ECE 665: Algorithms

Spring 2007

Homework 4 Supplement

5 You were asked to prove two things in class (Mar 6). First, prove the following claim by induction:

Lemma: For $a \geq 1$, $b > 1$, if $T(1) = f(1)$, $T(n) \leq a \cdot T(n/b) + f(n)$, then for $n = b^k$, we have that

$$T(n) \leq \sum_{i=0}^{k} a^i \cdot f\left(\frac{n}{b^i}\right).$$

Then, you were asked to prove that for the last part of the Master Theorem, it was sufficient to assume that $f(n) = \Omega(n^{\log_b a + \varepsilon})$ for some $\varepsilon > 0$. In fact, this is **not** true, for example, we can take $f(n) = n(2 - \cos 2\pi n)$ as a counterexample. (Thanks to Bo Jiang for pointing this out). What I had in mind was the simpler exercise of showing:

Lemma: If $f(n) = \alpha \cdot n^{\log_b a + \varepsilon}$ for some $\alpha < 1$, $a \geq 1$, $b > 1$, $\varepsilon > 0$, then $a \cdot f(n/b) \leq c \cdot f(n)$.

You may also show alternatively that:

Lemma: If for $a \geq 1$, $b > 1$, $a \cdot f(n/b) \leq c \cdot f(n)$ for some $c < 1$, then $f(n) = \Omega(n^{\log_b a + \varepsilon})$.