

## Homework #6

### 1. Groundwater and contaminant flow (2 points)

Two groundwater wells are located 500 m apart within the same aquifer. Well #1 has a water level at 373 ft above mean sea level. The second well's water level is 401 ft above mean sea level. The mean hydraulic conductivity in this aquifer is 0.3 m/day, and the porosity is 0.45. Determine the following:

a. Darcy's velocity between the wells

$$\begin{aligned}
 v_d &= -K \left( \frac{dh}{dx} \right) \\
 v_d &= -0.3 \frac{m}{d} \left( \frac{(373 \text{ ft} - 401 \text{ ft}) \frac{1m}{3.2808 \text{ ft}}}{500m} \right) \\
 &= 0.0051 \frac{m}{d} \\
 &= 1.87 \frac{m}{yr} \\
 &= 5.9 \times 10^{-8} \text{ m/s}
 \end{aligned}$$

b. True velocity between the wells

$$\begin{aligned}
 v_{true} &= \frac{1}{\eta} v_d \\
 v_{true} &= \frac{1}{0.45} 0.0051 \frac{m}{d} \\
 &= 0.0114 \frac{m}{d} \\
 &= 4.16 \frac{m}{yr} \\
 &= 1.32 \times 10^{-7} \frac{m}{s}
 \end{aligned}$$

c. The time it takes for water to travel between the two wells

$$\begin{aligned}
 t &= \frac{\Delta X}{v_{true}} \\
 &= \frac{500m}{4.16 \frac{m}{yr}} \\
 &= 120yr
 \end{aligned}$$

- d. The time it takes for a contaminant to travel between the two wells. Assume that the contaminant has an affinity for the soil such that at any given time, 80% of it will be adsorbed and 20% will be dissolved in the water.

The contaminant is therefore traveling at the “true” water velocity only 20% of the time, and it is stationary 80% of the time.

$$\begin{aligned}
 v_{contaminant} &= 0.20 * v_{true} \\
 &= 0.20 * 0.0114 \frac{m}{d} \\
 &= 0.00228 \frac{m}{d} \\
 &= 0.83 \frac{m}{yr}
 \end{aligned}$$

$$\begin{aligned}
 t_{contaminant} &= \frac{\Delta X}{v_{contaminant}} \\
 &= \frac{500m}{0.83 \frac{m}{yr}} \\
 &= 600yr
 \end{aligned}$$

- e. Calculate the partition coefficient (K) for this contaminant in units of L/kg. Use the information in part “d”, the porosity, and assume an aquifer soil particle density ( $\rho_s$ ) of 2.2 g/cm<sup>3</sup>.

There are many ways to solve this problem. Here is one of them, where I walk you through it step by step.

Your text (e.g., pg 95) defines the partition coefficient as:

$$K = \frac{q}{C} = \frac{\text{mass compound adsorbed per mass of adsorbent} \left( \frac{mg}{g} \right)}{\text{concentration compound in the aqueous phase} \left( \frac{mg}{L} \right)}$$

In the slides I expressed it in term of moles and used it in the context of soil as the adsorbent

$$K_d = \frac{C_s (\text{moles} / \text{kg} - \text{soil})}{C_w (\text{moles} / \text{L} - \text{water})}$$

In both cases the partition coefficient is the ratio of the amount of compound adsorbed to the solid to the concentration in the water that is in equilibrium with it. This is conceptually the same as the ratio in problem d. For example, problem “d” would suggest that out of a 100  $\mu\text{moles}$ , 80 are adsorbed to the soil at any given time and 20 are dissolved in the water. This ratio then differs from the classical  $K_d$  in its units. A dimensional analysis would tell us that for a total aquifer volume of 1 liter:

$$K_d = \frac{(80 \mu\text{moles} / \text{L-total}) (x^{L\text{-total} / \text{kg-soil}})}{(20 \mu\text{moles} / \text{L-total}) (y^{L\text{-total} / \text{L-water}})}$$

The problem is in determining the values of “x” (the number of liters of total volume occupied by a kilogram of aquifer soil, and “y” the liters of total volume occupied by a liter of aquifer water. Both pertain to the porosity as follows. The “y” value is simply the inverse of the porosity:

$$y^{(L\text{-total} / \text{L-water})} = \frac{1}{\eta} = \frac{1}{0.45} = 2.222$$

The “x” value is one minus the porosity and divided by the density

$$x^{(L\text{-total} / \text{kg-soil})} = \frac{1 - \eta}{\rho_s} = \frac{1 - 0.45}{2.2} = 0.25$$

And substituting in:

$$K_d = \frac{(80 \mu\text{moles} / \text{L-total}) (0.25^{L\text{-total} / \text{kg-soil}})}{(20 \mu\text{moles} / \text{L-total}) (2.22^{L\text{-total} / \text{L-water}})} = 0.45 \frac{\text{L}}{\text{kg}}$$

This assumes that the density I gave you is the specific density (without the pores) rather than the bulk soil density (which includes the empty pores).

Alternatively, you could take the equation (recall that  $v_{\text{water}}$  is the same as  $v_{\text{true}}$ ).

$$R \equiv \frac{v'_{\text{water}}}{v'_{\text{cont}}} = \frac{1}{0.2} = 5$$

And, we can rearrange:

$$R = 1 + K_d \left( \frac{\rho_s}{\eta(1-\eta)} \right) = 1 + K_d \left( \frac{\rho_b}{\eta} \right)$$

$$K_d = [R - 1] \left( \frac{\eta(1-\eta)}{\rho_s} \right) = [5 - 1] \left( \frac{0.45(1-0.45)}{2.2 \text{ g} / \text{cm}^3} \right) = 0.45 \text{ cm}^3 / \text{g} = 0.45 \text{ L} / \text{kg}$$

Note that if you presumed the density of 2.2 was a bulk density, you would have gotten:

$$K_d = [R - 1] \left( \frac{\eta}{\rho_b} \right) = [5 - 1] \left( \frac{0.45}{2.2 \text{ g/cm}^3} \right) = 0.82 \text{ L/kg}$$

## 2. Lake Modeling (1 point)

Julana Lake has a surface area of  $4.1 \times 10^6 \text{ m}^2$ . The average depth of the lake is 15 m. The lake is fed by a stream having a flow rate of  $2.02 \text{ m}^3/\text{s}$  and a phosphorus concentration of  $0.023 \text{ mg/L}$ . A wastewater treatment plant discharges into the lake at a rate of  $0.2 \text{ m}^3/\text{s}$  and a phosphorus concentration of  $1.1 \text{ mg/L}$ . Runoff from the homes along the lake adds phosphorus at an average rate of  $1.35 \text{ g/s}$ . The settling rate of phosphorus from the lake averages  $0.94 \text{ y}^{-1}$ . The river flows from the lake at a flow rate of  $2.42 \text{ m}^3/\text{s}$ . Assume evaporation and precipitation to negate each other. Assume steady state conditions apply.

- Make a clear sketch of the problem
- What is the concentration of phosphorus in the river flowing from the lake?
- Calculate the expected phosphorus concentration leaving the lake if there was a reduction in phosphorous concentration in the wastewater to  $0.2 \text{ mg/L}$ .

Given (from above): Surface area =  $4.1 \times 10^6 \text{ m}^2$ , depth = 15 m,  $Q_{\text{in}} = 2.02 \text{ m}^3/\text{s}$ ,  $P_{\text{in}} = 0.023 \text{ mg/L}$ ,  $Q_{\text{ww}} = 0.2 \text{ m}^3/\text{s}$ ,  $P_{\text{ww}} = 1.1 \text{ mg/L}$  or  $0.2 \text{ mg/L}$ , runoff =  $1.35 \text{ g/s}$ , settling rate =  $0.94 \text{ y}^{-1}$ ,  $Q_{\text{out}} = 2.42 \text{ m}^3/\text{s}$ , assuming evaporation and precipitation negate one another and system is steady state.

Solution:

- Based on a steady state mass balance approach

change = inflow – reaction - outflow

$$0 = P_{\text{in}}Q_{\text{in}} - k_s P_{\text{out}} \nabla - P_{\text{out}}Q_{\text{out}}$$

$$P_{\text{in}}Q_{\text{in}} = [(1.1 \text{ mg/L})(0.2 \text{ m}^3/\text{s}) + (0.023 \text{ mg/L})(2.02 \text{ m}^3/\text{s})] \left( \frac{1000 \text{ L}}{\text{m}^3} \right) \left( \frac{1}{1000 \text{ mg}} \right) + 1.35 \text{ g/s}$$

$$P_{\text{in}}Q_{\text{in}} = 0.22 \text{ g/s} + 0.04646 \text{ g/s} + 1.35 \text{ g/s} = 1.616 \text{ g/s} = 5.1 \times 10^7 \text{ g/yr}$$

$$k_s P_{\text{out}} \nabla = (0.94 \text{ y}^{-1}) \left( \frac{1 \text{ y}}{365 \text{ d}} \right) \left( \frac{1 \text{ d}}{24 \text{ h}} \right) (4.16 \times 10^6 \text{ m}^2) (15 \text{ m}) P_{\text{out}} = (1.833 \text{ m}^3/\text{s}) P_{\text{out}}$$

$$365 \text{ d} \quad 86400 \text{ s}$$

$$P_{\text{out}}Q_{\text{out}} = (2.42 \text{ m}^3/\text{s})P_{\text{out}}$$

Substituting

$$0 = 1.616 \text{ g/s} - (1.833 \text{ m}^3/\text{s})P_{\text{out}} - (2.42 \text{ m}^3/\text{s})P_{\text{out}}$$

$$1.616 \text{ g/s} = (4.253 \text{ m}^3/\text{s})P_{\text{out}}$$

$$P_{\text{out}} = 0.380 \text{ g/m}^3 = \mathbf{0.380 \text{ mg/L}}$$

Impact of a reduction in phosphorous concentration in the wastewater to 0.2 mg/L

$$P_{\text{in}}Q_{\text{in}} = [(0.2 \text{ mg/L})(0.2 \text{ m}^3/\text{s}) + (0.023 \text{ mg/L})(2.02 \text{ m}^3/\text{s})]\left(\frac{1000 \text{ L}}{\text{m}^3}\right)\left(\frac{1}{1000 \text{ mg}}\right) + 1.35 \text{ g/s}$$

$$P_{\text{in}}Q_{\text{in}} = 0.04 \text{ g/s} + 0.04646 \text{ g/s} + 1.35 \text{ g/s} = 1.436 \text{ g/s}$$

$$k_s P_{\text{out}} \nabla = (0.94 \text{ y}^{-1})\left(\frac{1 \text{ y}}{365 \text{ d}}\right)\left(\frac{1 \text{ d}}{86400 \text{ s}}\right)(4.16 \times 10^6 \text{ m}^2)(15 \text{ m})P_{\text{out}} = (1.833 \text{ m}^3/\text{s})P_{\text{out}}$$

$$P_{\text{out}}Q_{\text{out}} = (2.42 \text{ m}^3/\text{s})P_{\text{out}}$$

Substituting

$$0 = 1.436 \text{ g/s} - (1.833 \text{ m}^3/\text{s})P_{\text{out}} - (2.42 \text{ m}^3/\text{s})P_{\text{out}}$$

$$1.436 \text{ g/s} = (4.253 \text{ m}^3/\text{s})P_{\text{out}}$$

$$P_{\text{out}} = 0.338 \text{ g/m}^3 = \mathbf{0.338 \text{ mg/L}}$$