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Damage accumulation model for aluminium-closed cell foams subjected to fully reversed cyclic loading

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ABSTRACT Although metal foams are a relatively new material, substantial knowledge has been accumulated about their mechanical properties and behaviour under monotonic loads and tension–tension and compression–compression cyclic loads. However, there are very few reports of the behaviour of metal foams under tension–compression-reversed loading. In this paper, we examine some of the rare published data regarding the tension–compression cyclic response of metal foams, develop a statistical model of the fatigue lifetime and propose two damage accumulation models for aluminium-closed cell foams subjected to a fully reversed cyclic loading. In developing these models a fatigue analysis and a failure criterion for the material are needed; the fatigue models considered are the Coffin–Manson and the statistical Weibull model, and the failure criterion used is the one described by Ingraham *et al.* (Ingraham, M.D., DeMaria, C.J., Issen, K.A. and Morrison, D.J.L. (2009). Mater. Sci. Eng. A. 504:150–156). The models developed are compared with the experimental published data by Ingraham *et al.* (Ingraham, M.D., DeMaria, C.J., Issen, K.A. and Morrison, D.J.L. (2009). Mater. Sci. Eng. A. 504:150–156) and a final analysis was performed to determine whether it is preferable to use the total or plastic strain amplitude for the fatigue analysis.

Keywords damage accumulation; fatigue model; metal foams.

NOMENCLATURE

A, A_1, A_2 = Damage accumulation model parameters
B = Threshold value of lifetime
C = Threshold value of strain amplitude
c = Fatigue ductility exponent
D = Damage accumulation model
H_C = Compressive pre-peak slope
H_T = Tensile pre-peak slope
h = Function that provide the damage accumulation model
N = Number of cycles
N^* = Number of cycles at failure level
N_f = Number of cycles to failure
p = Probability of failure
R = Damage level
R_0 = Initial damage level
R^* = Damage level at failure
v_i = Variables of the damage accumulation model
β = Weibull shape parameter

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ε = Strain amplitude
ε_a = Total Strain amplitude
ε_{ae} = Elastic strain amplitude
ε_{pa} = Plastic strain amplitude
ε_f' = Fatigue ductility coefficient
δ = Scale factor
λ = Parameter defining the position of the corresponding

INTRODUCTION AND MOTIVATION

Metal foams are a relatively new material, very light and with good strength and stiffness to weight ratios[1–4] that make them useful for different applications in the automotive and aerospace industries, among others. The material has usually been used in applications in which the main demands are monotonic, and usually compressive, yet increasingly, applications are being identified where the material must resist cyclic loading, and the fatigue response of the material must therefore be more fully understood. In this paper, we propose a statistical model for the fatigue lifetime and a damage accumulation model for metal foams subjected to fully reversed cyclic loading. Towards this aim, an analysis of the fatigue behaviour and a failure criterion are necessary. The fatigue behaviour will be modelled using the Coffin–Manson model[5,6] and the statistical Weibull model,[7–10], with the failure criterion provided by Ingraham *et al*.[11] The model proposed in this paper allows us to obtain directly the damage accumulation in metal foams as a function of the number of cycles applied, the total or plastic strain amplitude and the initial value of the damage accumulation.

Fatigue behaviour of metal foams

In the last few years, several authors have dedicated substantial effort to determine the mechanical properties of metal foams. Regarding fatigue, nearly all of these efforts have been focused on tension–tension and compression–compression cyclic loading. The results of this research can be summarized as follows: for tension–tension, the specimen progressively elongates with increasing fatigue cycles until separation of the material (approximately at an accumulated strain of 1%).[1–4] In compression–compression fatigue loading, the material response is a progressive shortening to larger plastic strains (50%) with the fatigue life defined as the number of cycles at the point where the strain accumulation accelerates on a log–log plot of strain versus number of cycles (around 2% of accumulated strain).[1–4]

For fully reversed load cycles, the research and literature regarding fatigue and damage accumulation in metal foams are not as deep as in the cases of tension–tension and compression–compression. Ingraham *et al*.,[11] performed an experimental fatigue characterization of the response of closed cell aluminium foams to reversed cyclic loading. They introduced and defined the concept of a fatigue life criterion as follows: failure occurs in the cycle when the ratio H_C/H_T reaches the value 1.5, where H_C is the compressive pre-peak slope and H_T is the tensile pre-peak slope of the hysteresis loop. Using the data published by Ingraham *et al*.[11] we extend their treatment by introducing simple models for the rate of damage accumulation in the material and a statistical treatment of the fatigue lifetime.

The paper is structured as follows: first, the fatigue analysis model used in this research (Coffin–Manson model and the statistical Weibull model) is introduced, then the damage accumulation models will be obtained, next and using the Ingraham *et al*. [11] data, an example of application will be presented followed by a discussion of the results and finally the conclusions of the research are presented.

FATIGUE ANALYSIS MODEL

There exist three major approaches to analyzing fatigue in engineering materials: the stress-based approach, the strain-based approach and the fracture mechanics approach. Each one has specific advantages and limitations in practical applications. In this research we consider low-cycle fatigue with high-strain amplitudes, and therefore use the strain-based approach using the Coffin–Manson model and a statistical Weibull model.

Coffin–Manson model

The traditional deterministic model for low-cycle fatigue analysis uses the total strain amplitude which is obtained by summing the elastic strain amplitude (ε_{ea})[12], and the plastic strain amplitude (ε_{pa}) (Eq. (1))[13].

$$\varepsilon_a = \varepsilon_{ea} + \varepsilon_{pa}. \tag{1}$$

However, a previous analysis of the data used for this research,[11] showed that the applied strains are

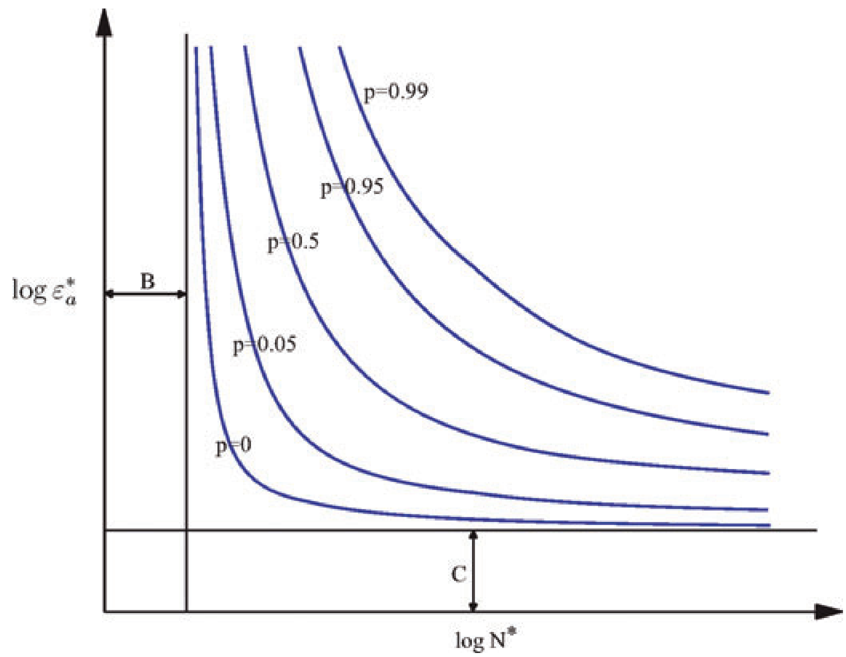

Fig. 1 Percentile curves representing the relationship between lifetime, N^* and strain range ε_a^*, in the $\varepsilon-N$ field for the fatigue model.

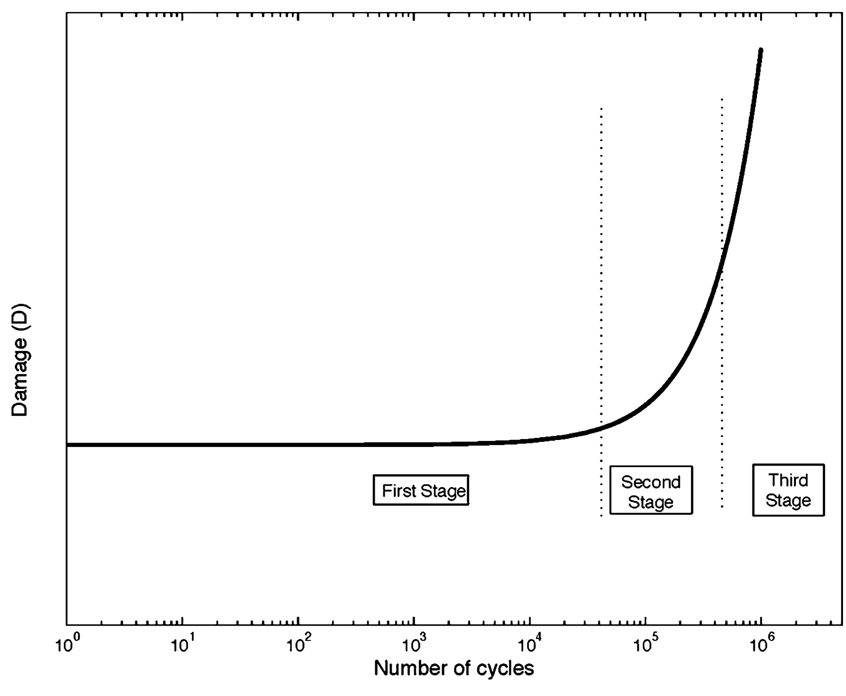

Fig. 2 Schematic damage accumulation curve.

predominantly in the plastic domain of the strain life curve. Thus a fatigue model can be developed based only on the plastic component of the applied strain amplitude. The model used to relate the applied plastic strain to the fatigue life is the Coffin–Manson model and can be expressed as

$$\varepsilon_{pa} = \varepsilon'_f (2N_f)^c. \tag{2}$$

where ε'_f is the fatigue ductility coefficient, c is the fatigue ductility exponent and N_f is the number of cycles to failure.

Statistical Weibull model for fatigue life analysis

The statistical and physical conditions that constrain the model are the weakest link principle, stability, limit behaviour, limited range and compatibility. These conditions ensure that the fatigue life of a system is governed by the shortest fatigue life of its constituent components (weakest link), that the distribution chosen for the fatigue life is valid over a broad range of specimen sizes with only the parameters being modified to account for size difference (stability), that the model displays a lower bound on

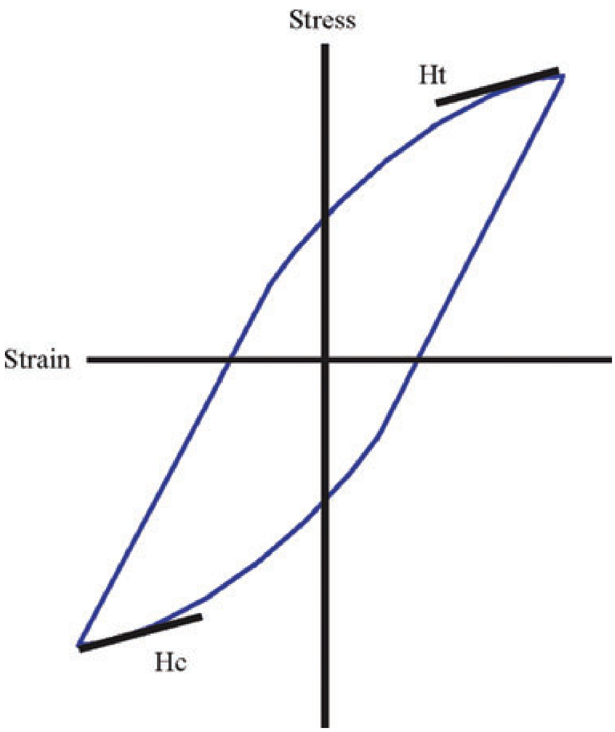

Fig. 3 Schematic definitions of H_C and H_T according to Ingraham *et al*.

Table 1 Strain-life data obtained experimentally by Ingraham *et al*.[11]

Total strain amplitude (%)	Plastic strain amplitude (%)	Cycles to failure (N^*)
0.050	0.013	251 989
0.050	0.013	248 336
0.050	0.013	264 911
0.075	0.029	42 924
0.075	0.029	33 760
0.075	0.026	41 407
0.100	0.045	13 798
0.100	0.045	12 871
0.100	0.045	11 352
0.100	0.043	16 700
0.100	0.044	11 500
0.100	0.045	6 200
0.175	0.104	710
0.175	0.103	720
0.175	0.102	855
0.250	0.166	515
0.250	0.165	350
0.250	0.161	155
0.500	0.393	20
0.500	0.388	55
0.500	0.376	130

the fatigue life and a fatigue threshold if appropriate (limit behaviour and limited range), and that the cumulative distribution functions (cdfs) of the fatigue life with respect to the applied strain and the applied strain with respect to fatigue life are consistent with one another (compatibility). These conditions are motivated by rigorous mathematics and physical reasoning.[9,14]

The only model that satisfies all of these conditions is the Weibull model.[7–10] Once the Weibull model has been chosen, the compatibility condition is applied to ensure that the cdf of the fatigue life, conditional upon the strain amplitude ($E(N^*;\, \varepsilon_a^*)$), is compatible with the cdf of the strain amplitude conditional upon the number of cycles ($F(\varepsilon_a^*;\, N^*)$):

$$E\left(N^*;\varepsilon_a^*\right) = F\left(\varepsilon_a^*;N^*\right). \tag{3}$$

Since the cdfs must be Weibull to satisfy the conditions of the model, we can rewrite Eq. (3) obtaining the following functional equation:

$$\left[\frac{\varepsilon_a^* - \lambda(N^*)}{\delta(N^*)}\right]^{\beta(N^*)} = \left[\frac{N^* - \lambda\left(\varepsilon_a^*\right)}{\delta\left(\varepsilon_a^*\right)}\right]^{\beta(\varepsilon_a^*)} \tag{4}$$

where $\lambda(N^*)$, $\delta(N^*)$, $\beta(N^*)$, $\lambda(\varepsilon_a^*)$, $\delta(\varepsilon_a^*)$ and $\beta(\varepsilon_a^*)$ are the unknown functions to be determined.

This problem has already been solved[15] and only has two general solutions. One of them has been discarded because it contains contradictions to basic principles of fracture mechanics, namely that the fatigue life should not become independent of the applied strain amplitude even when the fatigue threshold is approached. The remaining solution is

$$F\left(N^*;\varepsilon_a^*\right) = 1 - \exp\left\{-\left[\frac{\log\left(N/N_0\right)\log\left(\varepsilon_a^*\middle|\varepsilon_{a0}^*\right) - \lambda}{\delta}\right]^{\beta}\right\} \tag{5}$$

where

$$\lambda(N^*) = \left(\frac{\lambda}{N^* - B}\right); \quad \lambda\left(\varepsilon_a^*\right) = \left(\frac{\lambda}{\varepsilon_a^* - C}\right);$$
$$\delta(N^*) = \left(\frac{\delta}{N^* - B}\right); \quad \delta\left(\varepsilon_a^*\right) = \left(\frac{\delta}{\varepsilon_a^* - C}\right); \tag{6}$$
$$\beta(N^*) = \beta; \quad \beta\left(\varepsilon_a^*\right) = \beta;$$

are the unknown functions of the solution. From these expressions we obtain the cdf of the lifetime

$$F\left(N^*;\varepsilon_a^*\right) = 1 - \exp\left\{-\left[\frac{(N^* - B)\left(\varepsilon_a^* - C\right)}{\delta}\right]^{\beta}\right\} \tag{7}$$

in terms of the threshold value of lifetime $B = \log(N_0)$, the endurance limit $C = \log(\varepsilon_{a0})$, the Weibull shape parameter β of the cdf in the $\varepsilon - N$ field, the scale factor δ and

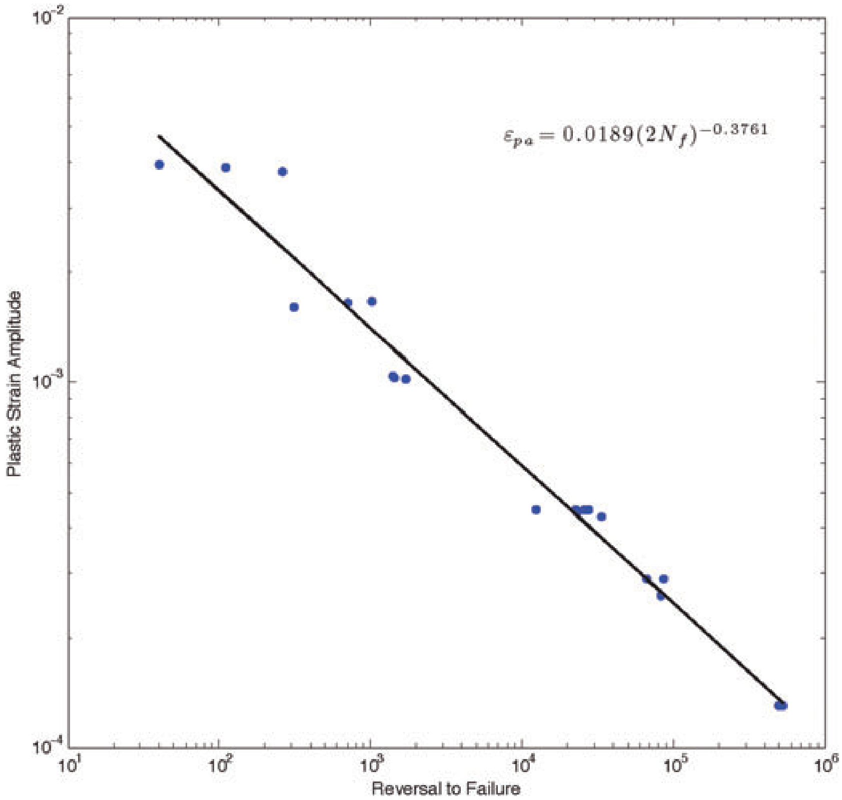

Fig. 4 *S–N* curve of the results published by Ingraham *et al.*[11] for plastic strain amplitude, using the Coffin–Manson model.

the parameter λ defining the position of the corresponding zero-percentile curve. After further manipulation, the following expression is obtained:

$$F\left(\log N^*; \log \varepsilon_a^*\right) = 1 - \exp\left\{-\left[\frac{\left(\log N^* - B\right)\left(\log \varepsilon_a^* - C\right)}{\delta}\right]^\beta\right\},$$

$$\log N^* \geq B + \frac{\lambda}{\log \varepsilon_a^* - C} \tag{8}$$

where B, C, λ, δ and β are the non-dimensional model parameters defined above. The parameters are divided in two categories, the threshold (B and C) and the Weibull parameters (β, δ and λ). The authors of the model proposed a two-stage parameter estimation procedure, first the threshold parameters are estimated using a constrained least squares method, and then the Weibull parameter can be estimated using a maximum likelihood method[7–10]

The percentile curves

$$N^* = \exp\left[B + \frac{\lambda + \delta(-\log(1-p))^{1/\beta}}{\log \varepsilon_a^* - C}\right] \tag{9}$$

are shown in Fig. 1. The zero-percentile curve represents the minimum number of cycles to produce failure for different values of ε_a^*, and the figure also shows the graphical interpretation of the model parameters B and C. Finally, the figure shows how the model directly provides, through the percentile curves, a statistical description of the strain-life curves for a given material.

DAMAGE ACCUMULATION

The fatigue analysis model provides only a description of the number of cycles to failure at a given strain amplitude but does not describe how the material damage state evolves from that in the virgin material up to the point of failure. A damage accumulation model is therefore necessary to describe the evolution of the damage state in the material with number of cycles. We show in this section that even very simple models for the rate of damage accumulation provide a good fit to the experimental data.

The damage accumulation model can be expressed in its most general form as

$$D = h\left(v_1, v_2, \ldots, v_n\right) \tag{10}$$

where D is the damage accumulation level, h is the function that provides the level of damage and v_i are the variables that affect the damage level of the material.

The damage accumulation model provides the damage evolution curve of the material. This curve will have the behaviour shown in Fig. 2, which consists of three regimes of damage accumulation. The first stage has very little or

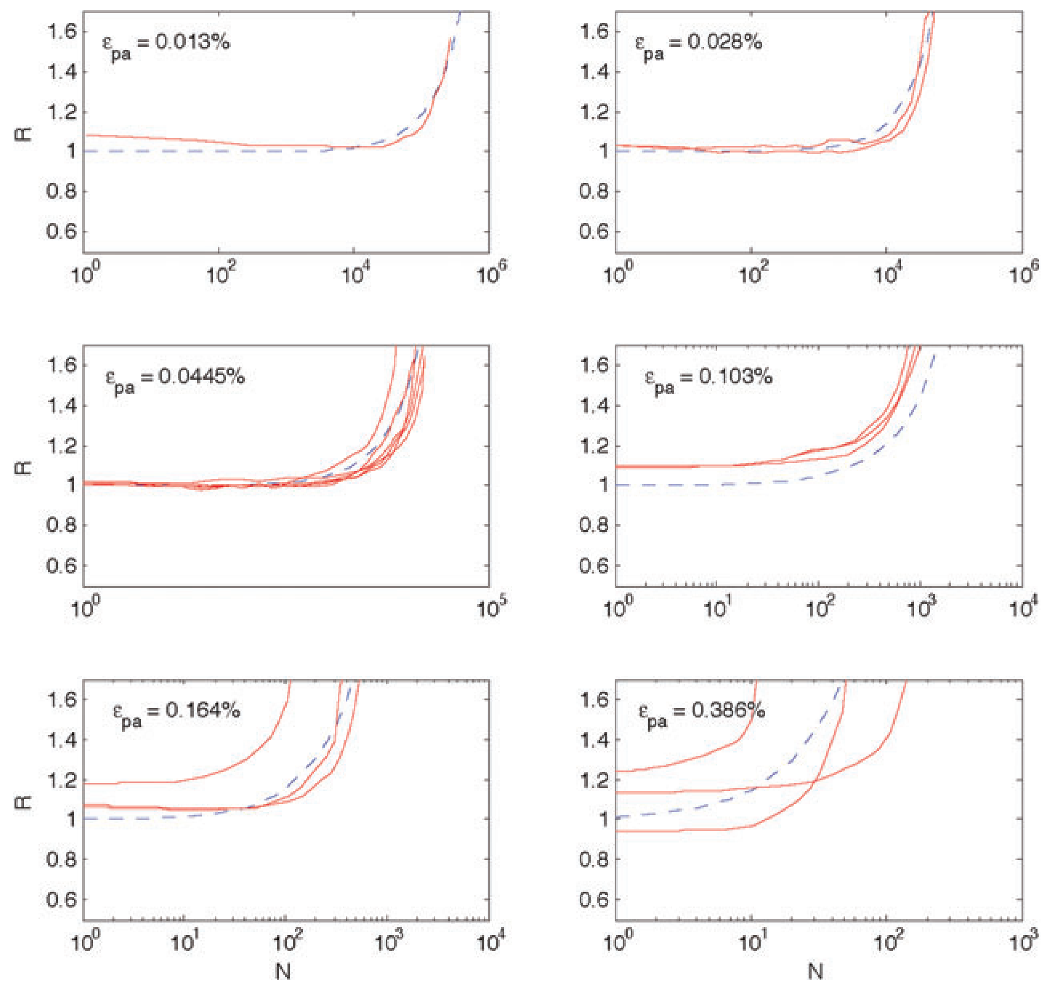

Fig. 5 Damage accumulation curves for different strain amplitudes obtained from the linear approach using the plastic strain amplitudes and the Coffin–Manson fatigue model.

zero slope over a long period of cycles. In the second regime the rate of damage accumulation starts to increase more rapidly, and in the final stage the maximum rate of damage accumulation is reached, and damage proceeds until material failure.

Damage accumulation in metal foams

In developing the damage accumulation model we make use of a set of published strain-life data and damage accumulation curves for a closed cell aluminium foam.[11] Since it is difficult to track standard measures of damage such as the length of the dominant fatigue crack in metal foams, a new definition of the damage state is required. In Ingraham *et al*.[11] the authors propose an innovative measure of the damage state in a closed cell aluminium foam undergoing cyclic loading.

Let H_C be the compressive pre-peak slope of the stress–strain curve, and let H_T be the tensile pre-peak slope of the stress–strain curve (Fig. 3). Ingraham *et al*. propose that the ratio $R = H_C/H_T$, which is initially close to unity, increases as the material becomes more damaged, and failure occurs when $R = 1.5$. This characterization provides a convenient scalar measure of the complicated damage process in metal foams, although it should be noted that without further testing its use should be restricted to the evaluation of damage under fully reversed cyclic loading.

Proposed damage accumulation model for metal foams

The damage accumulation model should be developed from fundamental mechanics or empirically by fitting an accumulation model to experimental data. In this case we choose an empirical approach given the relative paucity of data available for validating a mechanics-based model, and the very complicated mechanics occurring at the microscale during fatigue in metal foams.

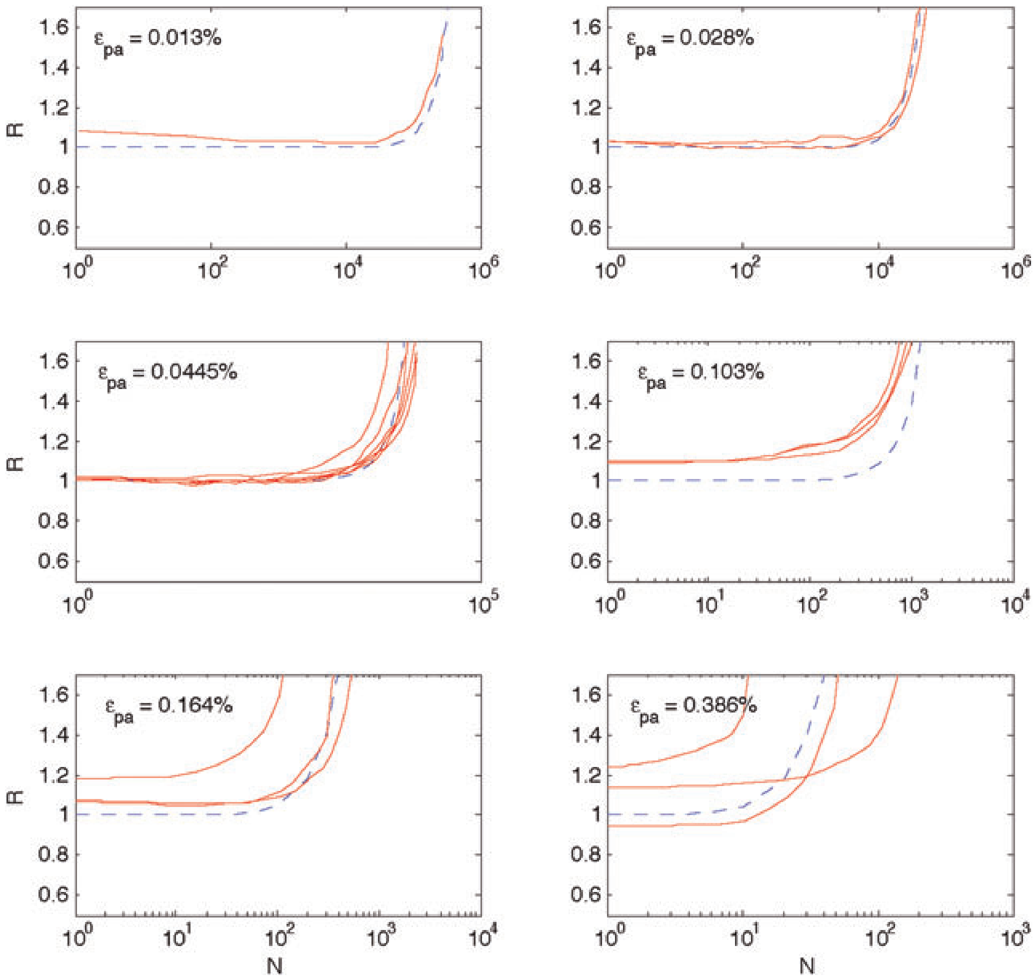

Fig. 6 Damage accumulation curves for different strain amplitudes obtained from the quadratic approach using the plastic strain amplitudes and the Coffin–Manson fatigue model.

Table 2 Weibull model parameters for plastic strain amplitudes

β	δ	λ	B	C
14.5	934	128	−44.4	−22.8

The general form of the damage accumulation model will be that of Eq. (10), and for this particular case the variables involved in the problem are the number of cycles (N), the initial damage (R_0) and the total applied strain amplitude (ε_a). The damage accumulation model, therefore, has the form

$$R = h\,(N, \varepsilon_a, R_0) \tag{11}$$

where R is the damage level and h is the function that provides the level of damage.

To fit the parameters of the damage accumulation curves, the selection of a failure criterion and a fatigue lifetime model are required. We choose the failure criterion $R = 1.5$ proposed by Ingraham *et al.*[11] and employ both the Coffin–Manson and statistical Weibull model as fatigue lifetime models. From the Coffin–Manson approach, we obtain the following relation for N^*

$$N^* = \frac{1}{2}\left(\frac{\varepsilon_{pa}}{\varepsilon'_f}\right)^{1/c} \tag{12}$$

where ε_{pa} is the plastic strain amplitude, ε^i_f is the fatigue ductility coefficient, c is the fatigue ductility exponent and N^* is the number of cycles to failure. On the other hand, from the Weibull model we have Eq. (9) that provides the lifetime N^* as a function of the Weibull and the threshold parameters. It is important to notice that the Coffin–Manson approach is only defined for plastic strain amplitude, whereas the Weibull model is valid for total and plastic strain amplitudes.

In this study, we propose linear and quadratic models for damage accumulation that interface with the Ingraham *et al.*[11] failure criterion, the Coffin–Manson and the statistical Weibull models.

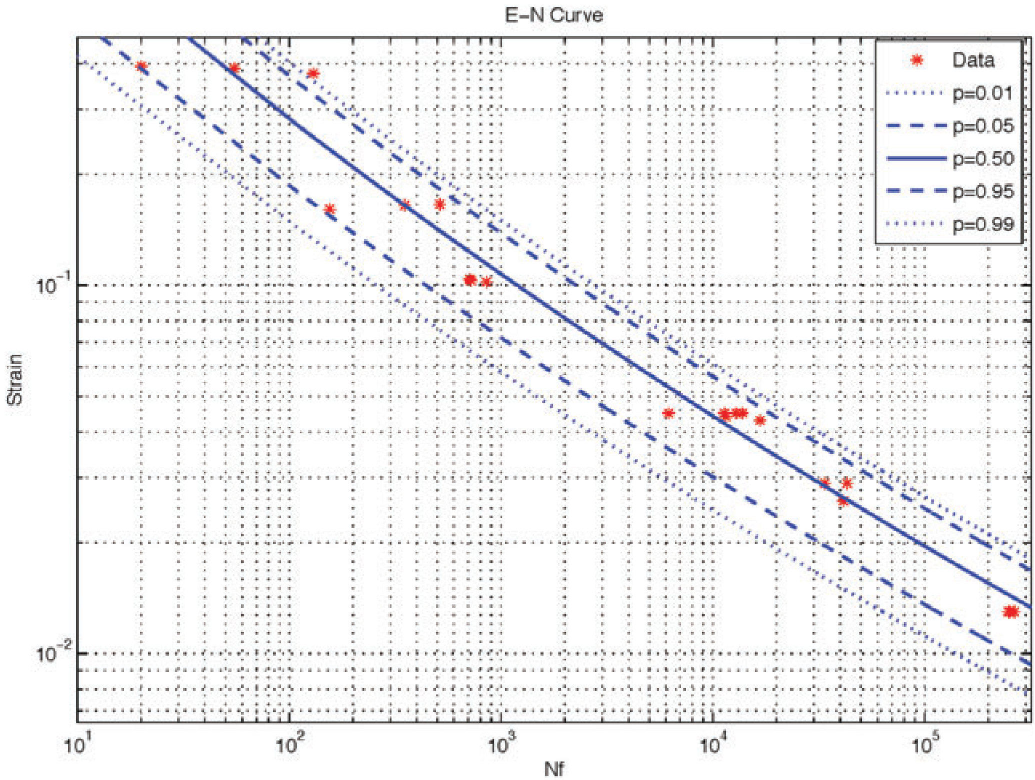

Fig. 7 ε–N field curves of the results published by Ingraham *et al.*[11] using the plastic strain amplitudes.

Linear damage accumulation model

A linear damage accumulation model has the general form

$$R = AN + R_0 \tag{13}$$

where A is the model parameter, R_0 the initial damage state and N the number of cycles. To estimate the parameter A, we use the failure criterion proposed by Ingraham *et al.*[11] Rewriting Eq. (13) at the failure state, we obtain

$$R^* = AN^* + R_0 \tag{14}$$

where R^* is the threshold value considered for failure ($R^* = 1.5$), and N^* is the number of cycles to failure. Equation (14) can be rewritten as

$$A = \frac{R^* - R_0}{N^*}. \tag{15}$$

Substituting Eq. (15) in Eq. (13), the general form of the model is expressed in terms of the initial damage state R_0, the lifetime N^* and the failure threshold R^*, as

$$R = \frac{R^* - R_0}{N^*} N + R_0. \tag{16}$$

As we can see in Eq. (16), the general form of the model is fully dimensionless. Using the failure threshold value $R^* = 1.5$ and the strain-lifetime expressions for the Coffin–Manson (Eq. (12)) and statistical Weibull (Eq. (**9**)) models we obtain the general expressions for the damage accumulation model

$$R = \frac{1.5 - R_0}{\frac{1}{2}\left(\frac{\varepsilon_{pa}}{\varepsilon'_f}\right)^{1/c}} N + R_0 \tag{17}$$

the Coffin–Manson model, and

$$R = \frac{1.5 - R_0}{\exp\left[B + \frac{\lambda + \delta(-\log(1-p))^{1/\beta}}{\log \varepsilon_a - C}\right]} N + R_0 \tag{18}$$

for the Weibull model.

Quadratic damage accumulation model

We now introduce a quadratic version of the damage accumulation model and show that it can also fit the experimental data well. The quadratic model has the general form

$$R = A_1 N + A_2 N^2 + R_0 \tag{19}$$

where A_1 and A_2 are the model parameters, R is the damage level, N is the number of cycles and R_0 is the initial damage level. Numerical experiments in curve fitting of the quadratic form to the experimental data showed that A_1 could be discarded because it is very small in comparison with the value of A_2, so that the damage is primarily determined by the quadratic term. The general formula

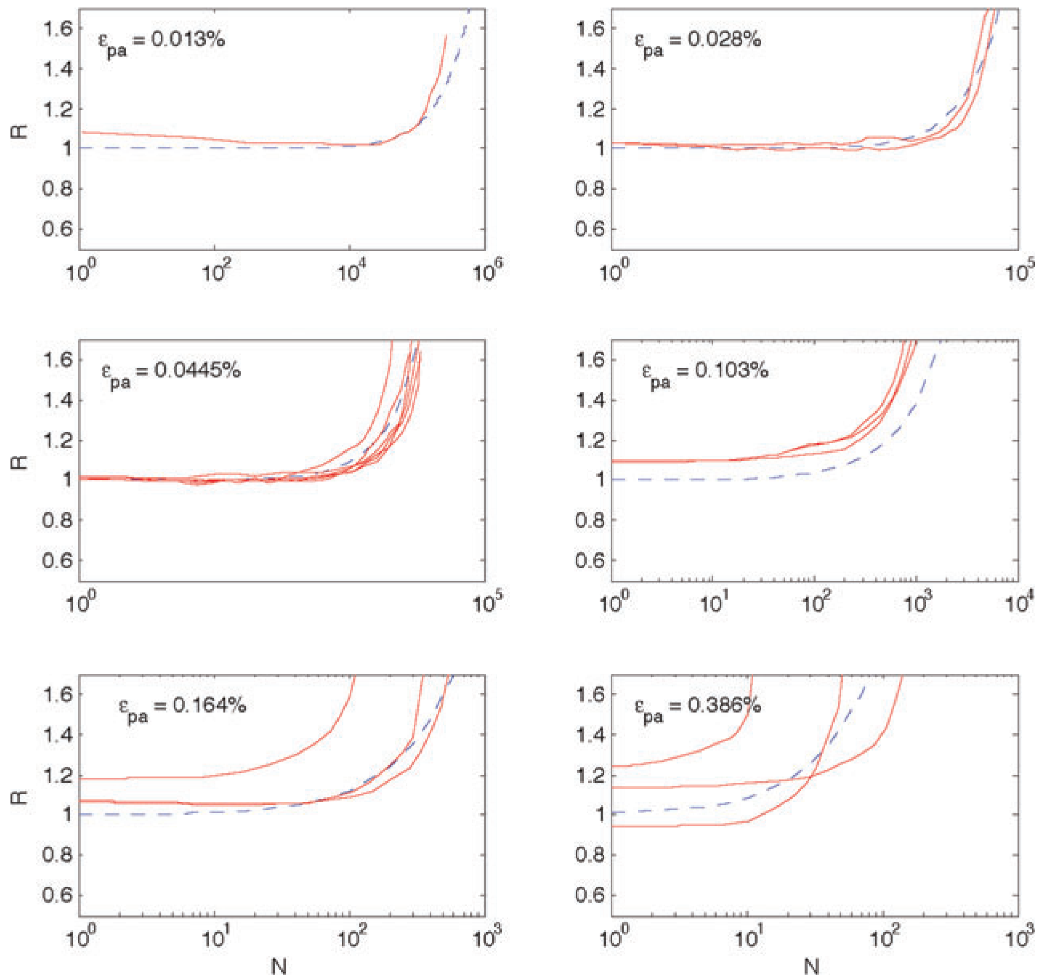

Fig. 8 Damage accumulation curves for different strain amplitudes obtained from the linear approach using the plastic strain amplitudes and the Weibull model.

becomes, after dropping the linear term

$$R = A_2 N^2 + R_0. \tag{20}$$

The A_2 parameter is estimated as follows. For the failure condition, Eq. (20) becomes

$$R^* = A_2 N^{*^2} + R_0 \tag{21}$$

where $R^* = 1.5$ corresponds to the failure criterion defined by Ingraham *et al.*,[11] and N^* is the lifetime. After a simple mathematical manipulation, the general form of the approach can be expressed by

$$R = \frac{R^* - R_0}{N^{*^2}} N^2 + R_0 \tag{22}$$

which is fully dimensionless. Replacing R^* by its value from the failure criteria ($R^* = 1.5$), and N^* by its expression in the Coffin–Manson and statistical Weibull model (Eqs(12) and (9)) we get the general expressions for the quadratic damage accumulation model

$$R = \frac{1.5 - R_0}{\left(\frac{1}{2}\left(\frac{\varepsilon_{pa}}{\varepsilon'_f}\right)^{1/c}\right)_2} N^2 + R_0 \tag{23}$$

for the Coffin–Manson approach, and

$$R = \frac{1.5 - R_0}{\left(\exp\left[B + \frac{\lambda + \delta\left(-\log(1-p)\right)^{1/\beta}}{\log \varepsilon_a - C}\right]\right)^2} N^2 + R_0 \tag{24}$$

for the Weibull model.

EXAMPLE APPLICATION

A practical application of the models, using the data published by Ingraham *et al.*[11] (see Table 1) is now presented. The procedure is divided in two steps; in the first, the fatigue analysis of the available data is performed, then,

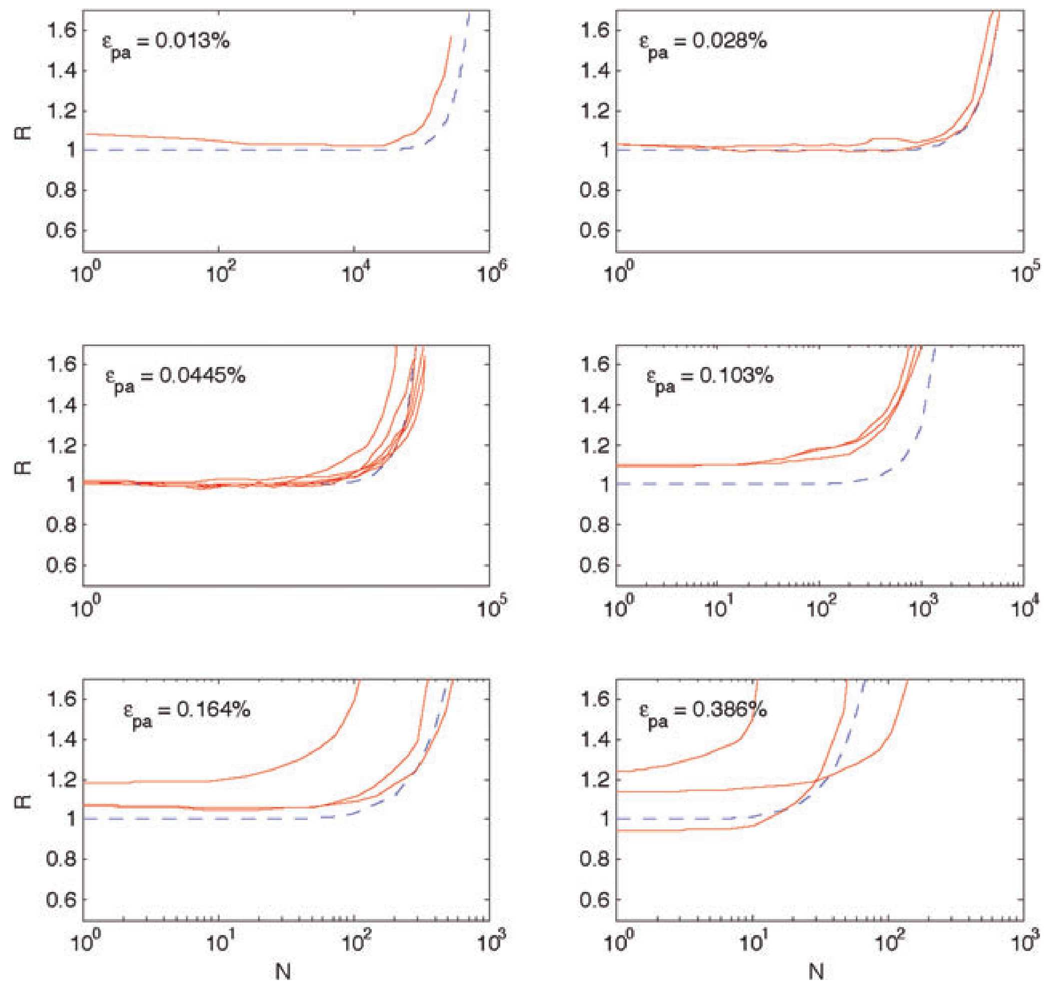

Fig. 9 Damage accumulation curves for different strain amplitudes obtained from the quadratic approach using the plastic strain amplitudes and the Weibull model.

Table 3 Weibull model parameters for total strain amplitudes

β	δ	λ	B	C
3.2997	8.9378	104.797	−12.9515	−7.4066

using the model proposed and the parameters obtained through the fatigue analysis, the damage accumulation curves are obtained.

We perform the modelling using both the plastic and total strain amplitudes as measures of the applied load to demonstrate the flexibility of the approach using the statistical Weibull model. Because of the requirements of the Coffin–Manson model, it is demonstrated only for the plastic strain amplitude.

The results of the various combinations of damage accumulation model (linear, quadratic), strain measure (plastic, total) and fatigue model (Coffin–Manson, Weibull) are presented in Figs 5, 6, 8, 9, 11 and 12, where the results of the modelling example are shown with dashed lines to be compared to the experimental results[11] shown as solid lines. Each panel in the figures represents a set of tests performed by Ingraham *et al*. at fixed applied total strain amplitude, as indicated in the figure.

Plastic strain amplitude

Application of the Coffin–Manson model for fatigue analysis behaviour

Figure 4 presents the results of the Coffin–Manson approach to modelling the relationship between the plastic strain amplitude and the fatigue lifetime. From this analysis we obtain the parameters of the model $\varepsilon_f^i = 0.0189$ and $c = -0.3761$.

Once we have estimates of the parameters, using Eqs (17) and (23) we can estimate the parameters of the linear and quadratic damage accumulation curves. The

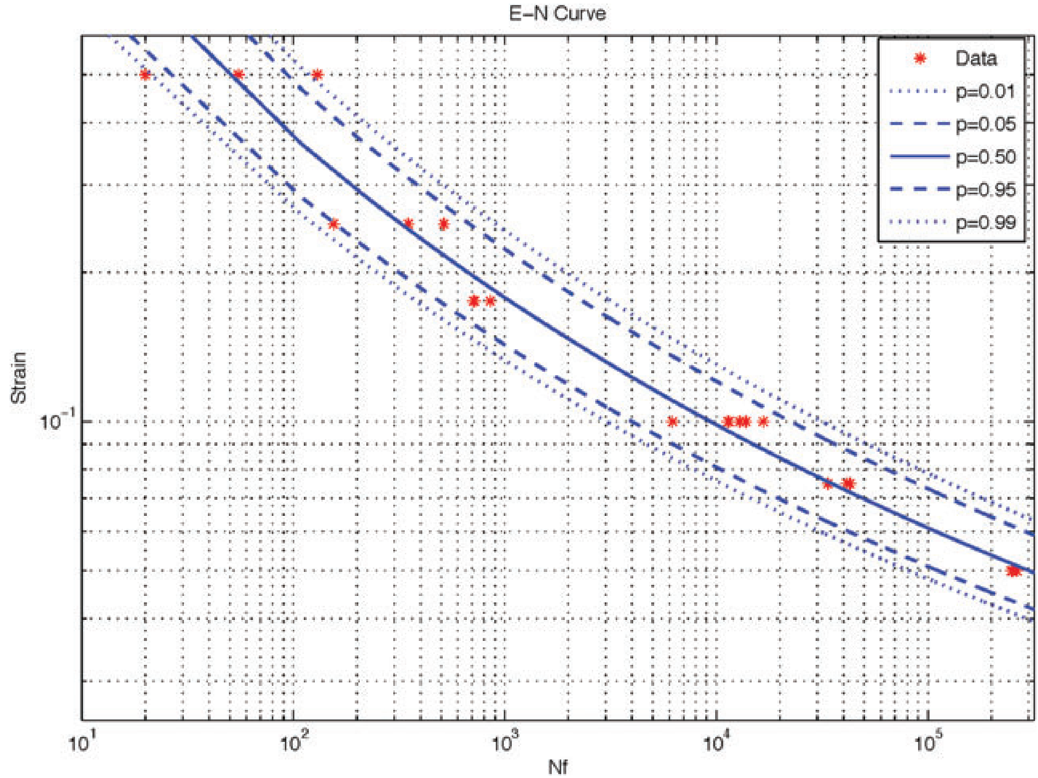

Fig. 10 $\varepsilon-N$ field curves of the results published by Ingraham *et al.*[11] using the total strain amplitudes.

curves are presented in the Figs 5 and 6 for the linear and the quadratic models respectively.

Application of the Weibull model for fatigue analysis behaviour

Using the published plastic strain amplitudes[11], the Weibull model parameters and the $\varepsilon-N$ field curve are presented in Table 2 and Fig. 7 respectively.

Using Eqs (18) and (24) with the values of the parameters shown in Table 2, we obtain the corresponding damage accumulation curves. The results are shown in Figs 8 and 9 for linear and quadratic approaches respectively.

Total strain amplitude

Using the model described in section 2 we are able to obtain the parameters of the Weibull model (Table 3), and the $\varepsilon - N$ curves that result from characterizing the applied deformation by the total, rather than the plastic, strain amplitude (Fig. 10).

Using the Eqs (18) and (24) for linear and quadratic models, we obtain the damage accumulation curves. Results are presented in Figs 11 and 12 for the linear and quadratic model respectively.

DISCUSSION

One choice that must be made in treating the problem of low-cycle fatigue in ductile materials is whether to consider the total or plastic strain amplitude as the controlling load parameter. Initially an analysis considering the plastic strain amplitude was performed using two different models to identify the fatigue behaviour; the Coffin–Manson and the statistical Weibull model. The results of these two analyses are presented in section 4.1, specifically in Figs 5 and 6, for the linear and quadratic damage accumulation models integrated with the Coffin–Manson model, and in Figs 8 and 9 for linear and quadratic damage accumulation models integrated with the Weibull model.

Upon first observation, the linear and quadratic approaches provide practically the same results, and agree well with the experimental results. However, we detect one important difference between both approaches. When the damage accumulation curve is approaching the failure threshold ($R = 1.5$), the linear model presents a lower slope than the experimental results. On the other hand, the damage accumulation curve obtained using the quadratic model has practically the same slope as the experimental results. This behaviour occurs for both models considered for fatigue analysis, and allows us to state that the quadratic approach provides better results since

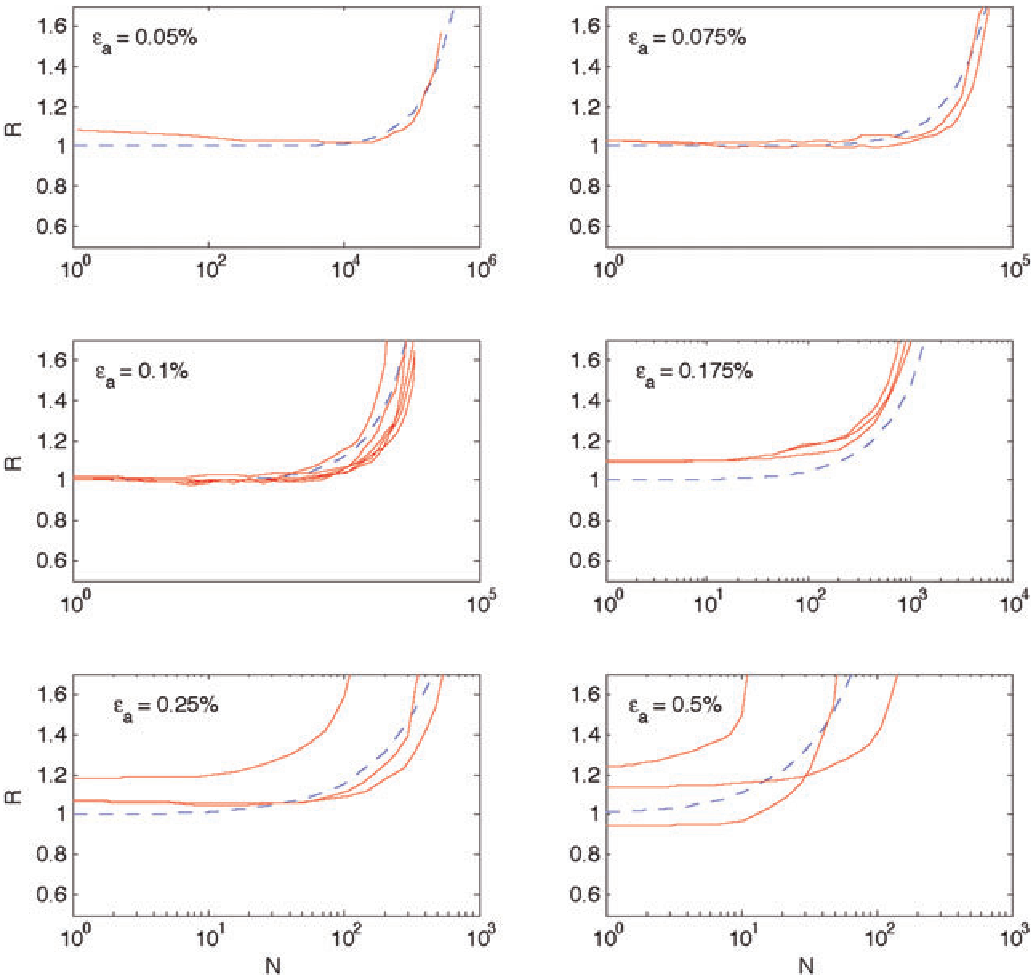

Fig. 11 Damage accumulation curves for different strain amplitudes obtained from the linear approach using the total strain amplitudes and the Weibull model.

it more accurately reproduces the rapid acceleration of damage accumulation that occurs as failure becomes incipient. Finally, we can see that there is an uncertainty related to the initial value of the damage parameter R which is scattered around $R = 1$. In our calculation, we have used $R_0 = 1$ exclusively, following the approach of Ingraham *et al*.[11] A comparison between the results obtained using the Coffin–Manson or statistical Weibull model shows that practically the same results were obtained for both models.

Since there appears to be essentially no difference between the results obtained using the Coffin–Manson and statistical Weibull models for plastic strain amplitude, we conduct the same procedure using the total strain amplitude as the measure for the applied load (for the Weibull model only). These results are presented in Figs 11 and 12. For these results we make the same observation as for the plastic strain amplitude models, namely that for the linear damage accumulation model, the slope of the damage accumulation curve obtained is lower than the experimental results in the neighbourhood of the failure threshold, whereas in the case of the quadratic model, the slopes of the experimental and the proposed model are practically the same. Also, the same uncertainty about the initial damage value R_0 is observed.

To determine whether it is better to use the total or the plastic strain amplitude, a comparison of the results obtained has been made. Through this analysis we determine that the choice of plastic or total strain amplitude as the load measure results in very small differences in the model quality. However the total strain amplitude results do appear slightly better, especially when we analyze in more detail the four first plots of each case (those corresponding to lower strain amplitudes). We observe that the total strain amplitude provides closer results to the experimental data than the results obtained using the

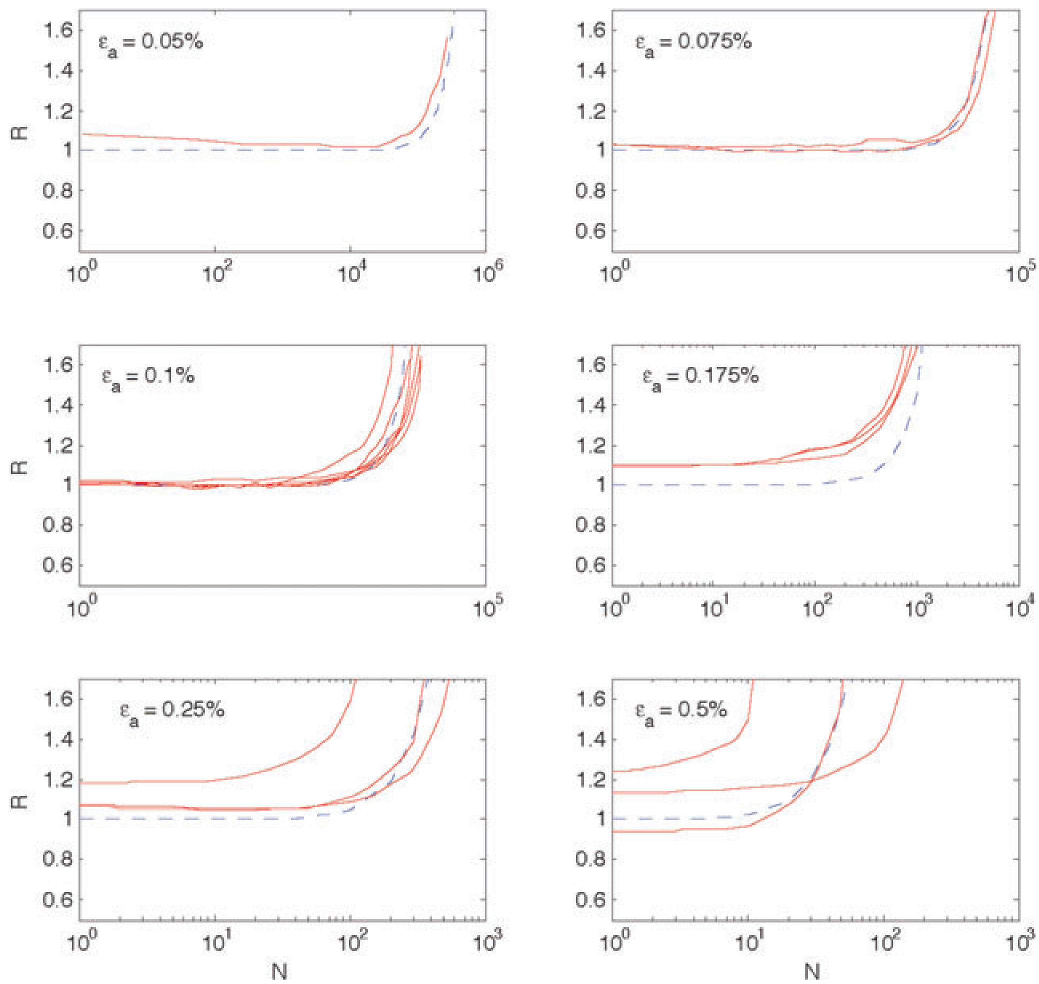

Fig. 12 Damage accumulation curves for different strain amplitudes obtained from the quadratic approach using the total strain amplitudes and the Weibull model.

plastic component of the applied strain. This result has the advantage of recommending the use of total strain amplitude in characterizing cyclic loading, which is advantageous since the total strain amplitude is easier to measure and control than plastic strain amplitude, which must be calculated after the test is completed (or at best after each cycle is completed).

Finally, we state an advantage of using the Weibull model instead of the traditional Coffin–Manson model, which is that use of the Weibull model allows the direct introduction of a probabilistic criterion for fatigue failure, and it can be used with the total rather than plastic strain amplitude avoiding the necessary calculations to obtain the plastic strain component. Also it is important to notice that if $\beta > 6$ the Weibull model becomes nearly equivalent to one in which the distribution of fatigue lifetimes follows a Gumbel distribution. An advantage to adopting a Gumbel distribution outright is that it results in a model with one less parameter than results from the Weibull model.

CONCLUSIONS

A statistical model for damage accumulation and fatigue lifetime in closed cell aluminium foam under fully reversed cyclic loading has been proposed. The procedure developed considers linear and quadratic damage accumulation models, Coffin–Manson and Weibull models for the fatigue life, and the total and plastic strain amplitudes as measures of the applied loading. The Weibull model developed by one of the authors in his previous work is found to be suitable for modelling the fatigue behaviour of metal foams for tension–compression loading cycles. Both linear and quadratic models for damage accumulation provide adequate predictions of the fatigue lifetime, but the quadratic model better approximates the rapid acceleration of damage accumulation when the fatigue life is approached. We find that either the total or plastic applied strain amplitude can be used to define the fatigue loading, but slightly better results are obtained when the total strain amplitude is used. This is advantageous because the total strain amplitude is easier to monitor during testing.

The importance of the procedure proposed is that it allows us to model statistically the fatigue behaviour of metal foams for the case of fully reversed cycles. In addition to this, a new procedure has been developed through which we are able to obtain the damage accumulation curve of aluminium-closed cell foams subjected to fully reversed cycles. In this approach the damage accumulation model and the fatigue lifetime models are completely and mathematically integrated. Finally, this new procedure allows us to use the total or the plastic strain amplitude as the controlling load parameter.

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REFERENCES

1 Ashby, M. F., Evans, A., Fleck, N. A., Gibson, L. J., Hutchinson, J. W. and Wadley, H. N. G. (2000) *Metal Foams: A Design Guide*. Butterworth-Heinemann, Woburn.
2 Gibson, L. J. and Ashby, M. (1997) *Cellular Solids: Structures and Properties*, 2nd Edition. Cambridge University Press, Cambridge, U.K/New York, U.S.A./Oakleig, Australia.
3 McCullogh, K. Y. G., Fleck, N. A. and Ashby M. F. (1999) The stress-life fatigue behaviour of aluminium alloy foams. *Fatigue Fract. Eng. Mater. Struct.* **23**, 199–208.
4 Sugimura, Y., Rabiei, A., Evans, A..G., Harte A. M., Fleck, N. A.,(1999) Compression fatigue of a cellular al alloy. *Mater. Sci. Eng. A.* **269**, 38–48.
5 Coffin, L. (1954) A study of the effect of cyclic thermal stresses on a ductile materials. *Trans. ASME.* **76**, 931–950.
6 Manson, S. (1965) Fatigue: A complex subject-some simple approximation. *Exp. Mech.* **5**, 193–226.
7 Castillo, E., Fernández-Canteli, A., Hadi, A. S. and López-Aenlle, M. (2006) A fatigue model with local sensitivity analysis. *Fatigue Fract. Eng. Mater. Struct*. **30**, 149–168.
8 Castillo, E., Fernández-Canteli, A., Pinto, H. and López-Aenlle, M. (2008) A general regression model for statistical analysis of strain life data. *Mater. Lett.* **62**, 3639–3642.
9 Castillo, E. and Fernández-Canteli, A. (2001) A general regression model for lifetime evaluation and prediction. *Int. J. Fract.* **107**, 117–137.
10 Pinto, H. (2009) Models for crack propagation and fatigue analysis based on strain and stress life data. Ph.D. thesis University of Cantabria, Santander, Spain.
11 Ingraham, M. D., DeMaria, C. J., Issen, K. A. and Morrison, D.J.L. (2009) Low cycle fatigue of aluminum foam. *Mater. Sci. Eng. A.* **504**, 150–156.
12 Basquin, O. (1910) The exponential law of endurance tests. *Am. Soc. Test Mater. Proc. ASTM* **10**, 625–630.
13 Morrow, J. D. (1965) Cyclic plastic strain energy and fatigue of metals. *Internal friction, damping and cyclic plasticity. ASTM* 45–86.
14 Galambos, J. (1987) *The Asymptotic Theory of Extreme Order Statistics*. (Edited by E. Robert), Krieger, Malabar, FL.
15 Castillo, E. and Galambos, J. (1987) Lifetime regression models based on a functional equation of physical nature. *J. Appl. Prob.* **24**, 160–169.