EOQ and Beyond

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Best Buy Example

- **Bust Buy**
  - **Annual Demand 2,500 computers**
  - **A computer costs $500**
  - **Holding cost, assume 15% annual interest**
  - **Transportation setup cost $3,000 per trip**
  - **How much is the total cost if BB orders**
    - **Once a year?**
    - **Twice a year?**
    - **…**

Gateway.

BEST BUY

Once a Year

- Order 2500 computers once in a year
- Transportation cost
  - $3,000
- Inventory cost
  - $187,500
- Total Cost
  - $190,500

11/15/2005

Twice a Year

- Order 1250 computers twice in a year
- Transportation cost
  - $6,000
- Inventory cost
  - $93,750
- Total Cost
  - $99,750

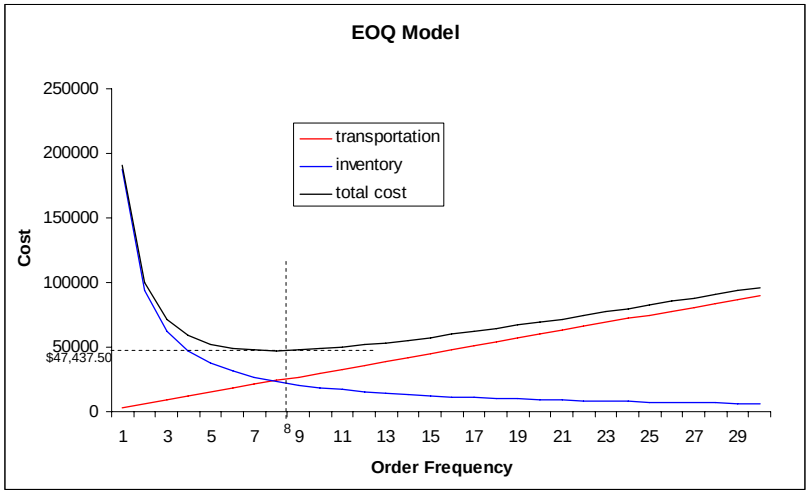
11/15/2005

Optimal Order?

Demand= 2500 Trip cost=3000 price=500 Interest=15%

Frequency	Quantity	Transportation	Inventory	Total cost
1	2500.00	$3,000.00	$187,500.00	$190,500.00
2	1250.00	$6,000.00	$93,750.00	$99,750.00
3	833.33	$9,000.00	$62,500.00	$71,500.00
4	625.00	$12,000.00	$46,875.00	$58,875.00
5	500.00	$15,000.00	$37,500.00	$52,500.00
6	416.67	$18,000.00	$31,250.00	$49,250.00
7	357.14	$21,000.00	$26,785.71	$47,785.71
8	312.50	$24,000.00	$23,437.50	$47,437.50
9	277.78	$27,000.00	$20,833.33	$47,833.33
10	250.00	$30,000.00	$18,750.00	$48,750.00
11	227.27	$33,000.00	$17,045.45	$50,045.45
12	208.33	$36,000.00	$15,625.00	$51,625.00
13	192.31	$39,000.00	$14,423.08	$53,423.08
14	178.57	$42,000.00	$13,392.86	$55,392.86
15	166.67	$45,000.00	$12,500.00	$57,500.00
16	156.25	$48,000.00	$11,718.75	$59,718.75
17	147.06	$51,000.00	$11,029.41	$62,029.41
18	138.89	$54,000.00	$10,416.67	$64,416.67
19	131.58	$57,000.00	$9,868.42	$66,868.42
20	125.00	$60,000.00	$9,375.00	$69,375.00
21	119.05	$63,000.00	$8,928.57	$71,928.57
22	113.64	$66,000.00	$8,522.73	$74,522.73
23	108.70	$69,000.00	$8,152.17	$77,152.17
24	104.17	$72,000.00	$7,812.50	$79,812.50
25	100.00	$75,000.00	$7,500.00	$82,500.00
26	96.15	$78,000.00	$7,211.54	$85,211.54
27	92.59	$81,000.00	$6,944.44	$87,944.44
28	89.29	$84,000.00	$6,696.43	$90,696.43
29	86.21	$87,000.00	$6,465.52	$93,465.52
30	83.33	$90,000.00	$6,250.00	$96,250.00

Optimal Order?



Formulation

- **Variables**
  - **A – Fixed cost per trip**
  - **D – Annual Demand**
  - **C – Cost per item**
  - **h – Holding cost h=i*C, interest rate i**
  - **Q – order quantity**
- **Total Cost**

$$Y(Q) = \frac{AD}{Q} + hQ$$

Best Answer

- **Best answer fund at**

$$dY(Q)/dQ = 0$$

$$-AD/Q^2 + h = 0$$

$$Q^* = \sqrt{AD/h}$$

- **Best Buy Example**
  - **Q* = sqrt(3000*2500/(0.15*500))**
  - **Q* ≅ 316**
  - **Frequency ≅ 8**

EOQ Takeways

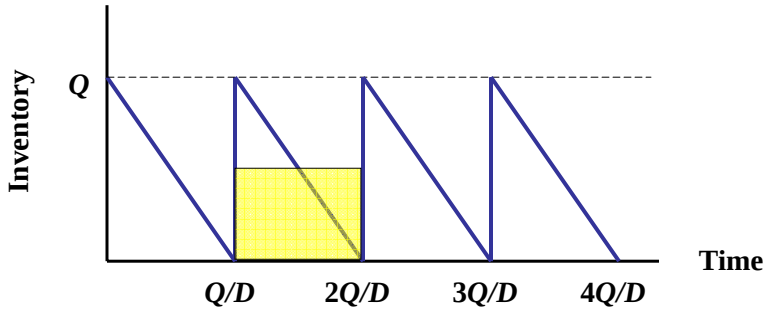
- Order quantity causes inventory
  - i.e. large size translates into more stock
- Under certain assumptions, the quantity that optimally balances inventory cost and transportation cost is given by:

$$Q^* = \sqrt{AD / h}$$

What's Wrong with EOQ?

- Constant inventory
  - Computers are sold over time
  - Inventory depletes
  - Until refill by next order
- Deterministic demand
  - Demand actually uncertain
  - Order more
    - stock cost goes up
    - risk of obsolescence
  - Order less
    - Loose business opportunity

Constant Inventory?



Average Inventory = Q/2

Re-Formulation

- **Total Cost**

$$Y(Q) = \frac{AD}{Q} + hQ/2$$

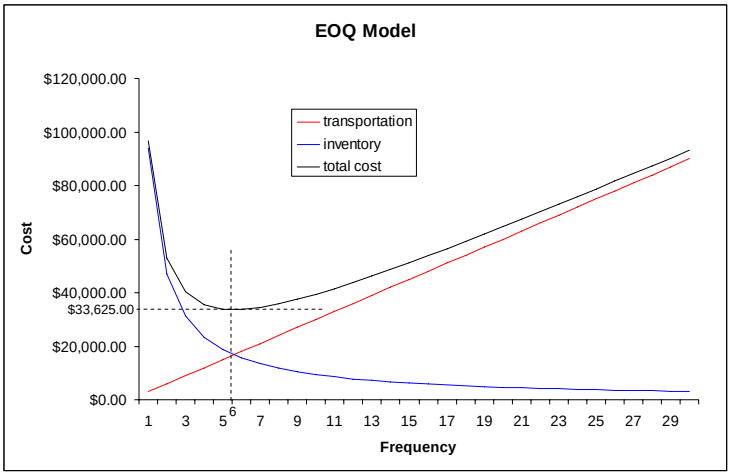
- **Best Answer**

$$dY(Q)/dQ = 0$$

$$-AD/Q^2 + h/2 = 0$$

$$Q^* = \sqrt{2AD/h}$$

Re-Formulation



Deterministic Demand?

- Newsvendor Problem
  - Newspapers – rapid obsolescence
  - Christmas trees – seasonal items
- Nature of the Problem
  - Demand unknown in advance – distribution
  - Order once per cycle
  - If not sold, depreciate or no value
  - If shortage, loose business
  - Order quantity difficult to determine

Formulation

X = demand (in units), a random variable.

$G(x) = P(X \le x)$, cumulative distribution function of demand (assumed continuous.)

$g(x) = \frac{d}{dx} G(x)$ = density function of demand.

c_o = cost (in dollars) per unit left over after demand is realized.

c_s = cost (in dollars) per unit of shortage.

Q = production/order quantity (in units); this is the decision variable.

Formulation

- **Cost Function:**

$$Y(x) = \text{expected overage} + \text{expected shortage cost}$$

$$= c_o E[\text{units over}] + c_s E[\text{units short}]$$

$$= c_o \int_0^\infty \max\{Q - x, 0\} g(x) dx + c_s \int_0^\infty \max\{x - Q, 0\} g(x) dx$$

$$= c_o \int_0^Q (Q - x) g(x) dx + c_s \int_Q^\infty (x - Q) g(x) dx$$

Formulation

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$$= c_o \int_0^Q (Q - x) g(x) dx + c_s \int_Q^\infty (x - Q) g(x) dx$$

Optimal Solution

- **Optimal Solution:** taking derivative of *Y*(*Q*) with respect to *Q*, setting equal to zero, and solving yields:

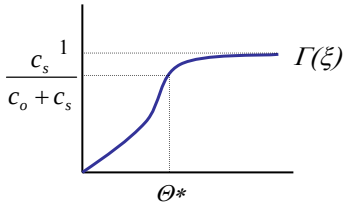
$$G(Q^*) = P\{X \le Q^*\} = \frac{c_s}{c_o + c_s}$$

Critical Ratio is probability stock covers demand

- **Notes:**

$Q^* \downarrow c_o$

$Q^* \uparrow c_s$



Example: Xmas Gift

- **Scenario:**
  - **Demand for Xmas gift is exponential with mean 1000 (i.e., $G(x) = P(X \le x) = 1 - e^{-x/1000}$).**
  - **Cost of gift is $10.**
  - **Selling price is $15.**
  - **Unsold gift can be sold off at $8.**

- **Model Parameters:**

  c_s = 15 – 10 = $5

  c_o = 10 – 8 = $2

Solution

- **Solution:**

$$G(Q^*) = 1 - e^{-\frac{Q}{1000}} = \frac{c_s}{c_o + c_s} = \frac{5}{2+5} = 0.714$$

$$Q^* = 1{,}253$$

- **Sensitivity:** If c_o = $10 (i.e., shirts must be discarded) then

$$G(Q^*) = 1 - e^{-\frac{Q}{1000}} = \frac{c_s}{c_o + c_s} = \frac{5}{10+5} = 0.333$$

$$Q^* = 405$$

Newsvendor Takeways

- Inventory is a hedge against demand uncertainty.
- Amount of protection depends on "overage" and "shortage" costs, as well as distribution of demand.
- If shortage cost exceeds overage cost, optimal order quantity generally increases.

Questions?

More Information:

http://www.factoryphysics.com/