# Homework #7

## *BOD Problems*

1. **BOD test modeling problem I**

    If the BOD of a municipal wastewater at the end of 7 days is 60.0 mg/L and the ultimate BOD is 85.0 mg/L, what is the rate constant?

    Given: 7 day BOD = 60.0; L = 85.0 mg/L.
    Solution:

    a. Setup Eqn from lecture #23, slide 21.

    $$BOD_t = L_o(1 - e^{-kt})$$

    $$60.0 = 85.0 \, (1 - e^{-k(7)})$$

    $$0.7059 = 1 - e^{-k(7)}$$

    $$-0.2941 = -e^{-k(7)}$$

    b. Divide through by -1 and take ln of both sides

    $$\ln (0.2941) = \ln (e^{-k(7)})$$

    $$-1.224 = -k(7)$$

    $$k = 0.1748 \text{ d}^{-1}$$

2. **BOD test modeling problem II**

    Assuming that the data in Problem 1 were taken at 25°C, compute the rate constant at 16°C.

    Given:  k = 0.1748 d$^{-1}$ at 25 ºC from Problem 1.
    Solution:

    a. First convert k at 25 ºC to k at 20 ºC

    $$0.1748 = k_{20} \, (1.056)^{25 - 20}$$

$$k_{20} = \frac{0.1748}{(1.056)^5} = \frac{0.1748}{1.3132} = 0.1331$$

b. Then convert to 16 °C

$$k_{16} = (0.1331)(1.135)^{16-20}$$

$$k_{16} = (0.1331)(0.6026) = 0.0802 \text{ d}^{-1}$$

## 3. BOD WW model

The $BOD_5$ of the raw Clarksville wastewater is 225 mg/L. A long term BOD test has revealed that this wastewater also has an ultimate carbonaceous BOD of 325 mg/L. Plant effluent testing has shown that the $BOD_5$ drops by 92% across the plant. Long term tests on the effluent also show that the BOD decay coefficient is half the original value in the raw wastewater.

    a. What is the BOD decay coefficient of the raw wastewater?

    b. What is the ultimate carbonaceous BOD of the effluent wastewater?

Answer:

$$y_5 = L_0\left(1 - e^{-k_b t}\right)$$
$$k_b = 0.236 \text{ d}^{-1}$$

Answer:

$$k_b = 0.5*(0.236) = 0.118 \text{ d}^{-1}$$
$$y_5 = 0.08*(225) = 18 \text{ mg/L}$$

$$y_5 = L_0\left(1 - e^{-k_b t}\right)$$
$$L_0 = {y_5}\Big/{\left(1 - e^{-k_b t}\right)}$$
$$= {18}\Big/{1 - e^{-0.118(5)}}$$
$$= 40.4 \text{ mg/L}$$

## 4. BOD Testing

a. What sample size (in percent) is required for a $BOD_5$ of 350.0 mg/L if the oxygen consumed is to be limited to 6.00 mg/L?

b. Assume a standard $BOD_5$ test is being done with a 300 mL sample bottle. Present your answer as both required volume of sample (in mL and as sample size in percent (as asked for in the text).

Given: $BOD_5$ = 350.0 mg/L; oxygen consumption = 6.00 mg/L.
Solution:

$$Relative\ sample\ size = \frac{6\ mg/L}{350\ mg/L} x100\% = \mathbf{1.71\%}$$

$$Absolute\ sample\ size = 1.71\%\ x\ 300\ mL = \mathbf{5.14\ mL}$$

## *Stream Modeling problems*

5. **Point of mixing problem.**
   The Waramurngundi tannery with a wastewater flow of 0.011 m³/s and a $BOD_5$ of 590 mg/L discharges into Djanggawul Creek. The creek has a 10-year, 7-day low flow of 1.7 m³/s. Upstream of the Waramurungundi tannery, the $BOD_5$ of the creek is 0.6 mg/L. The BOD rate constants (k) are 0.115 day⁻¹ for the Waramurungundi tannery and 3.7 day⁻¹ for the creek. Calculate the initial ultimate BOD after mixing.

Given: Tannery $Q_w$ = 0.011 m³/s, $BOD_5$ = 590 mg/L, Creek $Q_r$ = 1.7 m³/s, $BOD_5$ upstream of tannery = 0.6 mg/L, $k_{tannery}$ = 0.115 d⁻¹, $k_{creek}$ = 3.7 d⁻¹.
Solution:

a. Calculate the ultimate BOD of tannery wastewater using equation in lecture #23, slide 21

$$L_o = \frac{590\ mg/L}{1 - e^{(-0.115)(5)}} = \frac{590\ mg/L}{1 - 0.56} = 1{,}349.2\ mg/L$$

Thus, $L_w$ = 1,350 mg/L

b. Calculate the ultimate BOD of Djanggawul Creek

$$L_o = \frac{0.6\ mg/L}{1 - e^{(-3.7)(5)}} = \frac{0.6\ mg/L}{1 - (9.24x10^{-9})} = 0.6\ mg/L$$

Thus, $L_r$ = 0.6 mg/L

c. Calculate the initial ultimate BOD by simple mass balance

3

$$L_a = \frac{Q_w L_w + Q_r L_r}{Q_w + Q_r}$$

$$L_a = \frac{(0.011 \text{ m}^3/\text{s})(1{,}349.2 \text{ mg/L}) + (1.7 \text{ m}^3/\text{s})(0.6 \text{ mg/L})}{0.011 \text{ m}^3/\text{s} + 1.7 \text{ m}^3/\text{s}} = \frac{14.84 + 1.02}{1.711}$$

$L_a = 9.269$ or $9.3$ mg/L

## 6. Parameter Estimation Problem.

a. Compute the deoxygenation rate constant and reaeration rate constant (base e) for the following wastewater and stream conditions

| Source | k (day$^{-1}$) | Temp (°C) | H (m) | Velocity (m/s) | η |
|---|---|---|---|---|---|
| Wastewater | 0.25 | 20 | | | |
| Mill River | | 20 | 2.2 | 0.7 | 0.4 |

b. In addition, calculate the values of $k_d$ and $k_r$ if the temperature is 10°C.

Solution:

a. at 20C

i. Calculate the deoxygenation rate constant using Eqn in lecture #24

$$k_d = k + \frac{u}{h}\eta$$

$k_d = 0.25 + \dfrac{0.7}{2.2}(0.4) = $ **0.377 d$^{-1}$**

ii. Calculate the reaeration rate constant using Eqn in lecture #24

$$k_r = \frac{3.9u^{0.5}}{h^{1.5}} = \frac{3.9(0.7^{0.5})}{2.2^{1.5}} = 1.00 \ d^{-1}$$

b. at 10C

i. Adjust the deoxygenation rate constant to 10C

$$k_{t2} = k_{t1}(\theta)^{(t2-t1)}$$

For CBOD Often we use: θ=1.047;  D&M cite: 1.056 for 20-30C and 1.135 for 4-20C, so:

4

| | EPA | D&M | Units |
|---|---|---|---|
| Θ = | 1.047 | 1.135 | |
| $k_d$ = | 0.2383 | 0.1063 | day-1 |

ii. Adjust the reaeration constant to 10C

$$k_{t2} = k_{t1}(\theta)^{(t2-t1)}$$

for $k_r$ we use: θ=1.024

so kr = **0.789 day$^{-1}$**

## 7. River Model for DO I.

The initial ultimate BOD after mixing in the Bergelmir River is 12.0 mg/L. The DO in the Bergelmir River after the wastewater and river have mixed is at saturation. The river temperature is 10°C. At 10°C, the deoxygenation rate constant ($k_d$) is 0.30 day$^{-1}$, and the reaeration rate constant ($k_r$) is 0.40 day$^{-1}$. Determine the critical point ($t_c$) and the critical DO.

Given: $L_a$ = 12 mg/L, DO = saturation, river temp = 10 °C, $k_d$ = 0.30 d$^{-1}$, $k_r$ = 0.40 d$^{-1}$
Solution:

a. Since the DO in the river is at saturation after the wastewater and river have mixed, the initial deficit ($D_a$) is 0.0 mg/L.

b. calculate critical travel time from M&Z equation 7.17.

$$t_c = \frac{1}{k_r - k_d} \ln\left[\frac{k_r}{k_d}\left(1 - D_a \frac{k_r - k_d}{k_d L_a}\right)\right]$$

$t_c$ = **2.88 days**

c. The critical deficit is found using M&Z 7.15 with t = $t_c$

$$D_t = \frac{k_d L_a}{k_r - k_d}\left(e^{-k_d t} - e^{-k_r t}\right) + D_a\left(e^{-k_r t}\right)$$

$$D_t = \frac{0.30(12)}{0.40 - 0.30}\left(e^{-0.30(2.88)} - e^{-0.40(2.88)}\right) + 0\left(e^{-0.40(2.88)}\right)$$

$D_t$ = 3.78 mg/L
DO = Cs-Dt = 11.33-3.78 = **7.53 mg/L**
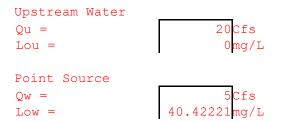
5

## 8. BOD and Toxicant River Model

Consider Monkey creek, a free-flowing stream with a mean water velocity of 0.1 ft/s. At milepoint zero, there is a discharge of 5 cfs of the Clarksville WWTP effluent. The total streamflow above this point is 20 cfs. Ignore any BOD in the upstream water. The Secchi depth for Monkey Creek is 3 ft, however, the average depth is 12 ft.

a. What is the $BOD_5$ of the river water at a point 5 miles downstream of the Clarksville outfall?

b. If the concentration of benzo[a]pyrene just above the Clarksville outfall is 23 x$10^{-6}$ mg/L, and there is no benzo[a]pyrene in the WWTP effluent, what will the concentration be 5 miles downstream? Consider that this compound undergoes photolysis ($k_{p0}$ = 1.2 d$^{-1}$). .

### Answer to a:

First, recall that the Clarksville effluent has an ultimate BOD of 40.4 mg/L:

Then consider the point source (Clarksville WWTP) and upstream flows and mix the two:

```
Upstream Water
Qu  =                      20 Cfs
Lou =                       0 mg/L


Point Source
Qw  =                       5 Cfs
Low =                 40.42221 mg/L
```

The combined flows and concentrations are:

```
Initial Values (from mass balance)
Lo  =                8.084441 mg/L
```

And the time of travel for 5 miles is:
```
t*  =      x/U  =      3.055556 D
```

So, now use the BOD plug flow model
$$L_t = L_0 e^{-k_L t^*}$$
$$= 8.08 e^{-0.118(3.06)}$$
$$= 5.64 \text{ mg/L}$$
and converting this ultimate BOD back into $BOD_5$, we get:

$$y_5 = L_0\left(1 - e^{-k_L t}\right)$$
$$= 5.64\left(1 - e^{-0.118(5)}\right)$$
$$= 2.5 \text{ mg/L}$$

**<u>Answer to b:</u>**
There are two major approaches to solving this problem.  First you can simply treat the photolysis rate constant as a fully-depth averaged value (i.e., all benzo[a]pyrene will degrade at the same average rate regardless of where it happens to be in the stream).  This is the same as saying that photolysis occurs throughout the depth of the water at the same rate as it does at the surface (represented by the $k_{po}$).  This is an oversimplification, but one that is appropriate for a "first level" approximation for the maximum amount of compound loss one could have.  Using this method, all you need is the rate constant, and the secchi depth becomes superfluous.

First, you should perform a mass balance at the point of mixing to determine the initial concentration.

$$C_0 = 18.4 \text{ ng/L}$$

Then, use the plug flow model with first order loss (in this case the loss is due only to photolysis).

$$C = C_o e^{-k_{po} t}$$
$$C = 18.4 e^{-1.2(3.06)} = 0.47 \ ng/L$$

In a more detailed analysis (of the type we cover in CEE 577), we would need to consider depth of the water and its cloudiness.  The light intensity drops with depth and is further diminished by the cloudiness that contributes to low Secchi Depth.

One approach that would be more accurate would be to divide the stream up into thin layers (say 0.5 ft each) and calculate a separate $k_p$ for each.  Then you could average these up.

Finally, use the plug flow model with first order loss (in this case the loss is due only to photolysis).

9. **River Model for DO II.**
   The town of Edinkira has filed a complaint with the state Department of Natural Resources (DNR) that the City of Quamta is restricting its use of the Umvelinqangi River because of the discharge of raw sewage.  The DNR water quality criterion for the Umvelinqangi River is 5.00 mg/L of DO.  Edinkira is 15.55 km downstream from Quamta.

a. What is the DO at Edinkira?

b. What is the critical DO and

c. Where (at what distance) downstream does the critical DO occur?

d. Is the assimilative capacity of the river restricted?

The following data pertain to the 7-day, 10-year (7Q10 – note the typo in your book) low flow at Quamta.

| Parameter | Wastewater | Umvelinqangi River | Units |
|---|---|---|---|
| Flow | 0.1507 | 1.08 | $m^3/s$ |
| $BOD_5$ | 128 | | mg/L |
| $BOD_u$ | | 11.4 | mg/L |
| DO | 1.00 | 7.95 | mg/L |
| k at 20°C | 0.4375 | | $day^{-1}$ |
| Velocity | | 0.390 | m/s |
| Depth | | 2.8 | m |
| Temperature | 16 | 16 | °C |
| Bed-activity coeff | | 0.200 | |

Given:  Table of data; Edinkira is 15.55 km downstream; standard = 5.00 mg/L

Part I.  DO at Edinkira

   a.  Calculate t

$$t = \frac{(15.55 \text{ km})(1{,}000 \text{ m/km})}{(0.390 \text{ m/s})(86{,}400 \text{ s/d})} = 0.462 \text{ d}$$

b. Calculate $k_d$

$$k_d = 0.4375 + \frac{0.390}{(2.80)}(0.200)$$

$$k_d = 0.4375 + 0.02785 = 0.4654 \text{ at 20 °C}$$

At 16 °C

$$k_d = 0.4654 \, (1.135)^{16-20}$$

$$k_d = 0.4654 \, (0.6026) = 0.2805 \text{ d}^{-1}$$

c. Calculate $k_r$

$$k_r = \frac{3.9(0.390)^{0.5}}{(2.80)^{1.5}} = 0.5212 \text{ at } 20\,°C$$

At 16 °C

$k_r = 0.5212\,(1.024)^{16-20}$

$k_r = 0.5212\,(0.9095) = 0.474\ d^{-1}$

d. Calculate $D_a$

From Table A-2 @ 16 °C $DO_s = 9.95$ mg/L

$$D_a = 9.95 - \left\{\frac{(0.1507)(1.00) + (1.08)(7.95)}{0.1507 + 1.08}\right\}$$

$D_a = 9.95 - 7.10 = 2.85$ mg/L

e. Calculate $L_w$

$$L_w = \frac{BOD_t}{(1 - e^{-(k)(t)})} = \frac{128}{(1 - e^{-(0.4375)(5)})}$$

$$L_w = \frac{128}{0.8878} = 144.17 \text{ mg/L}$$

f. Calculate $L_a$ from a simple mass balance

$$L_a = \frac{Q_w L_w + Q_r L_r}{Q_w + Q_r}$$

$$L_a = \frac{(0.1507)(144.17) + (1.08)(11.40)}{0.1507 + 1.08} = 27.65\ mg/L$$

g. Calculate Deficit (streeter phelps equation; equ #7.15)

9

$$D_t = \frac{k_d L_a}{k_r - k_d}\left(e^{-k_d t} - e^{-k_r t}\right) + D_a\left(e^{-k_r t}\right)$$

$$D = \frac{(0.2805)(27.65)}{0.474 - 0.2805} \left(e^{-(0.2805)(0.462)} - e^{-(0.474)(0.462)}\right) + 2.85\left(e^{-(0.474)(0.462)}\right)$$

$$D = \frac{7.7558}{0.1935} (0.8785 - 0.8033) + 2.85 (0.8033)$$

$$D = 3.014 + 2.289 = 5.303 \text{ or } 5.30 \text{ mg/L}$$

g. Calculate DO knowing $DO_s = 9.95$ mg/L

$$DO = 9.95 - 5.30 = \text{ }\boxed{\textbf{4.65 mg/L}}\text{ at Edinkira}$$

Part II.  Critical DO

.      a.Calculate $t_c$

$$t_c = \frac{1}{k_r - k_d}\ln\left[\frac{k_r}{k_d}\left(1 - D_a \frac{k_r - k_d}{k_d L_a}\right)\right]$$

$$t_c = \frac{1}{0.474 - 0.2805}\ln\left[\frac{0.474}{0.2805}\left(1 - 2.85\frac{0.474 - 0.2805}{(0.2805)(27.65)}\right)\right]$$

$$t_c = 5.168 \ln [1.6898 (0.9288)] = 2.33 \text{ d}$$

b.  Calculate $D_c$

$$D_c = \frac{(0.2805)(27.65)}{(0.474 - 0.2805)} \left(e^{-(0.2805)(2.33)} - e^{-(0.474)(2.33)}\right) + 2.85\left(e^{-(0.474)(2.33)}\right)$$

$$D_c = \frac{7.7558}{0.1935} (0.520188 - 0.3314) + 2.85 (0.3314)$$

$$D_c = 7.5669 + 0.9449 = 8.51 \text{ mg/L}$$

c. Calculate DO with Eqn 9-23 and $DO_s = 9.95$ mg/L

$$DO = 9.95 - 8.51 = \text{ }\boxed{\textbf{1.44 mg/L}}\text{ at critical point}$$

d. Distance to Critical Point is the product of the speed of the stream and the travel time

$$x = (v)(t) = (0.390 \text{ m/s})(86,400 \text{ s/d})(2.33 \text{ d})(1.0 \times 10^{-3} \text{ km/m})$$

$$x = \text{78.5 km} \text{ downstream from Quamta}$$

e. The assimilative capacity is restricted.

## 10. River Model for DO III.

Under the provisions of the Clean Water Act, the U.S. Environmental Protection Agency established a requirement that municipalities had to provide secondary tretment of their waste. This was defined to be treatment that resuted in an effluent $BOD_5$ that did not exceed 30 mg/L. The discharge from Quamta (Problem 9) is clearly in violation of that standard. Given the data in Problem 9, rework the problem, assuming that Quamta provides treatment to lower the $BOD_5$ to 30.00 mg/L.

Given: Data in Problem 9 but $BOD_5$ of Quamta wastewater is reduced to 30.00 mg/L
Solution:

Part I. DO at Edinkira

a. From Problem 9 the following remain the same: $t = 0.462$ d; $k_d = 0.2805$ d$^{-1}$; $k_r = 0.474$ d$^{-1}$; $D_a = 2.85$ mg/L

b. Recalculate $L_w$

$$L_w = \frac{BOD_t}{(1 - e^{-(k)(t)})} = \frac{30.00}{(1 - e^{-(0.4375)(5)})}$$

$$L_w = \frac{30.00}{0.8878} = 33.79 \text{ mg/L}$$

f. Calculate $L_a$ from the mass balance

$$L_a = \frac{(0.1507)(33.79) + (1.08)(11.40)}{0.1507 + 1.08} = 14.14 \text{ mg/L}$$

g. Calculate Deficit

$$D = \frac{(0.2805)(14.14)}{} (e^{-(0.2805)(0.462)} - e^{-(0.474)(0.462)}) + 2.85(e^{-(0.474)(0.462)})$$

$$0.474 - 0.2805$$

$$D = \frac{3.966}{0.1935}(0.8785 - 0.8033) + 2.85\,(0.8033)$$

$$D = 1.541 + 2.289 = 3.83 \text{ mg/L}$$

g. Calculate DO recognizing that $DO_s = 9.95$ mg/L

$$DO = 9.95 - 3.83 = \boxed{\textbf{6.12 mg/L}} \text{ at Edinkira}$$

Part II. Critical DO

a. Calculate $t_c$

$$t_c = \frac{1}{0.474 - 0.2805}\, ln\left[\frac{0.474}{0.2805}\left(1 - 2.85\frac{0.474 - 0.2805}{(0.2805)(14.14)}\right)\right]$$

$$t_c = 5.168 \, ln\,[1.6898\,(0.8610)] = 1.937 \text{ d}$$

b. Calculate $D_c$

$$D_c = \frac{(0.2805)(14.14)}{(0.474 - 0.2805)}(e^{-(0.2805)(1.937)} - e^{-(0.474)(1.937)}) + 2.85(10^{-(0.474)(1.937)})$$

$$D_c = \frac{3.966}{0.1935}(0.580732 - 0.399169) + 2.85\,(0.399169)$$

$$D_c = 3.72133 + 1.1376 = 4.8589 \text{ or } 4.86 \text{ mg/L}$$

c. Calculate DO and $DO_s = 9.95$ mg/L

$$DO = 9.95 - 4.86 = \boxed{\textbf{5.09 mg/L}} \text{ at critical point}$$

d. Distance to Critical Point

$$x = (v)(t) = (0.390 \text{ m/s})(86\,400 \text{ s/d})(1.937 \text{ d})(1.0 \times 10^{-3} \text{ km/m})$$

$$x = \boxed{\textbf{65.27 km}} \text{ downstream from Quamta}$$

e. The assimilative capacity is restricted but much less, so at Edinkira (6.12 - 5.00 = 1.12 mg/L above standard).

## 11. BOD Waste Load Allocation.

What amount of ultimate BOD (in kg/d) may Quamta (problem 9) discharge and still allow Edinkira 1.50 mg/L of DO above the DNR water quality criteria for assimilation of its waste?

Given: Data from Problem 9; allow 1.50 mg/L above DNR regulations at Edinkira
Solution:

a. Allowable deficit

$$D = 9.95 - (1.50 + 5.00) = 3.45 \text{ mg/L}$$

b. Allowable initial ultimate BOD

$$L_a = \frac{(D - D_a e^{(-k_r)(t)})(k_r - k_d)}{k_d(e^{-(k_d)(t)} - e^{-(k_r)(t)})}$$

$$L_a = \frac{(3.45 - 2.85(e^{(-0.474)(0.462)}))(0.474 - 0.2805)}{(0.2805)(e^{(-0.2805)(0.462)} - e^{(-0.474)(0.462)})}$$

$$L_a = \frac{(3.45 - 2.289)(0.1935)}{(0.2805)(0.07512)}$$

$$L_a = 10.66 \text{ mg/L}$$

c. Allowable ultimate BOD in discharge (solve mass balance for $L_w$)

$$L_a = \frac{Q_w L_w + Q_r L_r}{Q_w + Q_r}$$

rearranging

$$L_w = \frac{L_a(Q_w + Q_r) - Q_r L_r}{Q_w}$$

$$L_w = \frac{10.66(0.1507 + 1.08) - 1.08(11.40)}{0.1507}$$

$$L_w = 5.35 \text{ mg/L}$$

d. Allowable Mass Discharge in kg/d (Note: mg/L = g/m$^3$)

$$Q_w L_w = (0.1507 \text{ m}^3/\text{s})(5.35 \text{ mg/L})(86,400 \text{ s/d})(1 \times 10^{-3} \text{ kg/g})$$

$Q_wL_w = 69.67$  or  **70 kg/d**

## 12. Graphical Representation.

As an add-on to Problems 9, 10 and 11, produce a graph of dissolved oxygen concentration versus distance downstream from the wastewater discharge point, and show 3 plots on the graph, one for each problem. An efficient method to solve these problems is to create a generic Excel spreadsheet with cells for input of parameters in the problem and then cells for calculating the deficit and DO level as a function of travel time and travel distance. You will also find it convenient to set up your spreadsheet to calculate initial conditions for ultimate BOD and DO in the wastewater/river mixture, allowing for ease in changing loading conditions (as required in Problems 10 and 11).