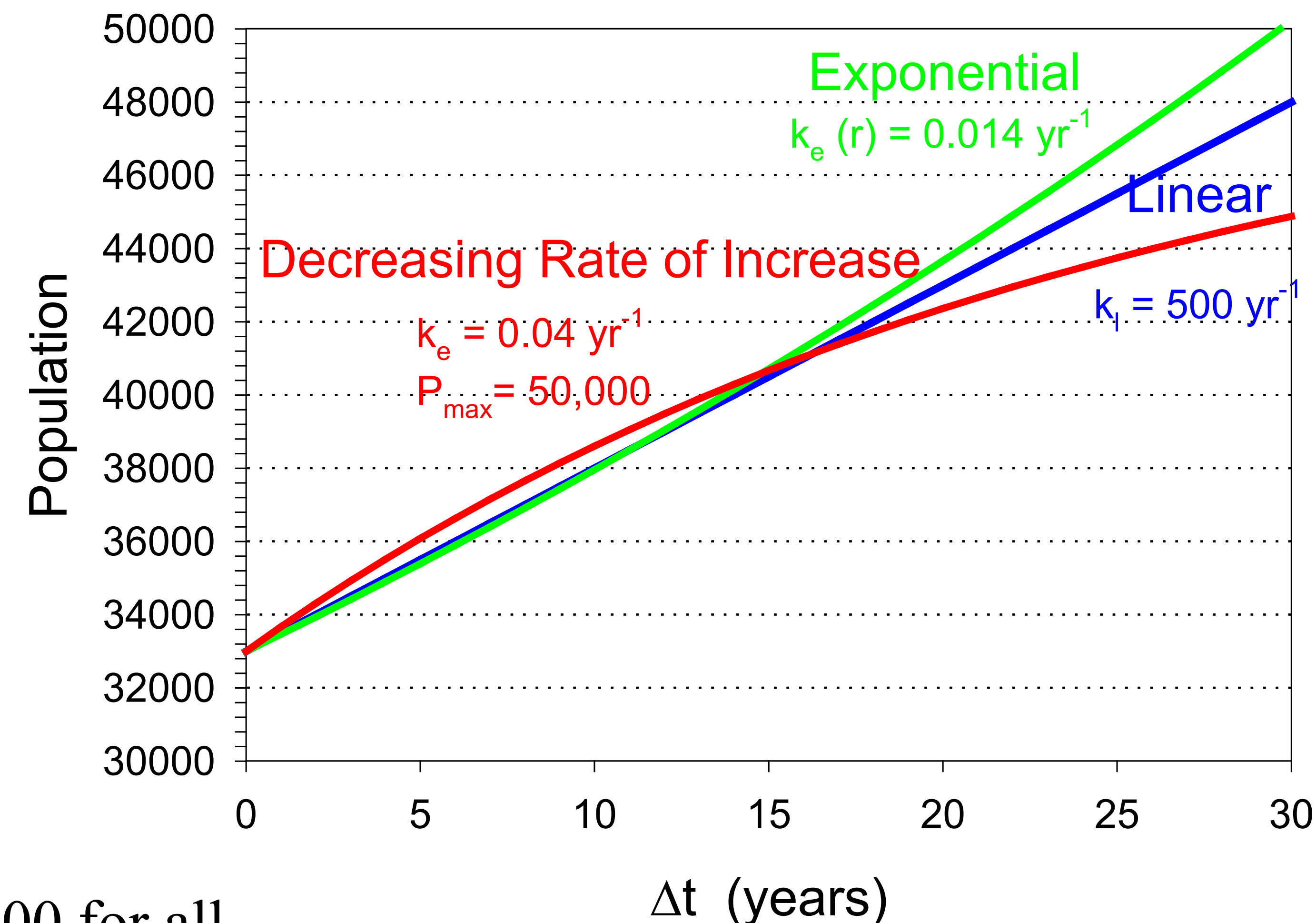
Decreasing Rate of Increase Model

$$P = P_o + (P_{max} - P_o)(1 - \exp(-k_e \Delta t))$$



Where: P_{max} is the limiting or saturation population

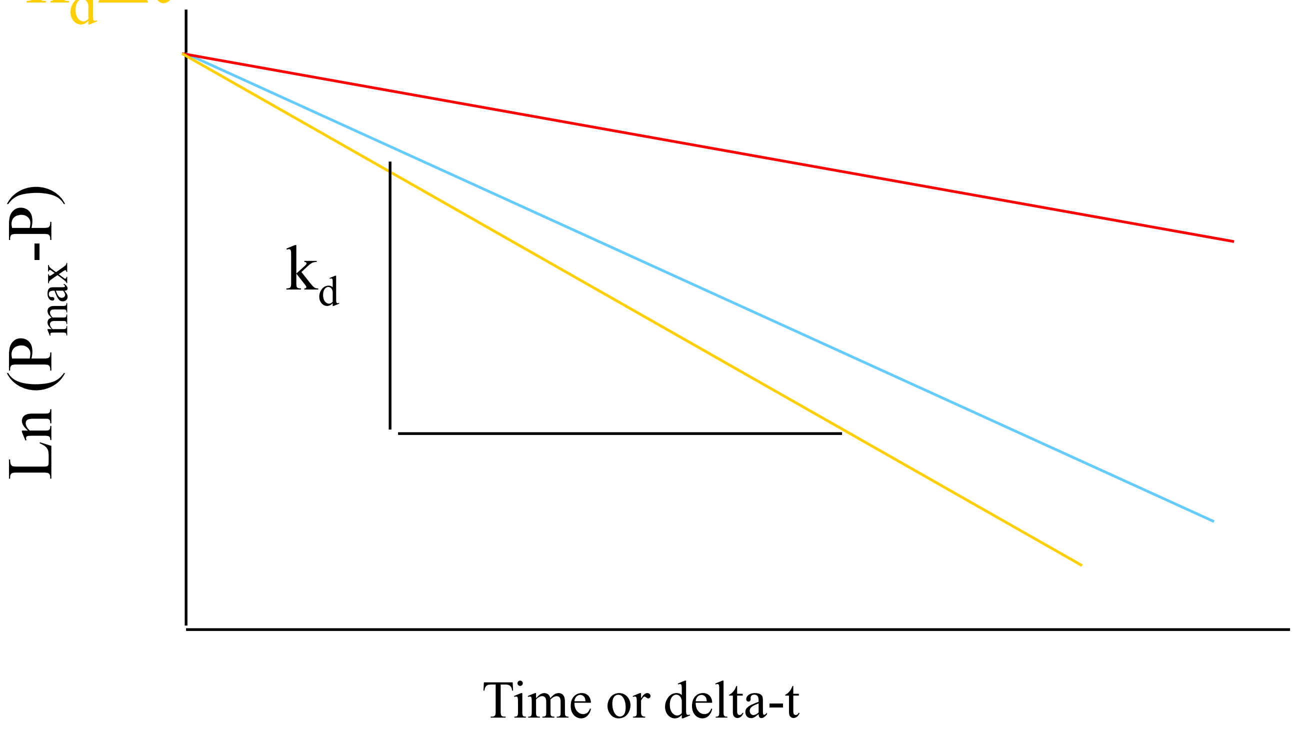
Comparison of Population Models



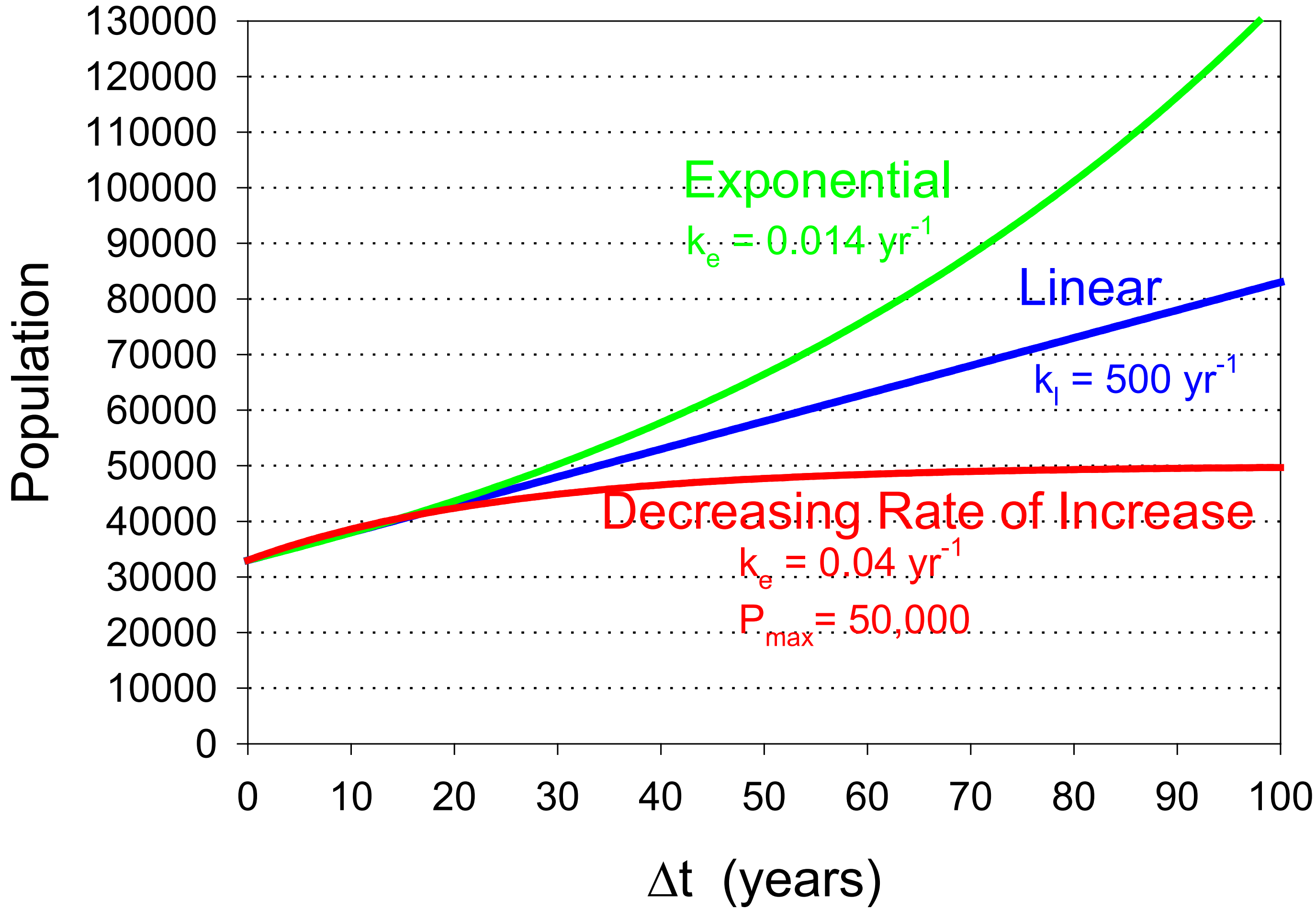
P_o = 33,000 for all

Parameter Estimation: Decreasing Rate of Increase Model

$$Ln(P_{max}-P) = Ln(P_{max}-P_o) - k_d \Delta t$$



Comparison of Population models: 100 yrs.



$P_o = 33{,}000$ for all

Example: World Population Growth

In 1950 the world population was estimated to be 2.5 billion. In 1990 it was estimated to be 5.5 billion. Assuming this growth rate will be sustained,

a) calculate the world population in the year 2000
b) determine the year the global population will reach 10 billion.

Solution

We must first calculate the population growth rate, r. To do this we convert Equation into log form:

$$\ln\left(\frac{P}{P_o}\right) = r\,\Delta t$$

Solution, cont.

Now, if we know the population at two different times in the past, we can calculate the population growth rate, r. We know that the population in 1950 was 2.5 billion, and that it was 5.5 billion in 1990. The time change, Δt, is 40 years. Thus, the population growth rate is,

$$r = \frac{\ln\left(\frac{P}{Po}\right)}{\Delta t} = \frac{\ln\left(\frac{5.5 \text{ x } 10^9}{2.5 \text{ x } 10^9}\right)}{40 \text{ years}}$$

$$r = 0.020 \text{ / yr}$$

Solution, cont.

Population in the year 2000?

$$P = P_o \, e^{r \Delta t} = 5.5 \text{ x } e^{0.020 \text{ x } (2000 - 1990)}$$

$$P = 6.7 \text{ x } 10^9 \text{ in } year\, 2000$$

Actual: 6.1 x 10^9

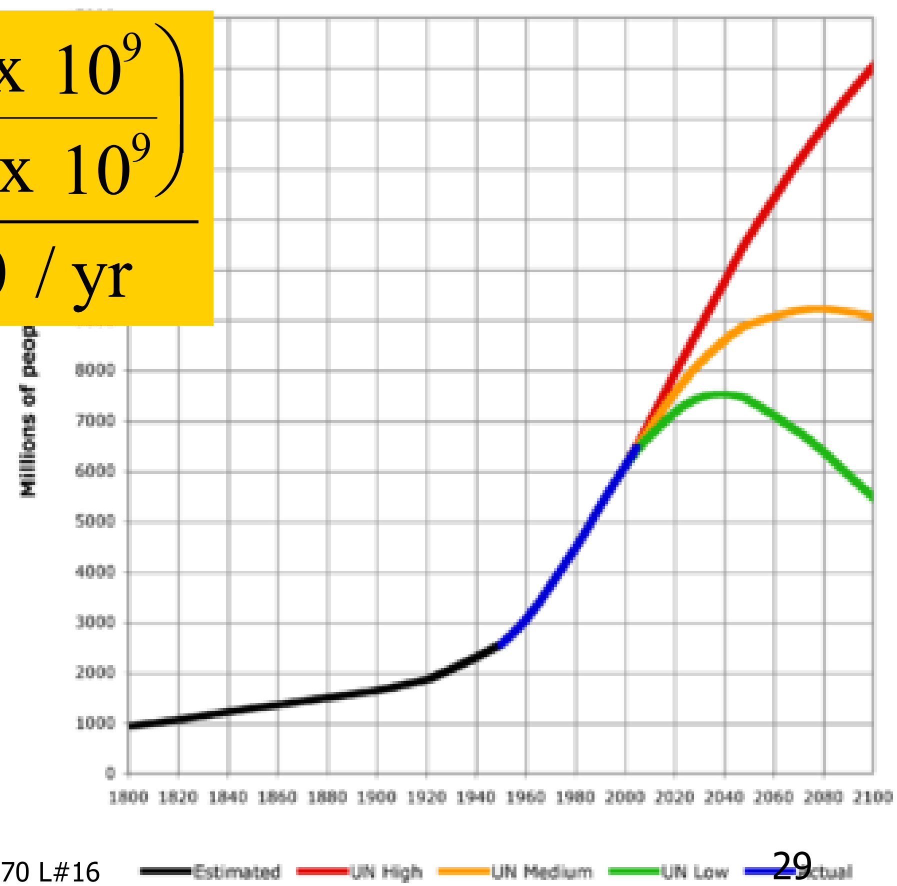
Solution, cont.

Year that the population will reach 10 billion

$$\Delta t = \frac{\ln\left(\frac{P}{P_o}\right)}{r} = \frac{\ln\left(\frac{10 \text{ x } 10^9}{5.5 \text{ x } 10^9}\right)}{0.020 \text{ / yr}}$$

$$\Delta t = 30 \text{ years}$$

So 1990 + 30 is A.D. 2020



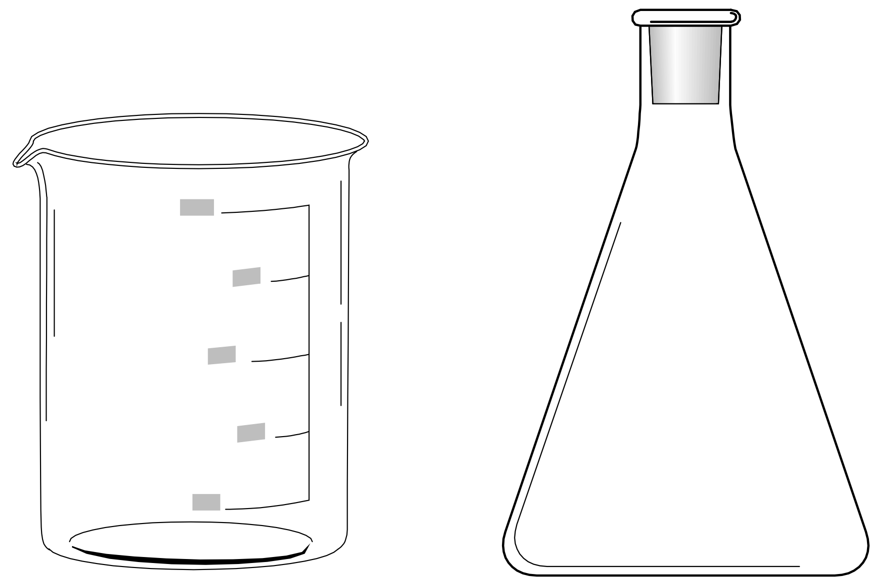
Oxygen Demand

- It is a measure of the amount of “reduced” organic and inorganic matter in a water
- Relates to oxygen consumption in a river or lake as a result of a pollution discharge
- Measured in several ways
  - BOD - Biochemical Oxygen Demand
  - COD - Chemical Oxygen Demand
  - ThOD - Theoretical Oxygen Demand

BOD with dilution

When BOD>8mg/L

$$BOD_t = \frac{DO_i - DO_f}{\left(\frac{V_s}{V_b}\right)}$$



Where

BOD_t = biochemical oxygen demand at t days, [mg/L]

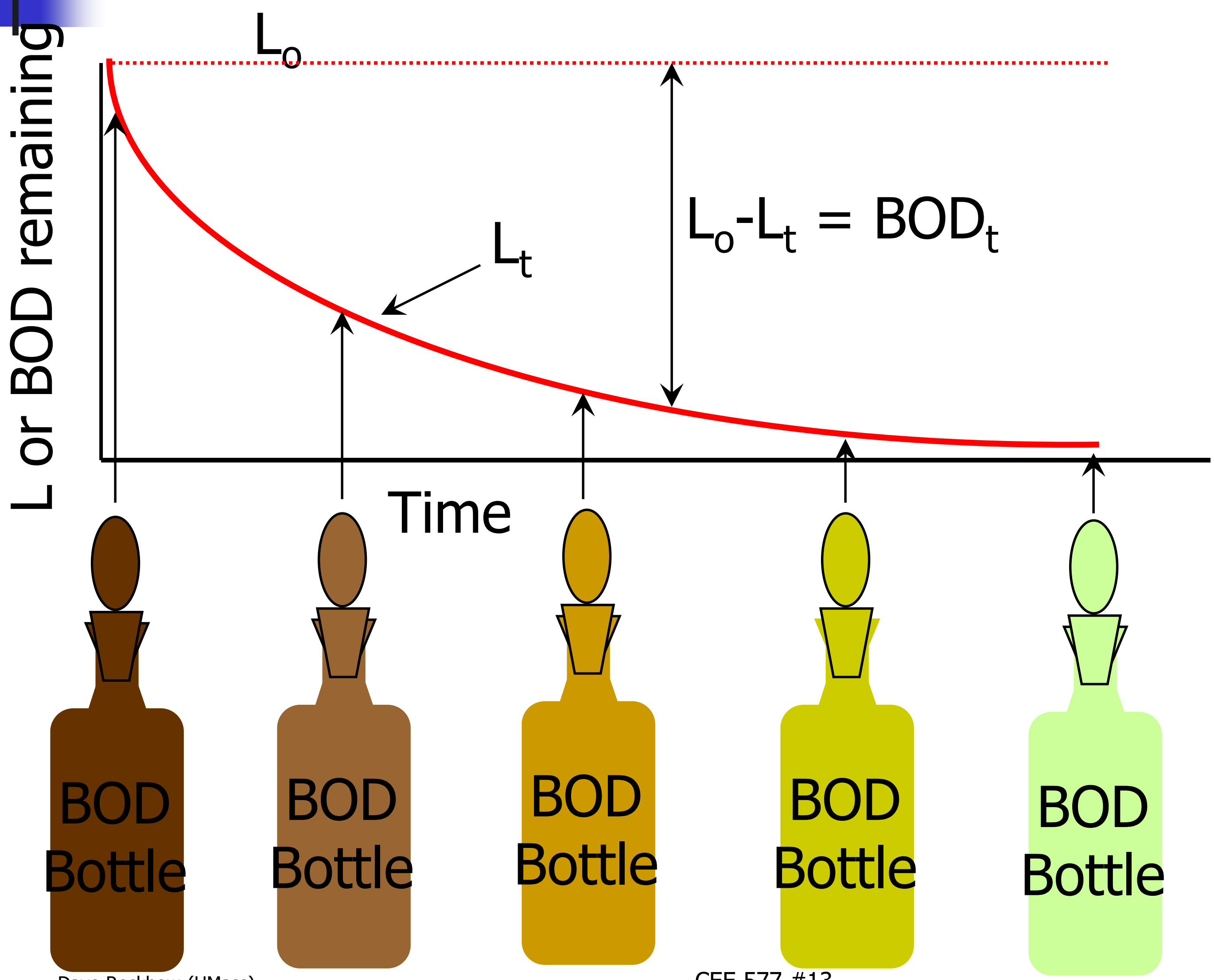
DO_i = initial dissolved oxygen in the sample bottle, [mg/L]

DO_f = final dissolved oxygen in the sample bottle, [mg/L]

V_b = sample bottle volume, usually 300 or 250 mL, [mL]

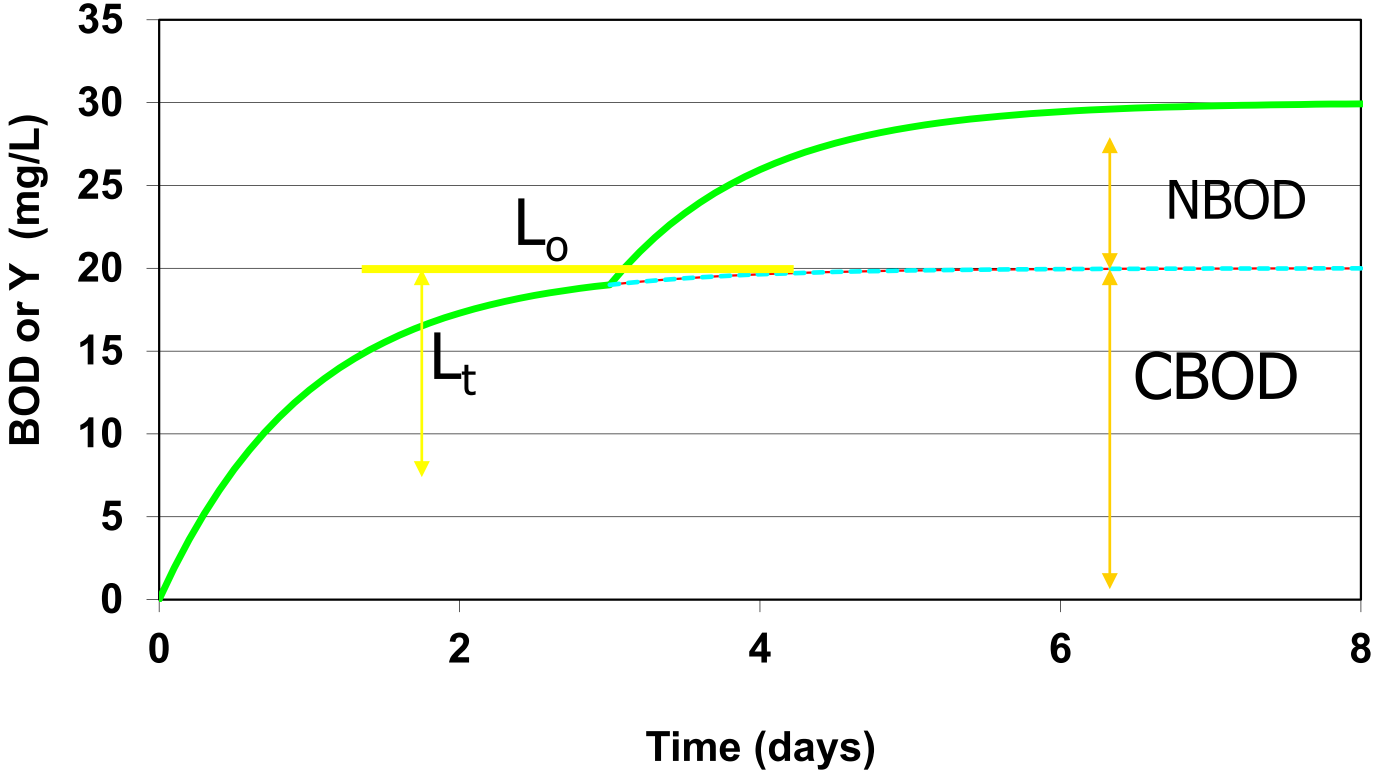
V_s = sample volume, [mL]

BOD - loss of biodegradable organic matter (oxygen demand)



The BOD bottle curve

- L=oxidizable carbonaceous material remaining to be oxidized



$$BOD_t \equiv y_t = L_o - L_t$$

BOD Modeling

"L" is modelled as a simple 1st order decay: $$\frac{dL}{dt} = -k_1 L$$

Which leads to: $$L = L_o e^{-k_1 t}$$

And combining with: $$BOD_t \equiv y_t = L_o - L_t$$

We get: $$BOD_t \equiv y_t = L_o(1 - e^{-k_1 t})$$

- To next lecture